

# DEM simulation of ballast oedometric test

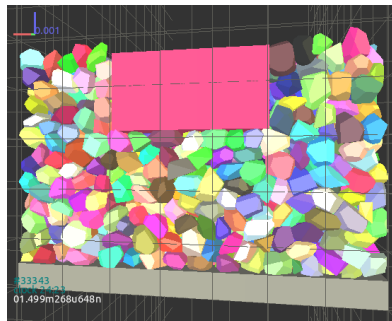
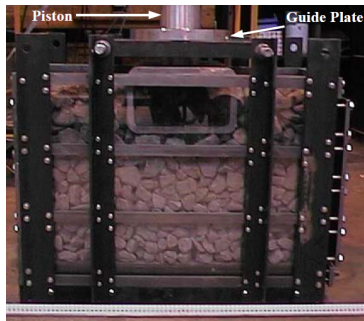
Jan Eliáš

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# Motivation

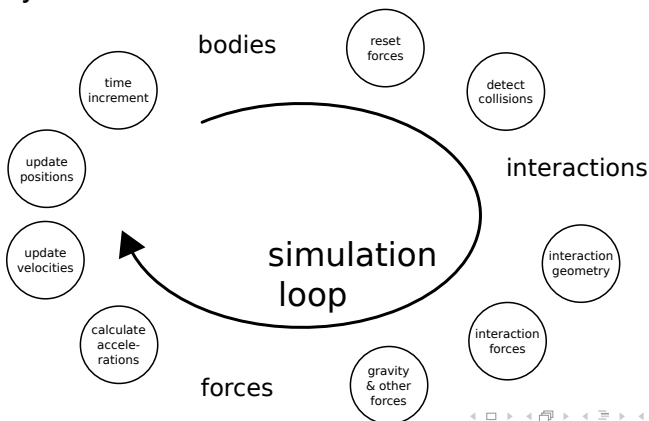
- simulate behavior of railway ballast
- simulate ballast-sleeper interaction
- use realistically shaped elements
- include crushing of particles



W. L. Lim, 2005, University of Nottingham

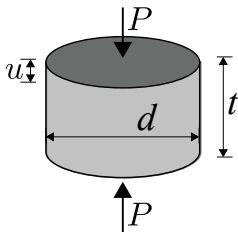
# Discrete Element Method

- originated in 1979 by Cundall & Strack
- bodies do not deform!
- approximation of interaction
- explicit dynamics



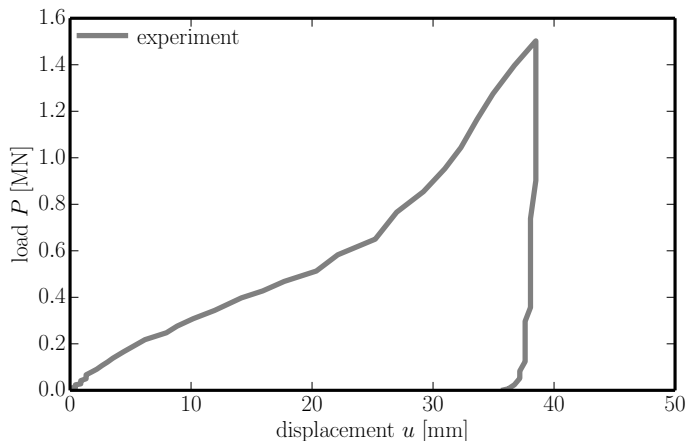
# Experiment: large oedometric test

- performed by Lim & McDowell  
W. L. Lim & G. R. McDowell, 2005, Granular Matter.
- cylinder of diameter  $d=300$  mm and depth  $t=150$  mm
- compressed by force 1.5 MN (mean stress = 21.2 MPa) and then unloaded



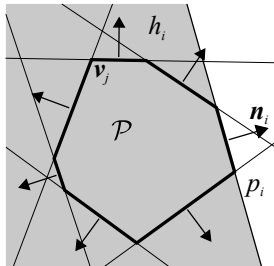
# Experimentally recorded data

- initially vibrated on vibration table
- extensive crushing occurred approx. from mean stress 1 MPa



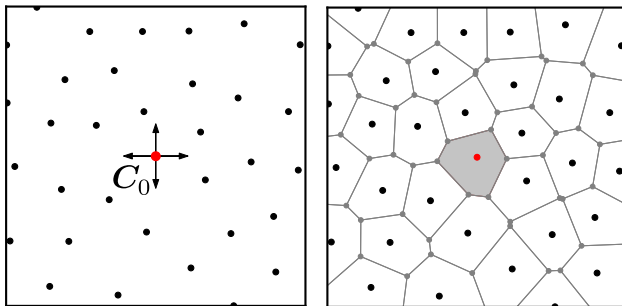
# Polyhedron

- convex polyhedron is intersection of half-spaces:  $\mathcal{P} = \cap_{i=1}^n h_i$
- half-space  $h_i$  is define by bounding plane  $p_i$ :  
$$p_i \equiv a_i x + b_i y + c_i z + d_i = 0$$
- $h_i$  is set of all points at the negative side of bounding plane:  
$$h_i = \{(x, y, z), \text{ where } a_i x + b_i y + c_i z + d_i \leq 0\}$$
- we use CGAL library to manipulate polyhedrons, compute convex-ghulls, etc.



# Randomly-shaped polyhedral ballast grains

- initial central nucleus  $C_0$  is placed at the origin
- nuclei are placed sequentially with random coordinates into the domain of size  $5 \times 5 \times 5$  units
- minimal mutual distance  $l_{\min}$  is restricted
- Voronoi tessellation is performed and cell associated with the central nucleus is taken

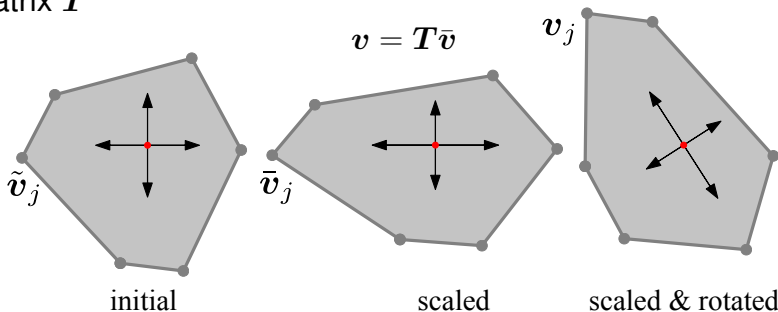


# Randomly-shaped polyhedral ballast grains

- cell is scaled by scaling factor  $s = (s_x, s_y, s_z)$

$$\bar{\mathbf{v}} = \mathbf{S}\tilde{\mathbf{v}}$$

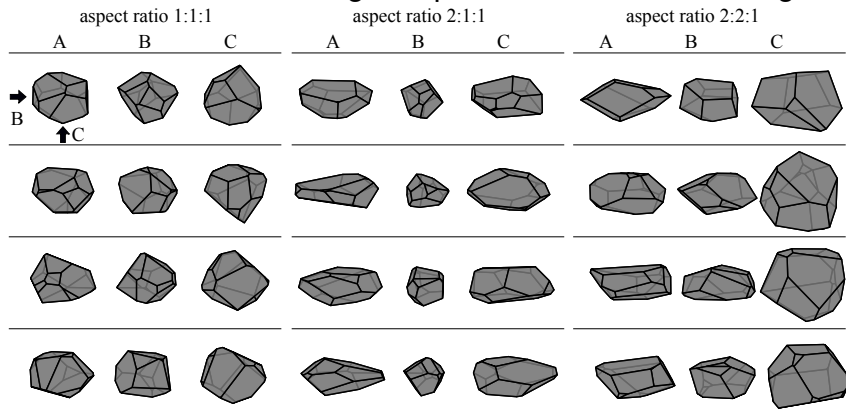
- finally, random orientation is applied using random rotation matrix  $T$





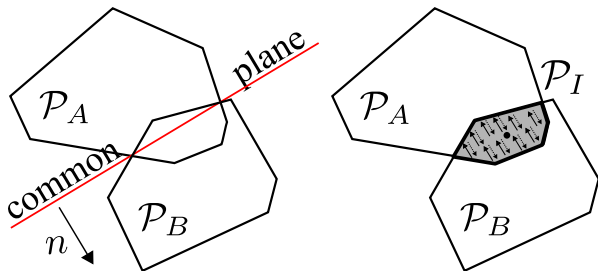
# Controlling polyhedral shape

- ratio between scaling components  $s_x : s_y : s_z$  controls grains shape
- absolute values of scaling components controls size of grains



# Contact between polyhedrons

- only repulsive force which occurs when polyhedrons intersect
- magnitude of force related to intersecting volume
- widely used **Common Plane Method** reduces polyhedron–polyhedron contact to two polyhedron–plane contacts
- we attempted to use “exact” intersecting volume and repulsive force linearly dependent on it



# Necessary algorithms

## Contact detection

fast identification if two polyhedrons overlap: is  $\mathcal{P}_A \cap \mathcal{P}_B$  empty?

## Magnitude of normal force

linearly dependent of intersecting volume:  $F_n = k_n V_I$ , where  $V_I$  is volume of  $\mathcal{P}_I = \mathcal{P}_A \cap \mathcal{P}_B$ . How to compute  $\mathcal{P}_I$ ?

## Magnitude of shear force

standard incremental algorithm + Coulomb friction

## Normal direction and point of action

normal direction determined by least-square fitting of polyhedron shells intersection, point of action assumed in centroid of  $\mathcal{P}_I$

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# Test of overlapping

- search in set of possible candidates for separation plane; if none found, overlapping confirmed  
test of separation plane  $s(a_s, b_s, c_s, d_s)$ :

$$\forall \mathbf{x}_A \in \mathcal{P}_A \quad : \quad a_s x_A + b_s y_A + c_s z_A \leq d_s$$

$$\forall \mathbf{x}_B \in \mathcal{P}_B \quad : \quad a_s x_B + b_s y_B + c_s z_B \geq d_s$$

- time can be saved by
  - saving and initial testing of separation plane from last step in case of no overlapping
  - saving of centroid  $x_c$  of intersecting polyhedron from last step and testing that it is inside both polyhedrons

$$\forall p_i(a_i, b_i, c_i, d_i) \in \mathcal{P}_A \text{ and } \mathcal{P}_B : a_i x_c + b_i y_c + c_i z_c \leq d_i$$

# Calculation of intersecting polyhedron

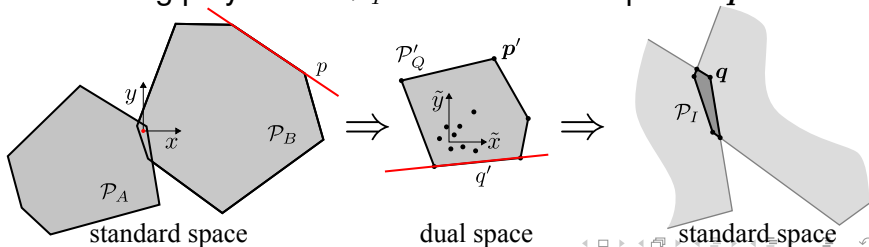
- through mapping between standard and dual space
- polyhedron planes  $p_i$  are projected to points  $p'$  in dual space

$$p \in \mathcal{P}_A \text{ and } \mathcal{P}_B \rightarrow \mathbf{p}' = (a/d, b/d, c/d)$$

- convex hull  $\mathcal{P}'_Q$  of dualized planes  $\mathbf{p}'$  is found
- bounding planes  $q'$  of  $\mathcal{P}'_Q$  are again dualized to standard space

$$q' \in \mathcal{P}'_Q \rightarrow \mathbf{q} = (a'/d', b'/d', c'/d')$$

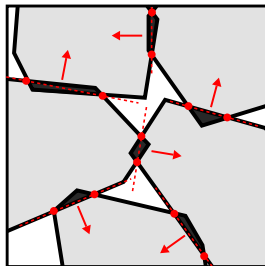
- intersecting polyhedron  $\mathcal{P}_I$  is convex hull of points  $\mathbf{q}$





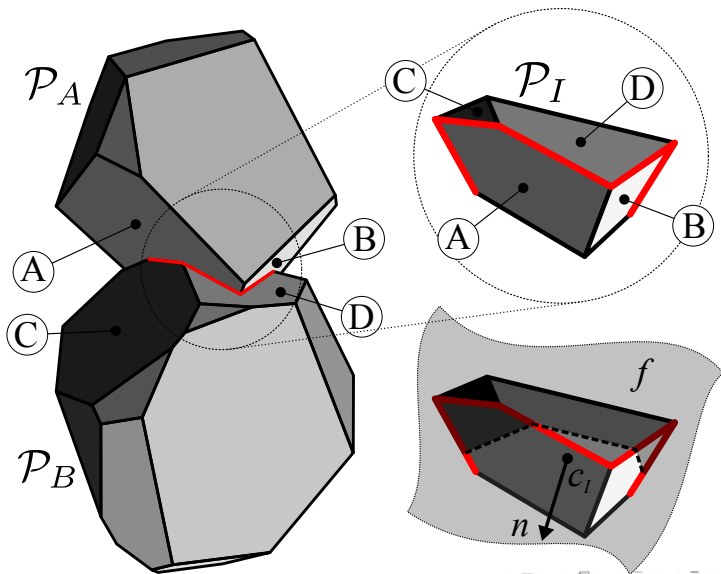
# Estimation of normal direction

- analogy to 2D case where one can connect shell intersections by line and assume normal direction perpendicular



- shell intersection in 3D is a continuous enclosed non-planar piece-wise linear line
- we adopt least square linear fitting of it by plane
- normal direction is assumed perpendicularly to this fitted plane

## 3D sketch



# Model parameters and compaction

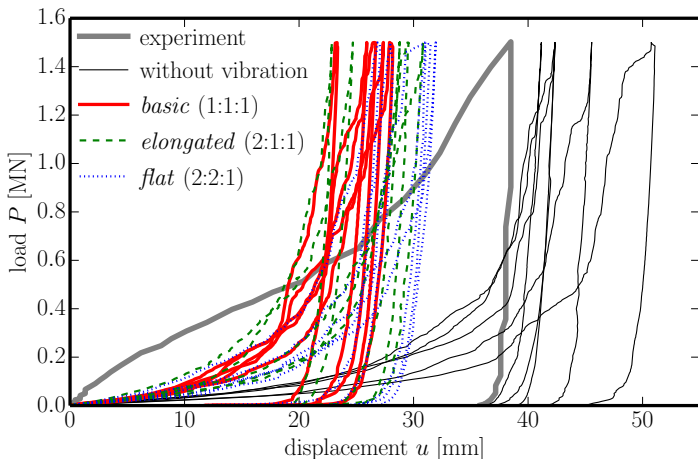
- chosen parameters of the materials

			ballast	steel
density	$\rho$	kg/m <sup>3</sup>	2600	7850
normal volumetric stiffness	$k_n$	N/m <sup>3</sup>	$2 \times 10^{13}$	$2 \times 10^{14}$
shear stiffness	$k_s$	N/m	$2 \times 10^8$	$2 \times 10^9$
angle of internal friction	$\phi$	-	0.6	0.4

- polyhedrons are generated sequentially with no overlapping in larger volume and left to freely fall into steel cylinder
- high level of compaction is hard to achieve in simulation
- following items should help
  - increased gravity  $5\times$
  - decreased angle of internal friction to 0.1
  - vibrating by adding changing horizontal acceleration

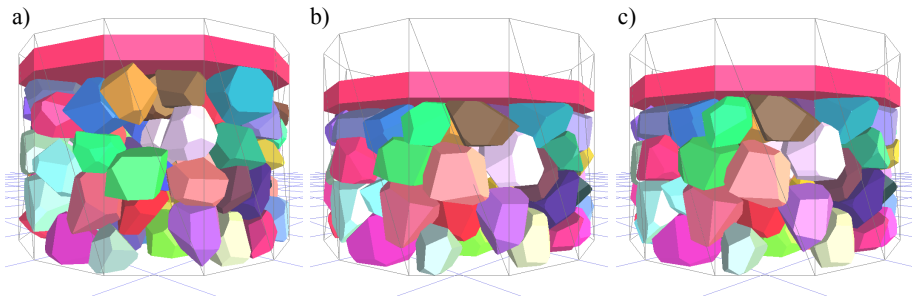
# Comparison to experiment

- no crushing present in the model, loading curvature mostly caused by low compaction level



# Views at different stages of simulation

- initial stage, after free fall and vibration (a)
- at the maximum load (b)
- after complete unloading (c)



# Crushing in numerical model

- crushing of grains needs to be included
- incorporated via splitting of polyhedron pieces by plane running through centroid
- criterion based on principal stresses inside the grain:

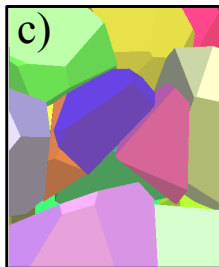
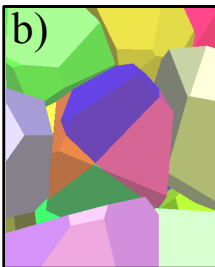
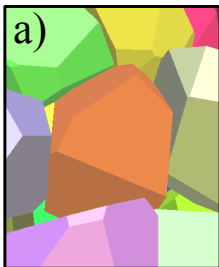
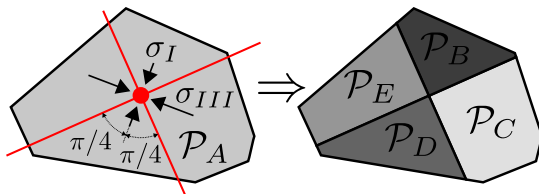
$$\sigma_{ij} = \sum_{k=1}^N F_i r_j \Rightarrow \sigma_I > \sigma_{II} > \sigma_{III}$$

- splitting stress  $\sigma_s = -\sigma_{III} + \sigma_I$
- size dependent strength  $f_t = f_{t0}/r_{eq}$ , where  $r_{eq}$  is equivalent radius (S. Lobo-Guerrero, L. E. Vallejo, 2006)

$$r_{eq} = \sqrt[3]{\frac{3V}{4\pi}}$$

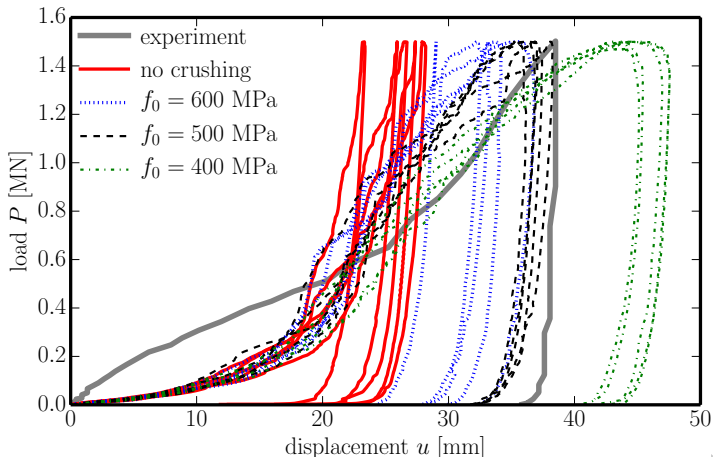
# Crushing in numerical model

- splitting into four pieces by planes parallel to plane  $\sigma_{II}$  and under angle  $\pi/4$  from  $\sigma_I$  and  $\sigma_{III}$  directions



# Comparison to experiment

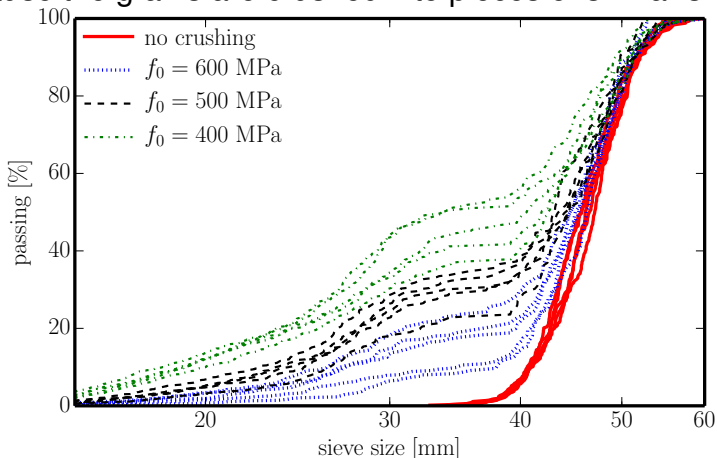
- three material strengths tested: 400, 500 and 600 MPa
- pieces with volumes lower than  $1 \text{ cm}^3$  were removed from the simulation





# Sieve curve after crushing

- includes small particles with volume under  $1 \text{ cm}^3$
- plateau between sieve sizes 30 and 40 mm is observed because the grains are crushed into pieces of similar sizes



# Conclusions

- simple method to generate convex randomly shaped grains
- possibility to control aspect ratio of grains
- repulsive force estimated from volume of intersecting polyhedron
- normal direction estimated from least-square fitting of shells intersection by plane
- model used to simulate large oedometric test performed on railway ballast
- crushing of grains can be simply done in geometrical sense, problem is to develop correct criterion
- many thanks to YADE and CGAL developers