

DEM simulation of ballast oedometric test

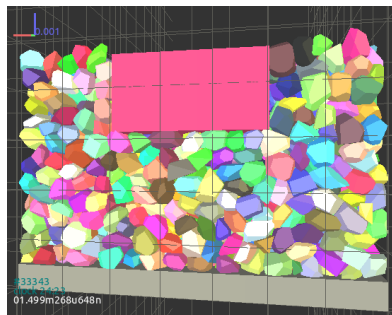
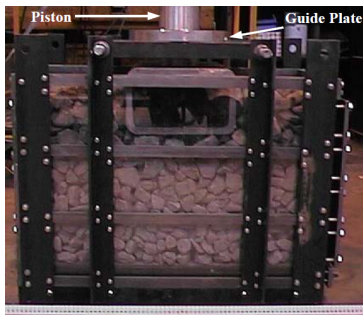
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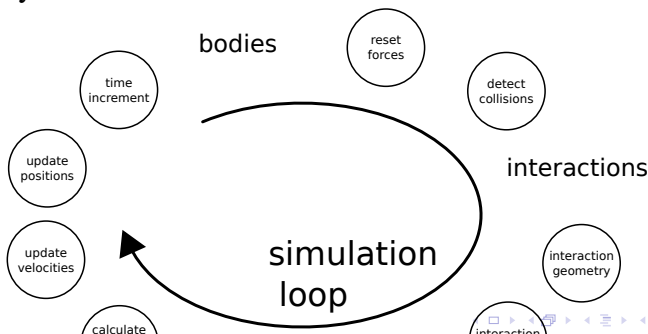
Motivation

- ▶ simulate behavior of railway ballast
- ▶ simulate ballast-sleeper interaction
- ▶ use realistically shaped elements
- ▶ include crushing of particles



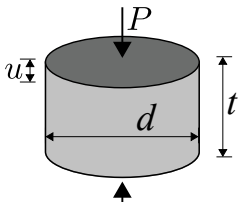
Discrete Element Method

- ▶ originated in 1979 by Cundall & Strack
- ▶ bodies do not deform!
- ▶ approximation of interaction
- ▶ explicit dynamics



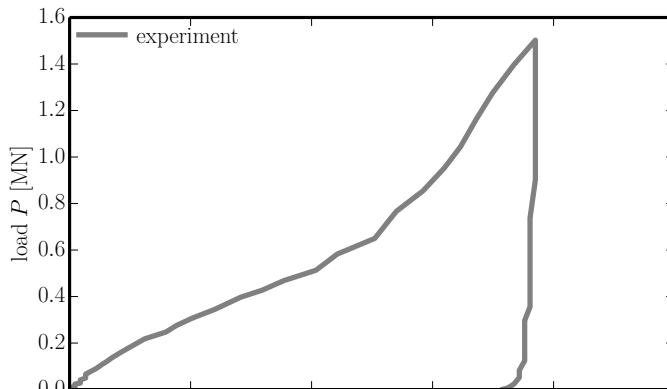
Experiment: large oedometric test

- ▶ performed by Lim & McDowell
- ▶ cylinder of diameter $d=300$ mm and depth $t=150$ mm
- ▶ compressed by force 1.5 MN (mean stress = 21.2 MPa) and then unloaded



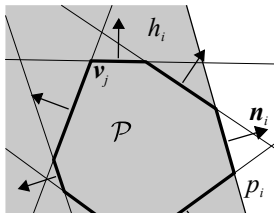
Experimentally recorded data

- ▶ initially vibrated on vibration table
- ▶ extensive crushing occurred approx. from mean stress 1 MPa



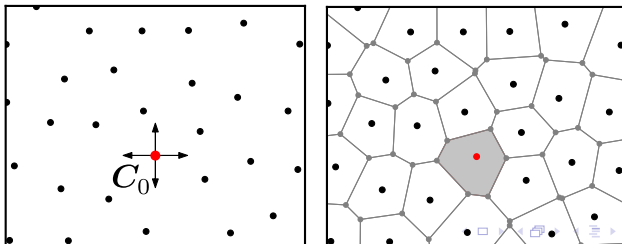
Polyhedron

- ▶ convex polyhedron is intersection of half-spaces: $\mathcal{P} = \cap_{i=1}^n h_i$
- ▶ half-space h_i is define by bounding plane p_i :
$$p_i \equiv a_i x + b_i y + c_i z + d_i = 0$$
- ▶ h_i is set of all points at the negative side of bounding plane:
$$h_i = \{(x, y, z), \text{ where } a_i x + b_i y + c_i z + d_i \leq 0\}$$
- ▶ we use CGAL library to manipulate polyhedrons, compute convex-g hulls, etc.



Randomly-shaped polyhedral ballast grains

- ▶ initial central nucleus C_0 is placed at the origin
- ▶ nuclei are placed sequentially with random coordinates into the domain of size $5 \times 5 \times 5$ units
- ▶ minimal mutual distance l_{\min} is restricted
- ▶ Voronoi tessellation is performed and cell associated with the central nucleus is taken

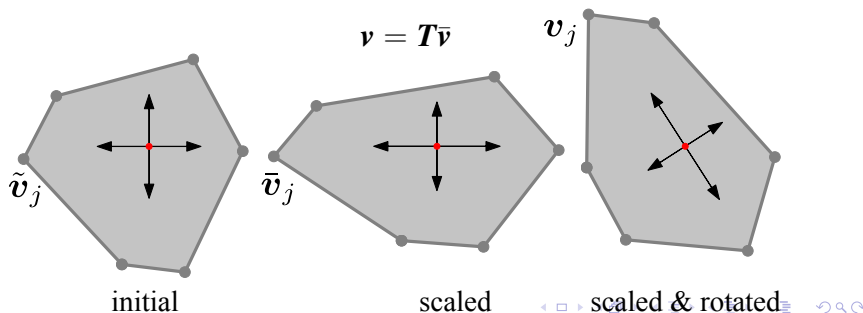


Randomly-shaped polyhedral ballast grains

- cell is scaled by scaling factor $\mathbf{s} = (s_x, s_y, s_z)$

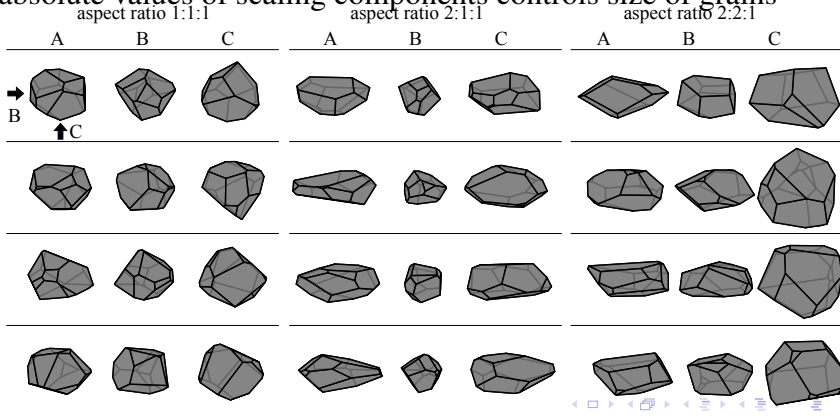
$$\bar{\mathbf{v}} = \mathbf{s}\tilde{\mathbf{v}}$$

- finally, random orientation is applied using random rotation matrix \mathbf{T}



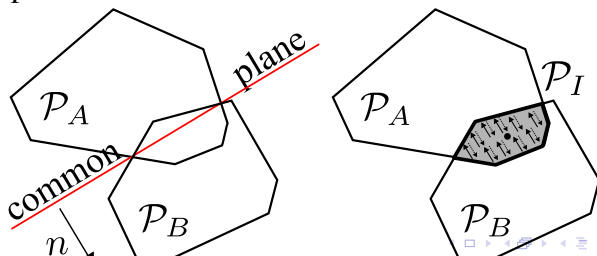
Controlling polyhedral shape

- ▶ ratio between scaling components $s_x : s_y : s_z$ controls grains shape
- ▶ absolute values of scaling components controls size of grains



Contact between polyhedrons

- ▶ only repulsive force which occurs when polyhedrons intersect
- ▶ magnitude of force related to intersecting volume
- ▶ widely used **Common Plane Method** reduces polyhedron–polyhedron contact to two polyhedron–plane contacts
- ▶ we attempted to use “exact” intersecting volume and repulsive force linearly dependent on it



Necessary algorithms

Contact detection

fast identification if two polyhedrons overlap: is $\mathcal{P}_A \cap \mathcal{P}_B$ empty?

Magnitude of normal force

linearly dependent of intersecting volume: $F_n = k_n V_I$, where V_I is volume of $\mathcal{P}_I = \mathcal{P}_A \cap \mathcal{P}_B$. How to compute \mathcal{P}_I ?

Magnitude of shear force

standard incremental algorithm + Coulomb friction

Normal direction and point of action

normal direction determined by least-square fitting of polyhedron shells intersection, point of action assumed in centroid of \mathcal{P}_I

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Test of overlapping

- search in set of possible candidates for separation plane; if none found, overlapping confirmed
- test of separation plane $s(a_s, b_s, c_s, d_s)$:

$$\forall \mathbf{x}_A \in \mathcal{P}_A : a_s x_A + b_s y_A + c_s z_A \leq d_s$$

$$\forall \mathbf{x}_B \in \mathcal{P}_B : a_s x_B + b_s y_B + c_s z_B \geq d_s$$

- time can be saved by
 - saving and initial testing of separation plane from last step in case of no overlapping
 - saving of centroid \mathbf{x}_c of intersecting polyhedron from last step and testing that it is inside both polyhedrons

$$\forall p_i(a_i, b_i, c_i, d_i) \in \mathcal{P}_A \text{ and } \mathcal{P}_B : a_i x_c + b_i y_c + c_i z_c \leq d_i$$

Calculation of intersecting polyhedron

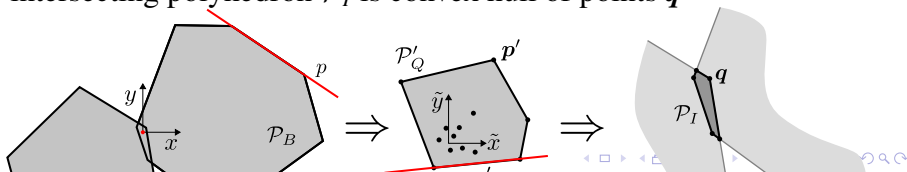
- ▶ through mapping between standard and dual space
- ▶ polyhedron planes p_i are projected to points \mathbf{p}' in dual space

$$p \in \mathcal{P}_A \text{ and } \mathcal{P}_B \rightarrow \mathbf{p}' = (a/d, b/d, c/d)$$

- ▶ convex hull \mathcal{P}'_Q of dualized planes \mathbf{p}' is found
- ▶ bounding planes q' of \mathcal{P}'_Q are again dualized to standard space

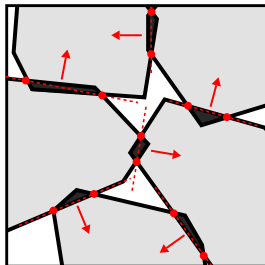
$$q' \in \mathcal{P}'_Q \rightarrow \mathbf{q} = (a'/d', b'/d', c'/d')$$

- ▶ intersecting polyhedron \mathcal{P}_I is convex hull of points \mathbf{q}



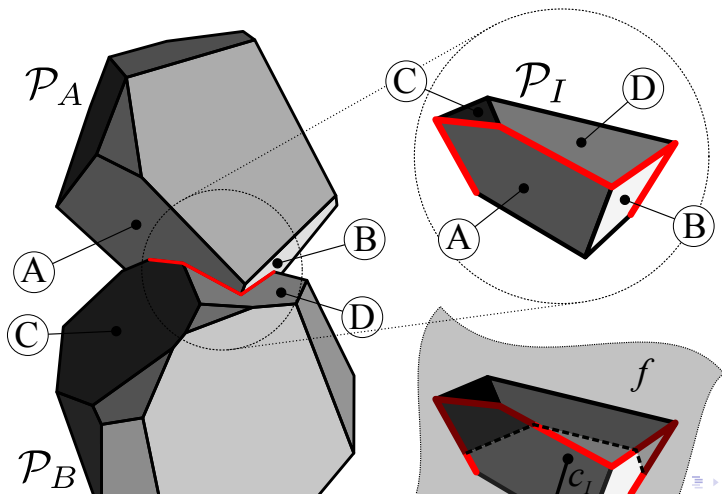
Estimation of normal direction

- ▶ analogy to 2D case where one can connect shell intersections by line and assume normal direction perpendicular



- ▶ shell intersection in 3D is a continuous enclosed non-planar piece-wise linear line
- ▶ we adopt least square linear fitting of it by plane

3D sketch



Model parameters and compaction

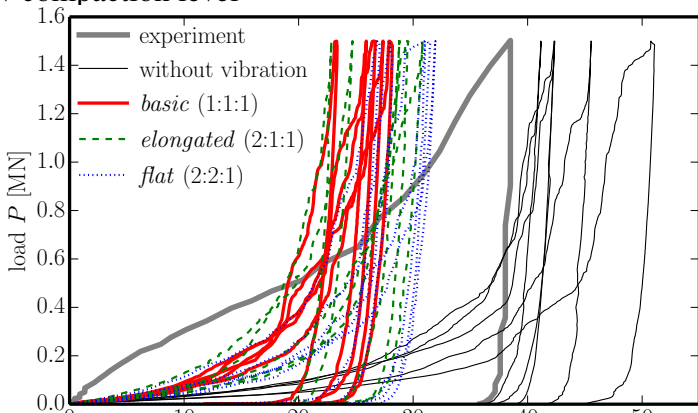
- ▶ chosen parameters of the materials

			ballast	steel
density	ρ	kg/m^3	2600	7850
normal volumetric stiffness	k_n	N/m^3	2×10^{13}	2×10^{14}
shear stiffness	k_s	N/m	2×10^8	2×10^9
angle of internal friction	ϕ	-	0.6	0.4

- ▶ polyhedrons are generated sequentially with no overlapping in larger volume and left to freely fall into steel cylinder
- ▶ high level of compaction is hard to achieve in simulation
- ▶ following items should help
 - ▶ increased gravity $5 \times$
 - ▶ decreased angle of internal friction to 0.1

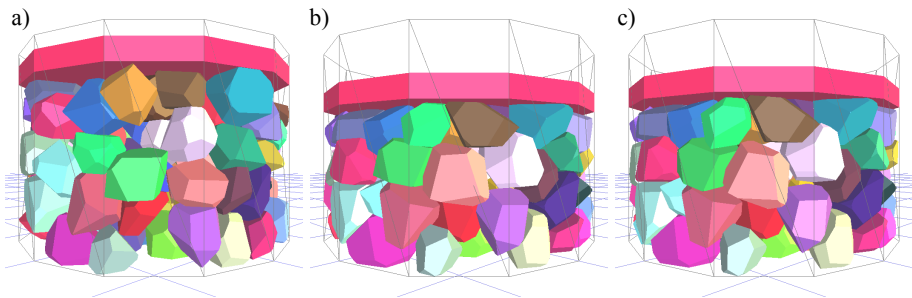
Comparison to experiment

- ▶ no crushing present in the model, loading curvature mostly caused by low compaction level



Views at different stages of simulation

- ▶ initial stage, after free fall and vibration (a)
- ▶ at the maximum load (b)
- ▶ after complete unloading (c)



Crushing in numerical model

- ▶ crushing of grains needs to be included
- ▶ incorporated via splitting of polyhedron pieces by plane running through centroid
- ▶ criterion based on principal stresses inside the grain:

$$\sigma_{ij} = \sum_{k=1}^N F_i r_j \Rightarrow \sigma_I > \sigma_{II} > \sigma_{III}$$

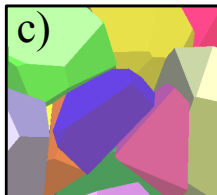
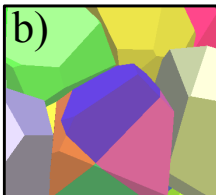
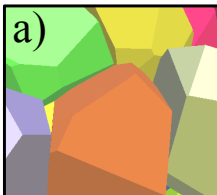
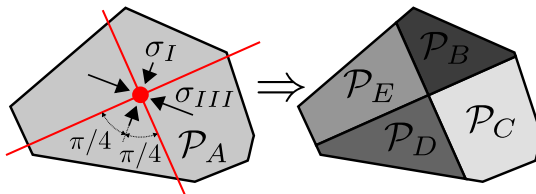
- ▶ splitting stress $\sigma_s = -\sigma_{III} + \sigma_I$
- ▶ size dependent strength $f_t = f_{t0}/r_{eq}$, where r_{eq} is equivalent radius (s.

Lobo-Guerrero, L. E. Vallejo, 2006)

$$r_{eq} = \sqrt[3]{\frac{3V}{4\pi}}$$

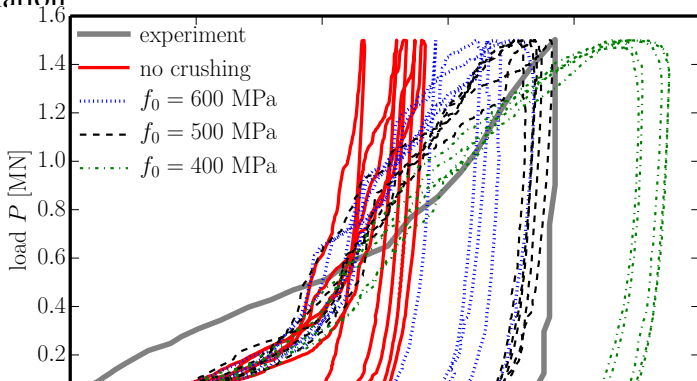
Crushing in numerical model

- ▶ splitting into four pieces by planes parallel to plane σ_{II} and under angle $\pi/4$ from σ_I and σ_{III} directions



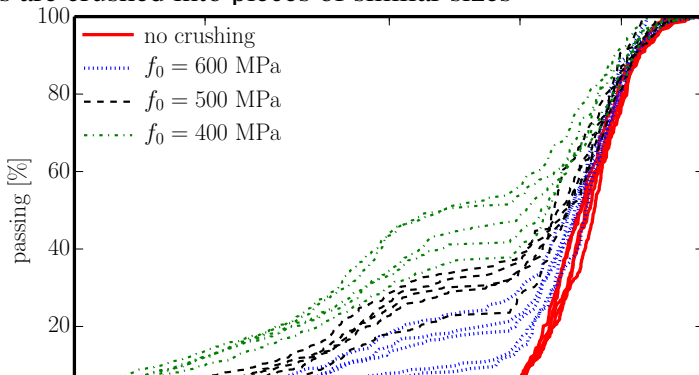
Comparison to experiment

- ▶ three material strengths tested: 400, 500 and 600 MPa
- ▶ pieces with volumes lower than 1 cm^3 were removed from the simulation



Sieve curve after crushing

- ▶ includes small particles with volume under 1 cm^3
- ▶ plateau between sieve sizes 30 and 40 mm is observed because the grains are crushed into pieces of similar sizes



Conclusions

- ▶ simple method to generate convex randomly shaped grains
- ▶ possibility to control aspect ratio of grains
- ▶ repulsive force estimated from volume of intersecting polyhedron
- ▶ normal direction estimated from least-square fitting of shells intersection by plane
- ▶ model used to simulate large oedometric test performed on railway ballast
- ▶ crushing of grains can be simply done in geometrical sense, problem is to develop correct criterion
- ▶ many thanks to YADE and CGAL developers