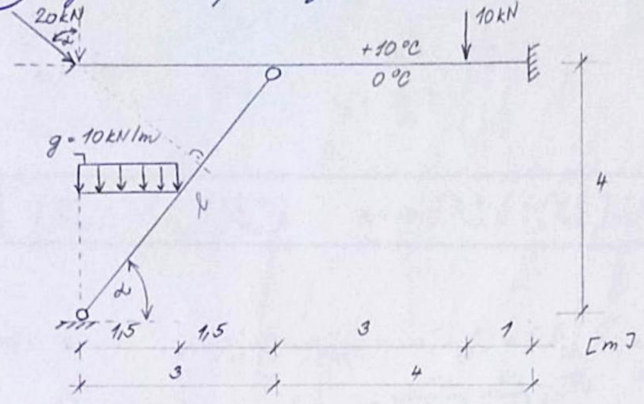
STATIKA II. (7 + 8.)

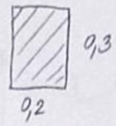
Silové + TEPLOTNÍ zatížení

Př.: Vykreslete průběhy vnitřních sil na daném rámu. Řešte pomocí ODM.



$\alpha_t = 1 \cdot 10^{-5}\ K^{-1}$

$E = 24\ GPa$



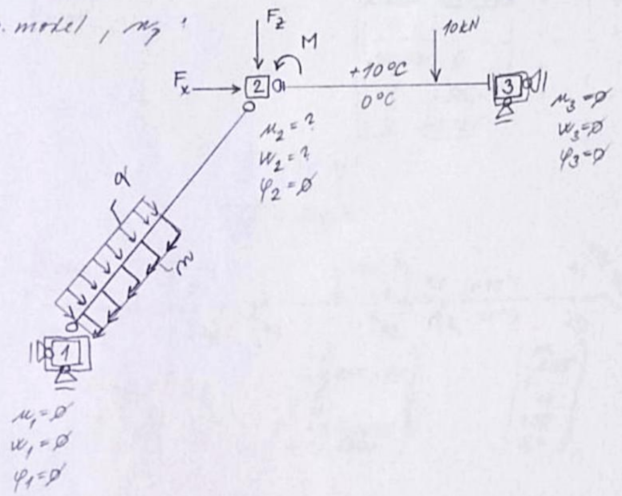
$A = 0{,}2 \cdot 0{,}3 = 0{,}06\ m$

$I = \frac{1}{12} b h^3 = \frac{1}{12} \cdot 0{,}2 \cdot 0{,}3^3 = 4{,}5 \cdot 10^{-4}\ m^4$

$EA = 24 \cdot 10^6 \cdot 0{,}06 = 1{,}44 \cdot 10^6\ kPa \cdot m^2$

$EI = 24 \cdot 10^6 \cdot 4{,}5 \cdot 10^{-4} = 10{,}8 \cdot 10^3\ kPa \cdot m^4$

1) výp. model, n_p :



$n_p = 2$

$l = \sqrt{3^2 + 4^2} = 5m$

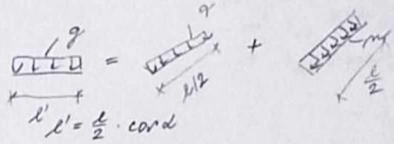
$\sin\alpha = 4/5 = 0{,}8$

$\cos\alpha = 3/5 = 0{,}6$

$F_x = 20 \cdot \sin\alpha = 20 \cdot 0{,}8 = 16\ kN$

$F_z = 20 \cdot \cos\alpha = 20 \cdot 0{,}6 = 12\ kN$

$M = F_z \cdot 3 = 12 \cdot 3 = 36\ kNm$



$l' = \frac{l}{2} \cdot \cos\alpha$

$l' = 2{,}5 \cdot 0{,}6 = 1{,}5\ m$

$G = 10 \cdot 1{,}5 = 15\ kN$

$Q = q \cdot l/2 = G \cdot \cos\alpha = 9\ kN$

$\hookrightarrow \underline{q = 3{,}6\ kN/m}$

$N = n \cdot l/2 = G \cdot \sin\alpha = 12\ kN$

$\hookrightarrow \underline{n = 4{,}8\ kN/m}$

$$[T_{ab}] = \left[\begin{array}{ccc|ccc} \cos & \sin & 0 & & & \\ -\sin & \cos & 0 & & \emptyset & \\ 0 & 0 & 1 & & & \\ \hline & & & \cos & \sin & 0 \\ & \emptyset & & -\sin & \cos & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

2) $\{r\}, \{S\}$:

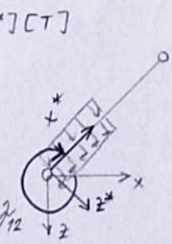
$\{r\} = \{u_2, w_2\}^T \qquad \{S\} = \{F_x; F_z\}^T = \{16; 12\}^T$

$[k_{ab}] = [T_{ab}]^T [k^*_{ab}] [T_{ab}]$

3) $[k^*_{ab}], \{\bar{R}^*_{ab}\}$ + $[k_{ab}], \{\bar{R}_{ab}\}$; úhel γ_{ab} .. pootočení vůči globální soustavě .. odměřujeme od kladné x ke kladné x* ve směru chodu hodinových ručiček !

Prut 1-2 : $\sin\gamma_{12} = -0{,}8$; $\cos\gamma_{12} = 0{,}6$

$[k] = [T^T][k^*][T]$



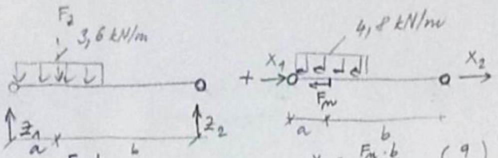
$$\begin{array}{c|cccccc|c} & u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2 & \\ \hline & . & . & . & -103680 & 138240 & . & X_1 \\ & . & . & . & 138240 & -184320 & . & Z_1 \\ [k_{12}] = & . & . & . & 0 & 0 & . & M_1 \\ & . & . & . & \boxed{103680} & \boxed{-138240} & . & X_2 \\ & . & . & . & \boxed{-138240} & \boxed{184320} & . & Z_2 \\ & . & . & . & 0 & 0 & . & M_2 \end{array}$$

$\frac{EA}{l} = 2{,}88 \cdot 10^5$

$\boxed{\{\bar{R}_{ab}\} = [T_{ab}]^T \cdot \{\bar{R}^*_{ab}\}}$

$$\{\bar{R}^*_{12}\} = \begin{Bmatrix} 9 \\ -6{,}75 \\ 0 \\ 3 \\ -2{,}25 \\ 0 \end{Bmatrix} \begin{matrix} X_1 \\ Z_1 \\ M_1 \\ X_2 \\ Z_2 \\ M_2 \end{matrix} \xrightarrow{[T]^T \cdot \{\bar{R}^*\}} \{\bar{R}_{12}\} = \begin{Bmatrix} 0 \\ -11{,}25 \\ 0 \\ \boxed{0} \\ \boxed{-3{,}75} \\ 0 \end{Bmatrix} \begin{matrix} X_1 \\ Z_1 \\ M_1 \\ X_2 \\ Z_2 \\ M_2 \end{matrix}$$

$$[T_{12}] = \left[\begin{array}{ccc|c} 0{,}6 & -0{,}8 & 0 & \\ 0{,}8 & 0{,}6 & 0 & \emptyset \\ 0 & 0 & 1 & \\ \hline & & & \end{array}\right]$$



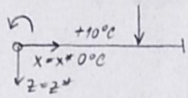
$z_1 = \frac{F_z \cdot b}{l}$ $\quad z_2 = \frac{F_z \cdot a}{l}$ $\quad \begin{Bmatrix} 0 \\ -6{,}75 \\ 0 \\ 0 \\ -8{,}25 \\ 0 \end{Bmatrix}$

$x_1 = \frac{F_m \cdot b}{l}$ $\quad x_2 = \frac{F_m \cdot a}{l}$ $\quad \begin{pmatrix} 9 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}$

– na oboustranně kloubově připojeném prutu → $\bar{R}^*$ výpočet jako na prostém nosníku

! $\boxed{\{\bar{R}_{ab}\} = [T_{ab}^T]\{\bar{R}^*_{ab}\} \longleftrightarrow \{\bar{R}^*_{ab}\} = [T_{ab}]\{\bar{R}_{ab}\}}$ $\Rightarrow \{\bar{R}_{ab}\} = [T_{ab}^{-1}]\{\bar{R}^*_{ab}\} = [T_{ab}^T]\{\bar{R}^*_{ab}\}$

Prut 2-3: $\sin\gamma_{23} = 0, \cos\gamma_{23} = 1 \rightarrow \{\bar{R}^*_{23}\} = \{\bar{R}_{23}\}$ a $[k^*_{ab}] = [k_{ab}]$



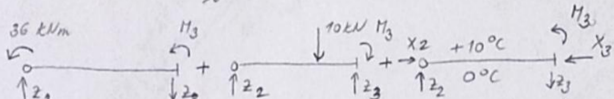
$$[k_{23}] = \begin{array}{c} \begin{array}{cccccc} u_2 & w_2 & \varphi_2 & u_3 & w_3 & \varphi_3 \end{array} \\ \begin{bmatrix} 360000 & 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & 506{,}25 & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot \\ -360000 & 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & -506{,}25 & \cdot & \cdot & \cdot & \cdot \\ 0 & -2025 & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \begin{array}{c} X_2 \\ Z_2 \\ M_2 \\ X_3 \\ Z_3 \\ M_3 \end{array}$$

$$\{\bar{R}^*_{23}\} = \{\bar{R}_{23}\} = \begin{Bmatrix} 72 \\ -15{,}709375 \\ 0 \\ -72 \\ 5{,}709375 \\ 16{,}8375 \end{Bmatrix} \begin{matrix} X_2 \\ Z_2 \\ M_2 \\ X_3 \\ Z_3 \\ M_3 \end{matrix} \qquad [T]^T\{\bar{R}^*\}$$

$\frac{EA}{l} = 3{,}6 \cdot 10^5$

$\frac{3EI}{l^3} = 506{,}25$

$[T_{23}] = \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \emptyset \\ 0 & 0 & 1 & \\ & & & \end{bmatrix}$



$\begin{Bmatrix} 0 \\ -13{,}5 \\ 0 \\ 0 \\ 13{,}5 \\ 18 \end{Bmatrix}$ $\quad \begin{Bmatrix} 0 \\ -0{,}859375 \\ 0 \\ 0 \\ -9{,}140625 \\ -6{,}5625 \end{Bmatrix}$ $\quad \begin{Bmatrix} 72 \\ -1{,}35 \\ 0 \\ -72 \\ 1{,}35 \\ 5{,}4 \end{Bmatrix}$

$+10 = \Delta t_h$

$0 = \Delta t_d$

$\Delta t_0 = \frac{10+0}{2} = 5$

$X_a = X_b = EA\alpha_t \Delta t$

$\Delta t_1 = \Delta t_d - \Delta t_h = -10°C < 0$

$\Rightarrow \bar{Z}_{ab}$ záměna ~~znaménko~~ $\bar{Z}_{ba}$

$\Rightarrow M_{ba}$ s opačným znam.

4) $[K], \{\bar{R}\}, \{F\}$: $\qquad \{F\} = \{S\} - \{\bar{R}\}$

$$[K] = \begin{array}{c} \begin{array}{cc} u_2 & w_2 \end{array} \\ \begin{bmatrix} 463\,680 & -138\,240 \\ -138240 & 184\,826{,}25 \end{bmatrix} \end{array} \begin{matrix} X_2 \\ Z_2 \end{matrix} \qquad \{\bar{R}\} = \begin{Bmatrix} 72 \\ -19{,}459\,375 \end{Bmatrix} \begin{matrix} X_2 \\ Z_2 \end{matrix} \qquad \{F\} = \begin{Bmatrix} -56 \\ 31{,}459\,375 \end{Bmatrix} \begin{matrix} X_2 \\ Z_2 \end{matrix}$$

5) $\{r\}$: $[K]\{r\} = \{F\} \longrightarrow \{r\} = [K^{-1}]\{F\}$ nebo soustava

$$\{r\} = \frac{1}{6{,}6589938 \cdot 10^{10}} \begin{bmatrix} 184\,826{,}25 & 138\,240 \\ 138\,240 & 463\,680 \end{bmatrix} \begin{Bmatrix} -56 \\ 31{,}459375 \end{Bmatrix}$$

$$\{r\} = \begin{Bmatrix} -9{,}012 \cdot 10^{-5} \\ 1{,}028 \cdot 10^{-4} \end{Bmatrix} \begin{matrix} u_2 \\ w_2 \end{matrix}$$

nebo: $463\,680\,u_2 - 138\,240\,w_2 = -56$ (1)

$-138\,240\,u_2 + 184\,826{,}25\,w_2 = 31{,}459375$ (2)

$(1) \cdot 138240 + (2) \cdot 463680$:

$0u_2 + 6{,}6589938 \cdot 10^{10} w_2 = 6\,845\,643$

$w_2 = 1{,}028 \cdot 10^{-4}$ rad

$u_2 = -9{,}012 \cdot 10^{-5}$ m

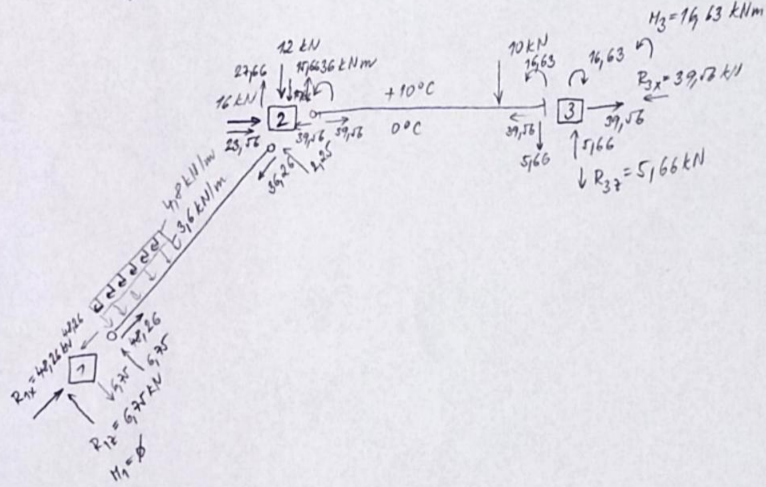
6) $\{r_{ab}\}$; $\{\hat{R}_{ab}\} \rightarrow \{R_{ab}\} = \{\bar{R}_{ab}\} + \{\hat{R}_{ab}\} \rightarrow \{R^*_{ab}\}$ $\qquad$; $\{\hat{R}_{ab}\} = [k_{ab}]\{r_{ab}\}$

$$\{r_{12}\} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -9{,}012 \cdot 10^{-5} \\ 1{,}028 \cdot 10^{-4} \\ 0 \end{pmatrix} \begin{matrix} u_1 \\ w_1 \\ \varphi_1 \\ u_2 \\ w_2 \\ \varphi_2 \end{matrix} \qquad \{r_{23}\} = \begin{pmatrix} -9{,}012 \cdot 10^{-5} \\ 1{,}028 \cdot 10^{-4} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} u_2 \\ w_2 \\ \varphi_2 \\ u_3 \\ w_3 \\ \varphi_3 \end{matrix}$$

$$\{\hat{R}_{12}\} = [k_{12}]\{r_{12}\} = \begin{Bmatrix} 23,555498 \\ -31,407331 \\ 0 \\ -23,555498 \\ 31,407331 \\ 0 \end{Bmatrix} \rightarrow \{R_{12}\} = \{\bar{R}_{12}\} + \{\hat{R}_{12}\} = \begin{Bmatrix} 23,555498 \\ -42,657331 \\ 0 \\ -23,555498 \\ 27,657331 \\ 0 \end{Bmatrix} \rightarrow \{R_{12}^*\} = [T_{12}]\{R_{12}\} = \begin{array}{c} \begin{Bmatrix} 48,259164 \\ -6,75 \\ 0 \\ -36,259164 \\ -2,25 \\ 0 \end{Bmatrix} \end{array} \begin{array}{c} x_1 \\ z_1 \\ M_1 \\ x_2 \\ z_2 \\ M_2 \end{array}$$

$$\{\hat{R}_{23}\} = [k_{23}]\{r_{23}\} = \begin{Bmatrix} -32,444502 \\ 0,052044 \\ 0 \\ 32,444502 \\ -0,052044 \\ -0,208176 \end{Bmatrix} \rightarrow \{R_{23}\} = \{\bar{R}_{23}\} + \{\hat{R}_{23}\} = \begin{Bmatrix} 39,555498 \\ -15,657331 \\ 0 \\ -39,555498 \\ 5,657331 \\ 16,629324 \end{Bmatrix} \begin{array}{c} x_2 \\ z_2 \\ M_2 \\ x_3 \\ z_3 \\ M_3 \end{array} \rightarrow \{R_{23}^*\} = \{R_{23}\}$$

7) vykreslení průběhů vnitřních sil :



$\boxed{1} \rightarrow R_{1x} = 48,26$ kN

$R_{1z} = 6,75$ kN

$M_1 = ∅$

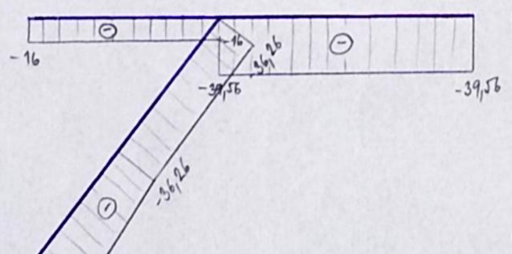
$\boxed{2}$ ✓ OK

$\boxed{3} \rightarrow R_{3x} = 39,56$ kN

$R_{3z} = 5,66$ kN

$M_3 = 16,63$ kNm

N



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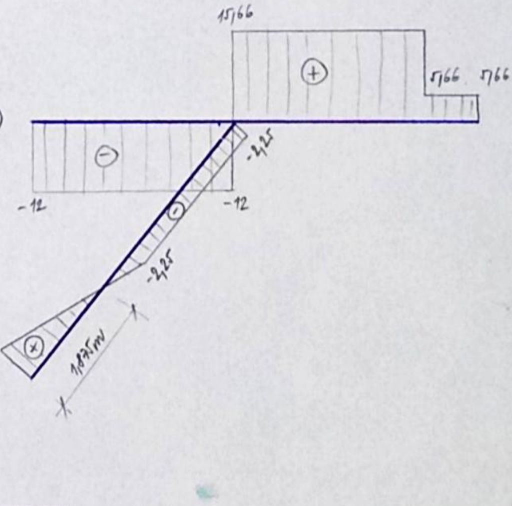

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