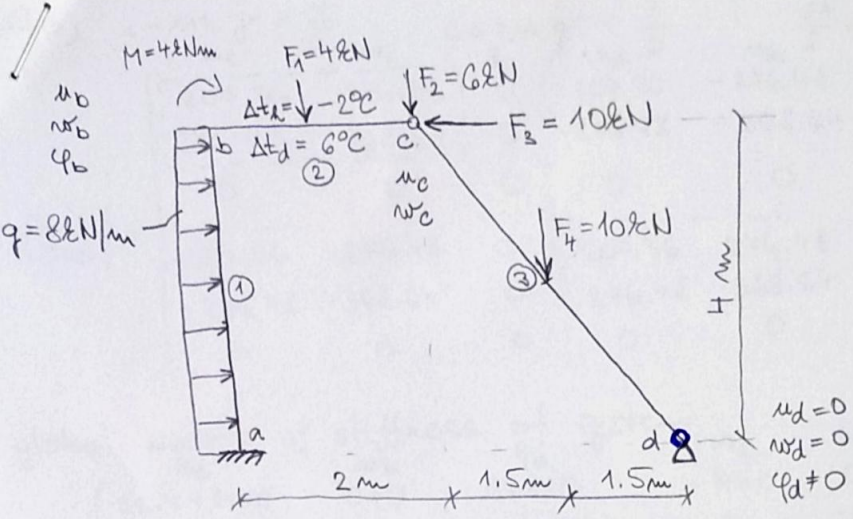
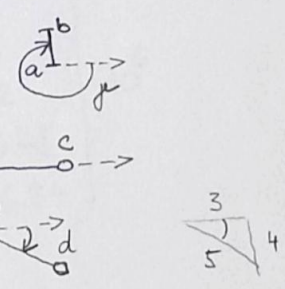




- ① $\square 0.4 \times 0.6$
 ②③ $\square 0.4 \times 0.3$
 $E = 24 \text{ GPa}$
 $\alpha_T = 10^{-5} \text{ K}^{-1}$
 coefficient of thermal expansion
 $A_1 = 0.24 \text{ m}^2$
 $A_{23} = 0.12 \text{ m}^2$
 $I_1 = 4.2 \cdot 10^{-3} \text{ m}^4$
 $I_{23} = 9 \cdot 10^{-4} \text{ m}^4$

- 1) $w_p = 5$
 2) $\{u\} = \{u_b; w_b; \phi_b; u_c; w_c\}^T$
 $\{S\} = \{0; 0; -4000; -10000; 6000\}^T$
 3) a) $[k_{ab}] = \text{tab. 11.4 a} \quad \mu = 270^\circ$
 b) $[k_{bc}] = \text{tab. 11.4 b} \quad \mu = 0^\circ$
 or tab 11.3 b
 c) $[k_{cd}] = \text{tab. 11.4 d} \quad \mu = 233.130^\circ$
 $A = \sin \mu = +\frac{4}{5}$
 $C = \cos \mu = +\frac{3}{5}$



ad a) $A = \sin 270^\circ = -1$
 $C = \cos 270^\circ = 0$

$[k_{ab}] = \begin{bmatrix} u_a & w_a & \phi_a & u_b & w_b & \phi_b \\ 32.4 & 0 & -64.8 & -32.4 & 0 & -64.8 \\ 0 & 1440 & 0 & 0 & -1440 & 0 \\ -64.8 & 0 & 142.8 & 64.8 & 0 & 86.4 \\ -32.4 & 0 & 64.8 & 32.4 & 0 & 64.8 \\ 0 & -1440 & 0 & 0 & 1440 & 0 \\ -64.8 & 0 & 86.4 & 64.8 & 0 & 142.8 \end{bmatrix} \cdot 10^6$

ad b)
 $[k_{bc}] = [k_{bc}^*] = \begin{bmatrix} u_b & w_b & \phi_b & u_c & w_c & \phi_c \\ 1440 & 0 & -16.2 & -1440 & 0 & 0 \\ 0 & 8.1 & -16.2 & 0 & -8.1 & 0 \\ 0 & -16.2 & 32.4 & 0 & 16.2 & 0 \\ -1440 & 0 & 0 & 1440 & 0 & 0 \\ 0 & -8.1 & 16.2 & 0 & 8.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot 10^6$

$$c) \Delta = \sin \mu = \frac{4}{5} \quad C = \cos \mu = \frac{3}{5} \quad \frac{EA}{L} = 546 \cdot 10^6$$

$$[k_{cd}] = \begin{bmatrix} \begin{matrix} 207.36 & 246.48 \\ 246.48 & 368.64 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} & \begin{matrix} -207.36 & -246.48 \\ -246.48 & -368.64 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} \\ \begin{matrix} -207.36 & -246.48 \\ -246.48 & -368.64 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 207.36 & 246.48 \\ 246.48 & 368.64 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 \end{matrix} \end{bmatrix} \cdot 10^6$$

$\begin{matrix} \mu_c \\ \mu_c \\ \varphi_c \\ \mu_d \\ \mu_d \\ \varphi_d \end{matrix}$

global matrix of stiffness of system

$$k = \begin{bmatrix} 32.4 + 1440 & 0 + 0 & 64.8 + 0 & -1440 & 0 \\ 0 + 0 & 1440 + 8.1 & 0 + (-16.2) & 0 & -8.1 \\ 64.8 + 0 & 0 + (-16.2) & 142.8 + 32.4 & 0 & 16.2 \\ -1440 & 0 & 0 & 1440 + 207.36 & 0 + 246.48 \\ 0 & -8.1 & 16.2 & 0 + 246.48 & 8.1 + 368.64 \end{bmatrix} \cdot 10^6$$

$\begin{matrix} \mu_b \\ \mu_b \\ \varphi_b \\ \mu_c \\ \mu_c \end{matrix}$

$$k = \begin{bmatrix} 1442.4 & 0 & 64.8 & -1440 & 0 \\ 0 & 1448.1 & -16.2 & 0 & -8.1 \\ 64.8 & -16.2 & 205.2 & 0 & 16.2 \\ -1440 & 0 & 0 & 1647.36 & 246.48 \\ 0 & -8.1 & 16.2 & 246.48 & 376.74 \end{bmatrix} \cdot 10^6$$

$$4) \{ \bar{R}_{ab}^* \} = \text{tab. 11.2 a (tab. 14.10 row 13)}$$

$$\{ \bar{R}_{bc}^* \}_{F_1} = (\text{tab. 11.2 c}) \text{ use tab. 14.11 row 2}$$

$$\{ \bar{R}_{cd}^* \} = \text{simply supported beam}$$

$$\{ \bar{R}_{bc}^* \}_L = \text{tab. 11.5 b}$$

$$\{ \bar{R}_{ab}^* \} = \{ 0; -16; 10.6; 0; -16; -10.6 \}^T \cdot 10^3$$

$\{ \bar{R}_{bc}^* \}_{F_1}$ bar bc is fixed-pinned but table is for pinned-fixed bar

how to get $\bar{R}_{bc}$?

$$\{ \bar{R}_{cb} \} = \begin{Bmatrix} \bar{X}_{cb} \\ \bar{Z}_{cb} \\ 0 \\ \bar{X}_{bc} \\ \bar{Z}_{bc} \\ \bar{\theta}_{bc} \end{Bmatrix} \Rightarrow \{ \bar{R}_{bc} \} = \begin{Bmatrix} \bar{X}_{bc} \\ \bar{Z}_{bc} \\ \bar{\theta}_{bc} \\ \bar{X}_{cb} \\ \bar{Z}_{cb} \\ 0 \end{Bmatrix} = \begin{Bmatrix} -\bar{X}_{bc} \\ \bar{Z}_{bc} \\ -\bar{\theta}_{bc} \\ -\bar{X}_{cb} \\ \bar{Z}_{cb} \\ 0 \end{Bmatrix}$$

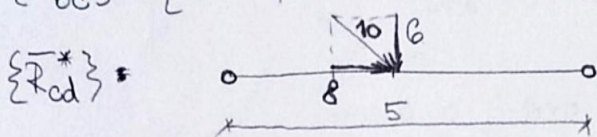
$$\{\bar{R}_{bc}^*\}_{\bar{r}} = \{0; -2.45; 1.5; 0; -1.25; 0\}^T \cdot 10^3$$

$$\{\bar{R}_{bc}^*\}_t: \Delta t_1 = 6 - (-2) = 8$$

$$\Delta t_0 = \frac{1}{2} (6 + (-2)) = 2$$

$$\{\bar{R}_{bc}^*\}_t = \{54.6; -4.32; 8.64; -54.6; +4.32; 0\}^T \cdot 10^3$$

$$\{\bar{R}_{bc}^*\} = \{54.6; -4.07; 10.14; -54.6; 3.07; 0\}^T \cdot 10^3$$



$$\{\bar{R}_{cd}^*\} = \{-4; -3; 0; -4; -3; 0\}^T \cdot 10^3$$

- primary vectors in global coordinates

$$\{\bar{P}_{ab}\} = \{-16; 0; 10.6; -16; 0; -10.6\}^T \cdot 10^3$$

$$\{\bar{P}_{bc}\} = \{\bar{R}_{bc}^*\} = \{54.6; -4.07; 10.14; -54.6; 3.07; 0\}^T \cdot 10^3$$

$$\{\bar{P}_{cd}\} = \{0; -5; 0; 0; 0; -5\}^T \cdot 10^3$$

- global primary vector of system

$$\{\bar{P}\} = \{-16 + 54.6; 0 + (-4.07); -10.6 + 10.14; -54.6 + 0; 3.07 + (-5)\}^T \cdot 10^3$$

$$\{\bar{P}\} = \{41.6; -4.07; -0.526; -54.6; -1.93\}^T \cdot 10^3$$

$$5) \{F\} = \{S\} - \{\bar{P}\}$$

$$\{F\} = \{-41.6; 4.07; -3.443; 44.6; 4.93\}^T \cdot 10^3$$

$$6) [k] \cdot \{u\} = \{F\}$$

$$\begin{bmatrix} 1442.4 & 0 & 64.8 & -1440 & 0 \\ 0 & 1448.1 & -16.2 & 0 & -8.1 \\ 64.8 & -16.2 & 205.2 & 0 & 16.2 \\ -1440 & 0 & 0 & 1647.36 & 246.48 \\ 0 & -8.1 & 16.2 & 246.48 & 346.74 \end{bmatrix} \cdot 10^6 \cdot \begin{Bmatrix} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \end{Bmatrix} = \begin{Bmatrix} -41.6 \\ 4.07 \\ -3.443 \\ 44.6 \\ 4.93 \end{Bmatrix} \cdot 10^3$$

$$\{u\} = \begin{Bmatrix} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \end{Bmatrix} = \begin{Bmatrix} 39.149 \\ 4.429 \\ -26.783 \\ 64.736 \\ -24.414 \end{Bmatrix} \cdot 10^{-6}$$

$$\{R\} = \{R\} + \{\hat{R}\}$$

$$\{\hat{R}\} = [k] \cdot \{u\}$$

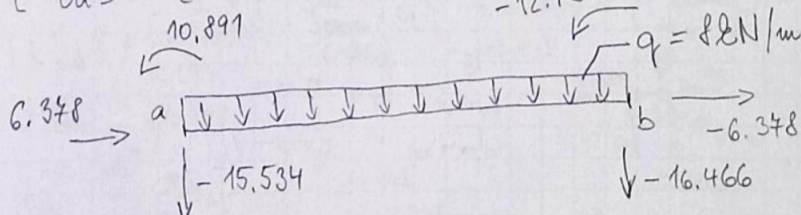
$$\{\hat{R}_{ab}\} = [k_{ab}] \{u_{ab}\} = \begin{bmatrix} -32.4 & 0 & -64.8 \\ 0 & -1440 & 0 \\ 64.8 & 0 & 86.4 \\ 32.4 & 0 & 64.8 \\ 0 & 1440 & 0 \\ 64.8 & 0 & 142.8 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 39,141 \\ 4,429 \\ -26,783 \end{Bmatrix} \cdot \frac{10^{-6}}{10} = \begin{Bmatrix} 466.398 \\ -6344.46 \\ 224,230 \\ -466.398 \\ 6344.46 \\ -2089,822 \end{Bmatrix}$$

$$\{\hat{R}_{bc}\} = [k_{bc}] \{u_{bc}\} = \begin{bmatrix} 1440 & 0 & 0 & -1440 & 0 & 0 \\ 0 & 8.1 & -16.2 & 0 & -8.1 & 0 \\ 0 & -16.2 & 32.4 & 0 & 16.2 & 0 \\ -1440 & 0 & 0 & 1440 & 0 & 0 \\ 0 & -8.1 & 16.2 & 0 & 8.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 39,141 \\ 4,429 \\ -26,783 \\ 64,436 \\ -24,414 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -41133.6 \\ 691,813 \\ -1383,626 \\ 41133.6 \\ -691,813 \\ 0 \end{Bmatrix} = \{\hat{R}_{bc}^*\}$$

$$\{\hat{R}_{cd}\} = [k_{cd}] \{u_{cd}\} = \begin{bmatrix} 207.36 & 246.48 & & & & \\ 246.48 & 368.64 & & & & \\ 0 & 0 & & & & \\ -207.36 & -246.48 & & & & \\ -246.48 & -368.64 & & & & \\ 0 & 0 & & & & \end{bmatrix} \begin{Bmatrix} 64,436 \\ -24,414 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 6466.314 \\ 8621.452 \\ 0 \\ -6466.314 \\ -8621.452 \\ 0 \end{Bmatrix}$$

$$\{\hat{R}_{ab}^*\} = \{6344.76; \textcircled{-15533.6}; 224,230; -6344.76; -466,398; -2089,822\}^T$$

$$\{\hat{R}_{cd}^*\} = \{10444.19; 0; 0; -10444.19; 0; 0\}^T$$

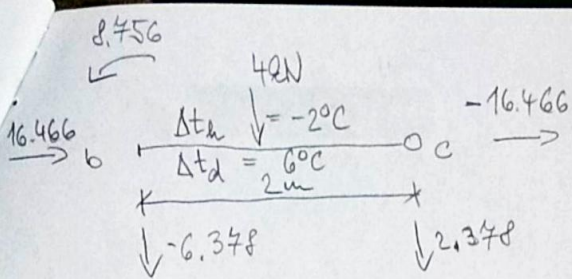


$$\{R_{ab}^*\} = \{\hat{R}_{ab}^*\} + \{\hat{R}_{cd}^*\} = \{6344.76; -15533.6; 10890,896; -6344.76; -16466.4; -12456,48\}^T$$

$$\sum F_x = 0 \quad 6.348 - 6.348 = 0 \quad \checkmark$$

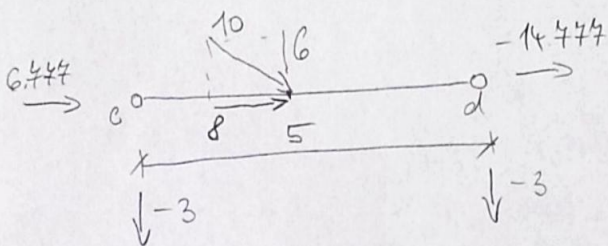
$$\sum F_z = 0 \quad -15.534 + 8 \cdot 4 - 16.466 = 0 \quad \checkmark$$

$$\sum M_a = 0 \quad 10,891 - 12,456 + 16,466 \cdot 4 - 8 \cdot 4 \cdot \frac{4}{2} = 0 \quad \checkmark$$



$$\{R_{bc}^*\} = \{R_{bc}^*\} + \{\hat{R}_{bc}^*\} = \{16.466, 4; -6.378, 187; 8.456, 374; -16.466, 4; 2.378, 187; 0\}^T$$

$$\begin{aligned} \sum F_x &= 0 & 16.466 - 16.466 &= 0 \quad \checkmark \\ \sum F_y &= 0 & -6.378 + 4 + 2.378 &= 0 \quad \checkmark \\ \sum M_a &= 0 & 8.456 - 2.378 \cdot 2 - 4 \cdot 1 &= 0 \quad \checkmark \end{aligned}$$



$$\{R_{cd}^*\} = \{R_{cd}^*\} + \{\hat{R}_{cd}^*\} = \{6.477, 19; -3000; 0; -14.777, 19; -3000; 0\}^T$$

$$\begin{aligned} \sum F_x &= 0 & 6.477 + 8 - 14.777 &= 0 \quad \checkmark \\ \sum F_y &= 0 & -3 + 6 - 3 &= 0 \quad \checkmark \\ \sum M_a &= 0 & 6 \cdot 2.5 - 3 \cdot 5 &= 0 \quad \checkmark \end{aligned}$$

$$\{R_{ab}\} = \{6.377, 76; 15.533, 6; 10.890, 896; 6.377, 46; -16.466, 398; -12.465, 4886\}^T$$

$$\{R_{bc}\} = \{16.466, 4; -6.378, 187; 8.456, 374; -16.466, 4; -2.378, 187; 0\}^T$$

$$\{R_{cd}\} = \{6.466, 314; 3.621, 8; -6.466, 314; -14.777, 19; -3.000, 0; -13.621, 8\}^T$$

