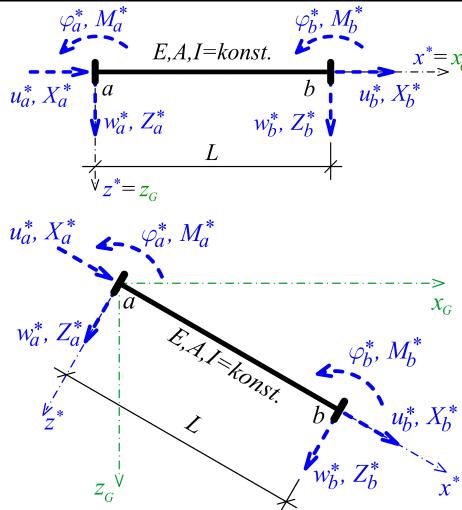
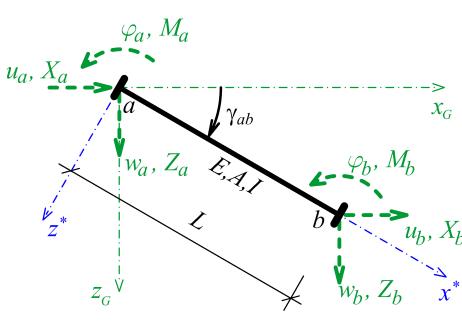


## Vetknutí - Vetknutí



$$[K_{a,b}^*] = \begin{bmatrix} u_a^* & w_a^* & \varphi_a^* & u_b^* & w_b^* & \varphi_b^* \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} X_a^* \\ Z_a^* \\ M_a^* \\ X_b^* \\ Z_b^* \\ M_b^* \end{bmatrix}$$



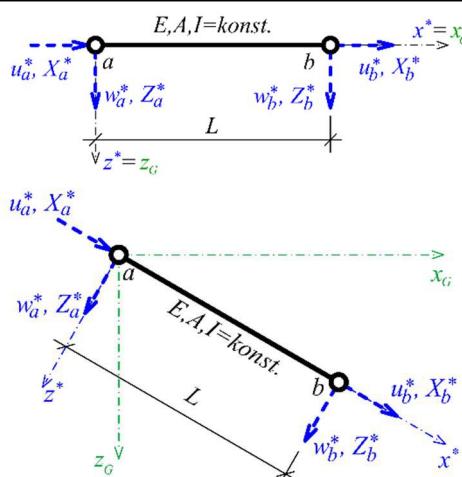
$$\mathbf{D} = \frac{EA}{L} - \frac{12EI}{L^3}$$

$$c = \cos(\gamma_{ab})$$

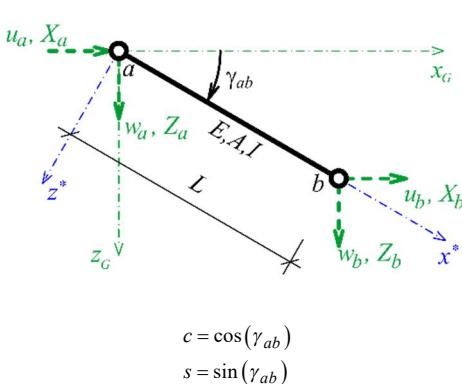
$$s = \sin(\gamma_{ab})$$

$$[K_{a,b}] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \left(\frac{12EI}{L^3} + \mathbf{D}c^2\right) & \mathbf{D}cs & \frac{6EI}{L^2}s & -\left(\frac{12EI}{L^3} + \mathbf{D}c^2\right) & -\mathbf{D}cs & \frac{6EI}{L^2}s \\ \mathbf{D}cs & \left(\frac{EA}{L} - \mathbf{D}c^2\right) & -\frac{6EI}{L^2}c & -\mathbf{D}cs & -\left(\frac{EA}{L} - \mathbf{D}c^2\right) & -\frac{6EI}{L^2}c \\ \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{4EI}{L} & -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{2EI}{L} \\ -\left(\frac{12EI}{L^3} + \mathbf{D}c^2\right) & -\mathbf{D}cs & -\frac{6EI}{L^2}s & \left(\frac{12EI}{L^3} + \mathbf{D}c^2\right) & \mathbf{D}cs & -\frac{6EI}{L^2}s \\ -\mathbf{D}cs & -\left(\frac{EA}{L} - \mathbf{D}c^2\right) & \frac{6EI}{L^2}c & \mathbf{D}cs & \left(\frac{EA}{L} - \mathbf{D}c^2\right) & \frac{6EI}{L^2}c \\ \frac{6EI}{L^2}s & -\frac{6EI}{L^2}c & \frac{2EI}{L} & -\frac{6EI}{L^2}s & \frac{6EI}{L^2}c & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{bmatrix}$$

## Kloub - Kloub

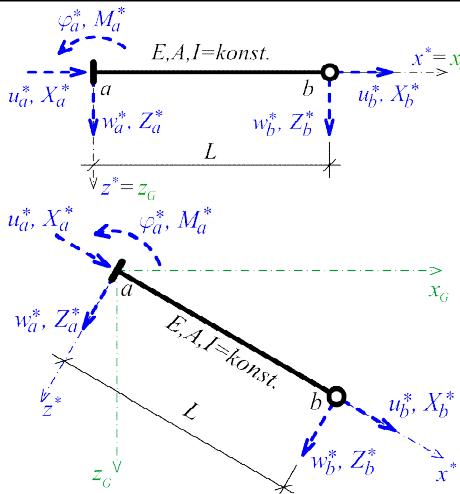


$$[K_{a,b}^*] = \begin{bmatrix} u_a^* & w_a^* & \varphi_a^* & u_b^* & w_b^* & \varphi_b^* \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_a^* \\ Z_a^* \\ M_a^* \\ X_b^* \\ Z_b^* \\ M_b^* \end{bmatrix}$$



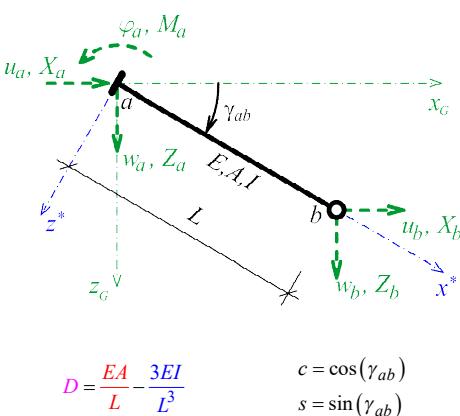
$$[K_{a,b}] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \frac{EA}{L}c^2 & \frac{EA}{L}cs & 0 & -\frac{EA}{L}c^2 & -\frac{EA}{L}cs & 0 \\ \frac{EA}{L}cs & \frac{EA}{L}s^2 & 0 & -\frac{EA}{L}cs & -\frac{EA}{L}s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L}c^2 & -\frac{EA}{L}cs & 0 & \frac{EA}{L}c^2 & \frac{EA}{L}cs & 0 \\ -\frac{EA}{L}cs & -\frac{EA}{L}s^2 & 0 & \frac{EA}{L}cs & \frac{EA}{L}s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{bmatrix}$$

## Vetknutí - Kloub



Lokální matice tuhosti

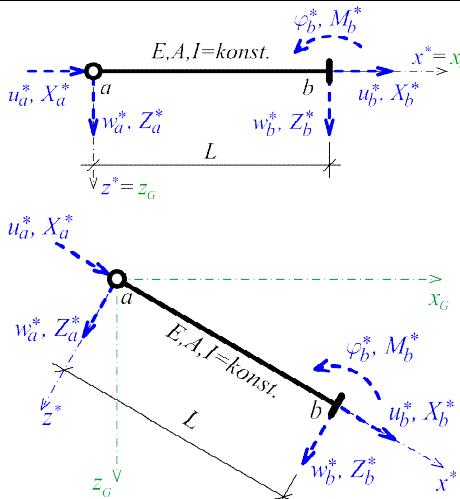
$$[K_{a,b}^*] = \begin{bmatrix} u_a^* & w_a^* & \varphi_a^* & u_b^* & w_b^* & \varphi_b^* \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} & 0 & \frac{3EI}{L^2} & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} X_a^* \\ Z_a^* \\ M_a^* \\ X_b^* \\ Z_b^* \\ M_b^* \end{matrix}$$



Globální matice tuhosti

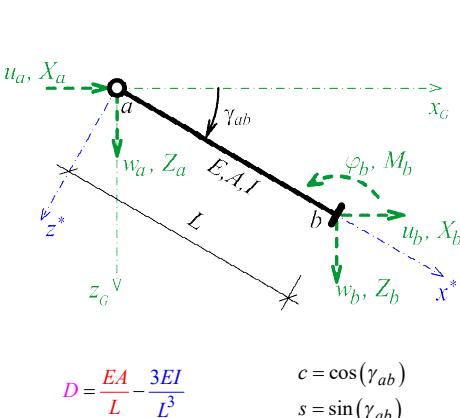
$$[K_{a,b}] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \left(\frac{3EI}{L^3} + Dc^2\right) & Dcs & \frac{3EI}{L^2}s & -\left(\frac{3EI}{L^3} + Dc^2\right) & -Dcs & 0 \\ Dcs & \left(\frac{EA}{L} - Dc^2\right) & -\frac{3EI}{L^2}c & -Dcs & -\left(\frac{EA}{L} - Dc^2\right) & 0 \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & \frac{3EI}{L} & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & 0 \\ -\left(\frac{3EI}{L^3} + Dc^2\right) & -Dcs & -\frac{3EI}{L^2}s & \left(\frac{3EI}{L^3} + Dc^2\right) & Dcs & 0 \\ -Dcs & -\left(\frac{EA}{L} - Dc^2\right) & \frac{3EI}{L^2}c & Dcs & \left(\frac{EA}{L} - Dc^2\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{matrix}$$

## Kloub - Vetknutí



Lokální matice tuhosti

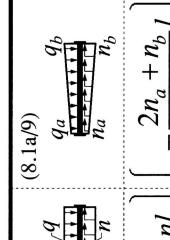
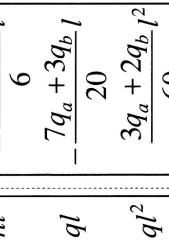
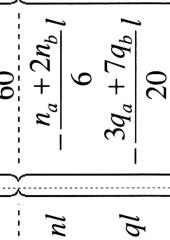
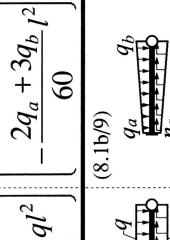
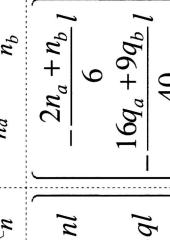
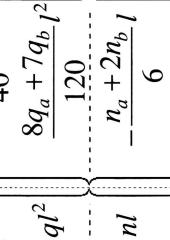
$$[K_{a,b}^*] = \begin{bmatrix} u_a^* & w_a^* & \varphi_a^* & u_b^* & w_b^* & \varphi_b^* \\ \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & -\frac{3EI}{L^2} & 0 & 0 & \frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix} \begin{matrix} X_a^* \\ Z_a^* \\ M_a^* \\ X_b^* \\ Z_b^* \\ M_b^* \end{matrix}$$



Globální matice tuhosti

$$[K_{a,b}] = \begin{bmatrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \left(\frac{3EI}{L^3} + Dc^2\right) & Dcs & 0 & -\left(\frac{3EI}{L^3} + Dc^2\right) & -Dcs & \frac{3EI}{L^2}s \\ Dcs & \left(\frac{EA}{L} - Dc^2\right) & 0 & -Dcs & -\left(\frac{EA}{L} - Dc^2\right) & -\frac{3EI}{L^2}c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{3EI}{L^3} + Dc^2 & -Dcs & 0 & \left(\frac{3EI}{L^3} + Dc^2\right) & Dcs & -\frac{3EI}{L^2}s \\ -Dcs & -\left(\frac{EA}{L} - Dc^2\right) & 0 & Dcs & \left(\frac{EA}{L} - Dc^2\right) & \frac{3EI}{L^2}c \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & 0 & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & \frac{3EI}{L} \end{bmatrix} \begin{matrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{matrix}$$

Tabulka 8.1 Primární lokální vektory koncových sil prutu  $\{R_{a,b}^*\}$ 

<b>(a)</b>	$\bar{X}_{a,b}^*$	$\bar{Z}_{a,b}^*$	$\bar{M}_{a,b}^*$	$\bar{X}_{b,a}^*$	$\bar{Z}_{b,a}^*$	$\bar{M}_{b,a}^*$
						
(8.1a/1)	$F_x$	$F_z$	$M$	(8.1a/2)	$F_x$	$F_z$
(8.1a/3)	$F_x$	$F_z$	$M$	(8.1a/4)	$F_x$	$F_z$
(8.1a/5)	$M$	$M$	$M_1$	(8.1a/6)	$M$	$M_2$
(8.1a/7)	$M$	$M$	$M_1$	(8.1a/8)	$M$	$M$
(8.1a/9)	$q_a$	$q_b$	$n_a$	(8.1a/10)	$q_a$	$q_b$
Obdělník, průřez: $\Delta T_0 = (\Delta T_d + \Delta T_h)/2$ $\Delta T_1 = \Delta T_d - \Delta T_h$						
<b>(b)</b>	$\bar{X}_{a,b}^*$	$\bar{Z}_{a,b}^*$	$\bar{M}_{a,b}^*$	$\bar{X}_{b,a}^*$	$\bar{Z}_{b,a}^*$	$\bar{M}_{b,a}^*$
(8.1b/1)	$F_x$	$F_z$	$M$	(8.1b/2)	$F_x$	$F_z$
(8.1b/3)	$F_x$	$F_z$	$M$	(8.1b/4)	$F_x$	$F_z$
(8.1b/5)	$M$	$M$	$M$	(8.1b/6)	$M$	$M$
(8.1b/7)	$M$	$M$	$M$	(8.1b/8)	$M$	$M$
(8.1b/9)	$q_a$	$q_b$	$n_a$	(8.1b/10)	$q_a$	$q_b$
Obdělník, průřez: $\Delta T_0 = (\Delta T_d + \Delta T_h)/2$ $\Delta T_1 = \Delta T_d - \Delta T_h$						

**Tabulka 8.1** Primární lokální vektory koncových sil prutu  $\{R_{a,b}^*\}$  (pokračování)

Oprava: Ing. Zbyněk Vlk, Ph.D. - 2025

## Transformace vektorů

z lokálních do globálních souřadnic

$$\{\bar{\mathbf{R}}_{a,b}^*\} \rightarrow \{\bar{\mathbf{R}}_{a,b}\}$$

$$\{\bar{\mathbf{R}}_{a,b}\} = \begin{Bmatrix} \bar{X}_a^* \cos \gamma_{ab} - \bar{Z}_a^* \sin \gamma_{ab} \\ \bar{X}_a^* \sin \gamma_{ab} + \bar{Z}_a^* \cos \gamma_{ab} \\ \bar{M}_a^* \end{Bmatrix} = \begin{Bmatrix} \bar{X}_b^* \cos \gamma_{ab} - \bar{Z}_b^* \sin \gamma_{ab} \\ \bar{X}_b^* \sin \gamma_{ab} + \bar{Z}_b^* \cos \gamma_{ab} \\ \bar{M}_b^* \end{Bmatrix}$$

z globálních do lokálních souřadnic

$$\{\mathbf{r}_{a,b}\} \rightarrow \{\mathbf{r}_{a,b}^*\}$$

$$\{\mathbf{r}_{a,b}^*\} = \begin{Bmatrix} u_a \cos \gamma_{ab} + w_a \sin \gamma_{ab} \\ -u_a \sin \gamma_{ab} + w_a \cos \gamma_{ab} \\ \varphi_a \end{Bmatrix} = \begin{Bmatrix} u_b \cos \gamma_{ab} + w_b \sin \gamma_{ab} \\ -u_b \sin \gamma_{ab} + w_b \cos \gamma_{ab} \\ \varphi_b \end{Bmatrix}$$