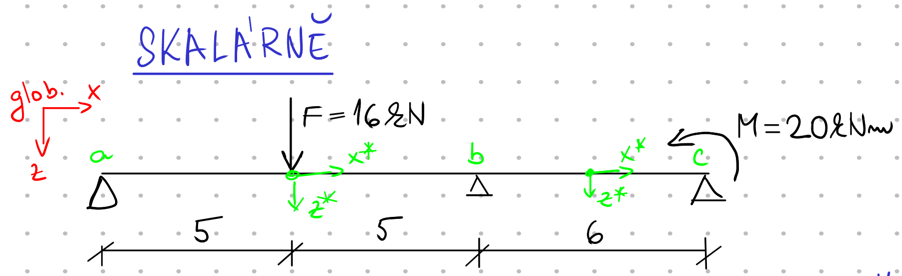
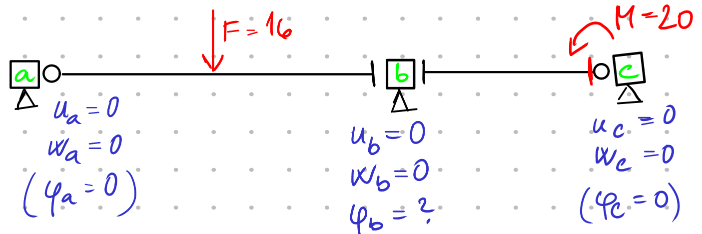
SKALÁRNĚ



$E = 30\,GPa$

$I = 2\cdot 10^{-4}\,m^4$

$EI = 30\cdot 10^{6}\cdot 2\cdot 10^{-4} = 6000\,kNm^2$



Výpočetní model

- stupeň přetvárné neurčitosti $n_p = 1$
- neznámé deformace φ_b

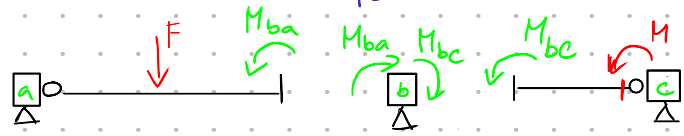
Podmínky rovnováhy

$\varphi_b: \ \sum M_{bi} = 0$

$M_{ba} + M_{bc} = 0$

$\overline{M}_{ba} + \widehat{M}_{ba} + \overline{M}_{bc} + \widehat{M}_{bc} = 0$

$k\cdot\varphi_b$



lokální souř. systém ≡ globální

prut a-b

primární vektor koncových sil 8.1c/2

$$\{\overline{R}^*_{ab}\} = \{\overline{R}_{ab}\} = \begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{5}{16}F_z \\ 0 \\ -\frac{1}{2}F_x \\ -\frac{11}{16}F_z \\ -\frac{3}{16}F_z l \end{Bmatrix} = \begin{Bmatrix} \overline{X}_{ab} \\ \overline{Z}_{ab} \\ \overline{M}_{ab} \\ \overline{X}_{ba} \\ \overline{Z}_{ba} \\ \boxed{\overline{M}_{ba}} \end{Bmatrix} = \begin{Bmatrix} \\ \\ \\ \\ \\ -30 \end{Bmatrix}$$

(8.1c/2) F_x, F_z, $l/2$, $l/2$

matice tuhosti

$$[k_{ab}] = [k^*_{ab}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \boxed{1800} \end{bmatrix} \end{array} \begin{array}{l} \widehat{X}_{ab} \\ \widehat{Z}_{ab} \\ \widehat{M}_{ab} \\ \widehat{X}_{ba} \\ \widehat{Z}_{ba} \\ \widehat{M}_{ba} \end{array}$$

$$[K_{a,b}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \left[\begin{array}{ccc|ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & \frac{3EI}{L^2}s \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & -\frac{3EI}{L^2}s \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 & Dcs & \left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & 0 & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & \frac{3EI}{L} \end{array}\right] \end{array} \begin{array}{l} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{array}$$

prut b-c

primární vektor 8.1b/5

$$\{\overline{R}_{bc}\} = \{\overline{R}^*_{bc}\} = \begin{Bmatrix} 0 \\ -\frac{3}{2}\frac{M}{l} \\ \frac{1}{2}M \\ 0 \\ \frac{3}{2}\frac{M}{l} \\ M \end{Bmatrix} = \begin{Bmatrix} \overline{X}_{bc} \\ \overline{Z}_{bc} \\ \boxed{\overline{M}_{bc}} \\ \overline{X}_{cb} \\ \overline{Z}_{cb} \\ \overline{M}_{cb} \end{Bmatrix} = \begin{Bmatrix} \\ \\ 10 \\ \\ \\ \\ \end{Bmatrix}$$

(8.1b/5) M

$$[k_{bc}] = [k^*_{bc}] = \begin{array}{c} \begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \end{array} \\ \begin{bmatrix} & & & & & \\ & & & & & \\ & & \boxed{3000} & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{array} \begin{array}{l} \widehat{X}_{bc} \\ \widehat{Z}_{bc} \\ \widehat{M}_{bc} \\ \widehat{X}_{cb} \\ \widehat{Z}_{cb} \\ \widehat{M}_{cb} \end{array}$$

$$[K_{a,b}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \left[\begin{array}{ccc|ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & \frac{3EI}{L^2}s & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & \frac{3EI}{L} & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & 0 \\ \hline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & -\frac{3EI}{L^2}s & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c & Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{array}$$

$$\overline{M}_{ba} + \widehat{M}_{ba} + \overline{M}_{bc} + \widehat{M}_{bc} = 0$$

$$-30 + 1800\varphi_b + 10 + 3000\varphi_b = 0$$

$$1800\varphi_b + 3000\varphi_b = -(-30+10)$$

$$4800\varphi_b = 20$$

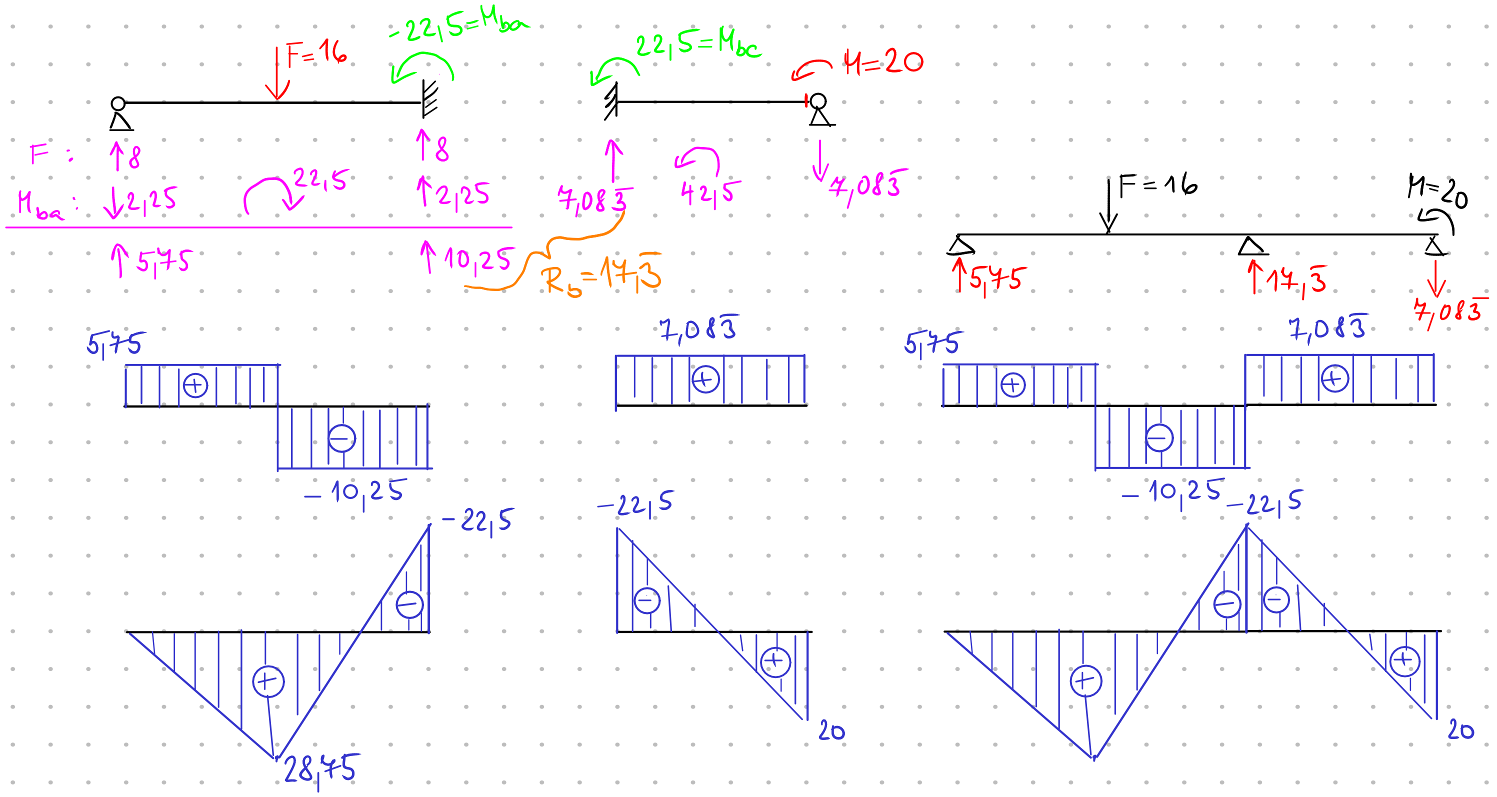
$$\varphi_b = \frac{20}{4800} = \underline{4{,}1\overline{6}\cdot 10^{-3}\,rad}$$

Dopočet sekundárních a celkových výsledných sil

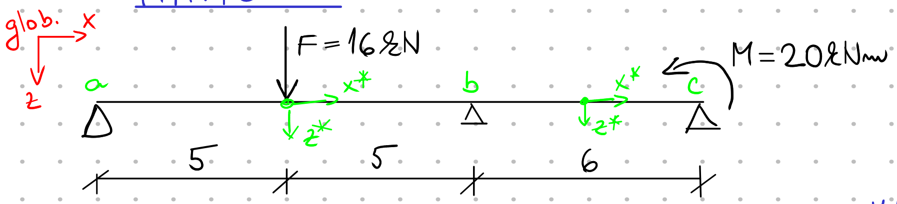
$$M_{ba} = \bar{M}_{ba} + \hat{M}_{ba} = -30 + 1800\,\varphi_b = -30 + 1800 \cdot 4{,}1\bar{6} \cdot 10^{-3} = -30 + 7{,}5 = \underline{-22{,}5 \text{ kNm}}$$

$$M_{bc} = \bar{M}_{bc} + \hat{M}_{bc} = 10 + 3000\,\varphi_b = 10 + 3000 \cdot 4{,}1\bar{6} \cdot 10^{-3} = 10 + 12{,}5 = \underline{22{,}5 \text{ kNm}}$$

Reakce a vykreslení



MATICOVĚ



$E = 30\ GPa$

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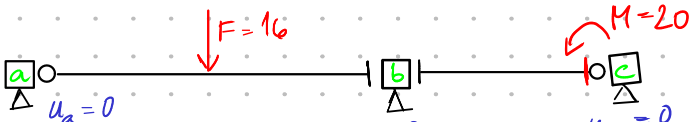


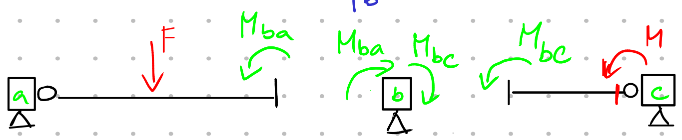
$u_a = 0$
$w_a = 0$
$(\varphi_a = 0)$

$u_b = 0$
$w_b = 0$
$\varphi_b = ?$

$u_c = 0$
$w_c = 0$
$(\varphi_c = 0)$

Výpočetní model

- stupeň přetvárné neurčitosti
  $n_p = 1$
- vektor neznámých deformací
  $\{r\} = \{\varphi_b\}$
- vektor styčníkových zatížení
  $\{S\} = \{0\}$



lokální souř. systém ≡ globální

prut a-b

primární vektor koncových sil 8.1c/2

(8.1c/2)

$$\{\bar{R}^*_{ab}\} = \begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{5}{16}F_z \\ 0 \\ -\frac{1}{2}F_x \\ -\frac{11}{16}F_z \\ -\frac{3}{16}F_z l \end{Bmatrix} = \begin{Bmatrix} \bar{X}_{ab} \\ \bar{Z}_{ab} \\ \bar{M}_{ab} \\ \bar{X}_{ba} \\ \bar{Z}_{ba} \\ \boxed{\bar{M}_{ba}} \end{Bmatrix} = \begin{Bmatrix} \\ \\ \\ \\ \\ -30 \end{Bmatrix}$$

$$\{\bar{R}^*_{ab}\} = \{\bar{R}_{ab}\}$$

matice tuhosti

$$[k_{ab}] = [k^*_{ab}] = \begin{array}{c} \begin{matrix} u_a\, w_a\, \varphi_a\, u_b\, w_b & \varphi_b \end{matrix} \\ \begin{bmatrix} & \\ & \\ & \\ & \\ & \\ & \boxed{1800} \end{bmatrix} \end{array} \begin{matrix} \hat{X}_{ab} \\ \hat{Z}_{ab} \\ \hat{M}_{ab} \\ \hat{X}_{ba} \\ \hat{Z}_{ba} \\ \hat{M}_{ba} \end{matrix}$$

$$[K_{a,b}] = \begin{array}{c} \begin{matrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{matrix} \\ \left[\begin{array}{ccc|ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & \frac{3EI}{L^2}s \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & -\frac{3EI}{L^2}s \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 & Dcs & \left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & 0 & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & \frac{3EI}{L} \end{array}\right] \end{array} \begin{matrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{matrix}$$

prut b-c

primární vektor 8.1b/5

(8.1b/5)

$$\{\bar{R}_{bc}\} = \begin{Bmatrix} 0 \\ -\frac{3}{2}\frac{M}{l} \\ \frac{1}{2}M \\ 0 \\ \frac{3}{2}\frac{M}{l} \\ M \end{Bmatrix} = \begin{Bmatrix} \bar{X}_{bc} \\ \bar{Z}_{bc} \\ \boxed{\bar{M}_{bc}} \\ \bar{X}_{cb} \\ \bar{Z}_{cb} \\ \bar{M}_{cb} \end{Bmatrix} = \begin{Bmatrix} \\ \\ 10 \\ \\ \\ \\ \end{Bmatrix}$$

$$\{\bar{R}_{bc}\} = \{\bar{R}^*_{bc}\}$$

$$[k_{bc}] = [k^*_{bc}] = \begin{array}{c} \begin{matrix} u_b\, w_b & \varphi_b & u_c\, w_c\, \varphi_c \end{matrix} \\ \begin{bmatrix} & & \\ & & \\ & \boxed{3000} & \\ & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{matrix} \hat{X}_{bc} \\ \hat{Z}_{bc} \\ \hat{M}_{bc} \\ \hat{X}_{cb} \\ \hat{Z}_{cb} \\ \hat{M}_{cb} \end{matrix}$$

$$[K_{a,b}] = \begin{array}{c} \begin{matrix} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{matrix} \\ \left[\begin{array}{ccc|ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & \frac{3EI}{L^2}s & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & \frac{3EI}{L} & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & 0 \\ \hline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & -\frac{3EI}{L^2}s & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c & Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{matrix} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{matrix}$$

Řešení soustavy rovnic

$$[k] \cdot \{r\} = \{F\} \qquad \{F\} = \{S\} - \{\bar{R}\} = \{0\} - \{-30 + 10\}$$

$$\overset{\varphi_b}{[1800 + 3000]}\,\hat{M}_b \cdot \{\varphi_b\} = \{20\}\,\hat{M}_b$$

matice tuhosti konstrukce

$$\underline{\varphi_b = 4{,}16 \cdot 10^{-3}\ rad}$$

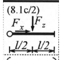
Dopočet koncových sil maticově

– potřebujeme dopočítat vektory a celé sloupečky matic

prut a–b o—|

primární vektor koncových sil 8.1c/2

(8.1c/2)



$$\{R^*_{ab}\} = \begin{Bmatrix} -\frac{1}{2}F_x \\ -\frac{5}{16}F_z \\ 0 \\ -\frac{1}{2}F_x \\ -\frac{11}{16}F_z \\ -\frac{3}{16}F_z l \end{Bmatrix} = \begin{Bmatrix} \bar{X}_{ab} \\ \bar{Z}_{ab} \\ \bar{M}_{ab} \\ \bar{X}_{ba} \\ \bar{Z}_{ba} \\ \boxed{\bar{M}_{ba}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -5 \\ 0 \\ 0 \\ -11 \\ -30 \end{Bmatrix}$$

$$\{R^*_{ab}\} = \{\bar{R}_{ab}\}$$

matice tuhosti

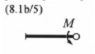
$$[k_{ab}] = [k^*_{ab}] = \begin{array}{c} \begin{array}{cc} u_a\, w_a\, \varphi_a\, u_b\, w_b & \varphi_b \end{array} \\ \left[\begin{array}{cc} & 0 \\ & -180 \\ & 0 \\ & 0 \\ & 180 \\ & \boxed{1800} \end{array}\right] \end{array} \begin{array}{c} \hat{X}_{ab} \\ \hat{Z}_{ab} \\ \hat{M}_{ab} \\ \hat{X}_{ba} \\ \hat{Z}_{ba} \\ \hat{M}_{ba} \end{array}$$

$$[K_{a,b}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \left[\begin{array}{ccc:ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & \frac{3EI}{L^2}s \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hdashline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & -\frac{3EI}{L^2}s \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 & Dcs & \left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & 0 & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & \frac{3EI}{L} \end{array}\right] \end{array} \begin{array}{c} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{array}$$

prut b–c |—o

primární vektor 8.1b/5

(8.1b/5)



$$\{\bar{R}_{bc}\} = \begin{Bmatrix} 0 \\ -\frac{3}{2}\frac{M}{l} \\ \frac{1}{2}M \\ 0 \\ \frac{3}{2}\frac{M}{l} \\ M \end{Bmatrix} = \begin{Bmatrix} \bar{X}_{bc} \\ \bar{Z}_{bc} \\ \boxed{\bar{M}_{bc}} \\ \bar{X}_{cb} \\ \bar{Z}_{cb} \\ \bar{M}_{cb} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -5 \\ 10 \\ 0 \\ 5 \\ 0 \end{Bmatrix}$$

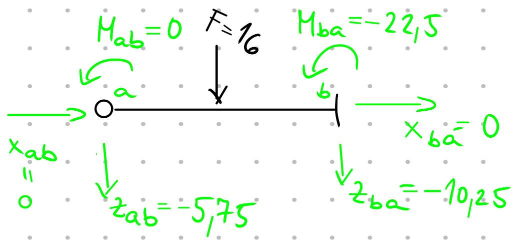
$$\{\bar{R}_{bc}\} = \{\bar{R}^*_{bc}\}$$

$$[k_{bc}] = [k^*_{bc}] = \begin{array}{c} \begin{array}{ccc} u_b\, w_b & \varphi_b & u_c\, w_c\, \varphi_c \end{array} \\ \left[\begin{array}{ccc} & 0 & \\ & -500 & \\ & \boxed{3000} & \\ & 0 & \\ & 500 & \\ & 0 & \end{array}\right] \end{array} \begin{array}{c} \hat{X}_{bc} \\ \hat{Z}_{bc} \\ \hat{M}_{bc} \\ \hat{X}_{cb} \\ \hat{Z}_{cb} \\ \hat{M}_{cb} \end{array}$$

$$[K_{a,b}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \left[\begin{array}{ccc:ccc} \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & \frac{3EI}{L^2}s & -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & 0 \\ Dcs & \left(\frac{EA}{L}-Dc^2\right) & -\frac{3EI}{L^2}c & -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & 0 \\ \frac{3EI}{L^2}s & -\frac{3EI}{L^2}c & \frac{3EI}{L} & -\frac{3EI}{L^2}s & \frac{3EI}{L^2}c & 0 \\ \hdashline -\left(\frac{3EI}{L^3}+Dc^2\right) & -Dcs & -\frac{3EI}{L^2}s & \left(\frac{3EI}{L^3}+Dc^2\right) & Dcs & 0 \\ -Dcs & -\left(\frac{EA}{L}-Dc^2\right) & \frac{3EI}{L^2}c & Dcs & \left(\frac{EA}{L}-Dc^2\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} X_a \\ Z_a \\ M_a \\ X_b \\ Z_b \\ M_b \end{array}$$

$$\{R^*_{ab}\} = \{R_{ab}\} = \{\bar{R}_{ab}\} + \{\hat{R}_{ab}\} = \{\bar{R}_{ab}\} + [k_{ab}]\cdot\{r_{ab}\} \qquad \{r_{ab}\} = \{u_a, w_a, \varphi_a, u_b, w_b, \varphi_b\}^T$$

$$\{R_{ab}\} = \begin{Bmatrix} 0 \\ -5 \\ 0 \\ 0 \\ -11 \\ -30 \end{Bmatrix} + \left[\begin{array}{cc} \cdots\cdots & 0 \\ & -180 \\ & 0 \\ & 0 \\ & 180 \\ & 1800 \end{array}\right] \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4{,}1\overline{6}\cdot 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -5{,}75 \\ 0 \\ 0 \\ -10{,}25 \\ -22{,}5 \end{Bmatrix} \begin{array}{c} X_{ab} \\ Z_{ab} \\ M_{ab} \\ X_{ba} \\ Z_{ba} \\ M_{ba} \end{array}$$



$$\{R_{bc}\} = \begin{Bmatrix} 0 \\ -5 \\ 10 \\ 0 \\ 5 \\ 0 \end{Bmatrix} + \left[\begin{array}{ccc} \cdot\;\cdot & 0 & \cdots \\ & -500 & \\ & 3000 & \\ & 0 & \\ & 500 & \\ & 0 & \end{array}\right] \cdot \begin{Bmatrix} 0 \\ 0 \\ 4{,}1\overline{6}\cdot 10^{-3} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{array}{c} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{array} = \begin{Bmatrix} 0 \\ -7{,}08\overline{3} \\ 22{,}5 \\ 0 \\ 7{,}08\overline{3} \\ 0 \end{Bmatrix} \begin{array}{c} X_{bc} \\ Z_{bc} \\ M_{bc} \\ X_{cb} \\ Z_{cb} \\ M_{cb} \end{array}$$