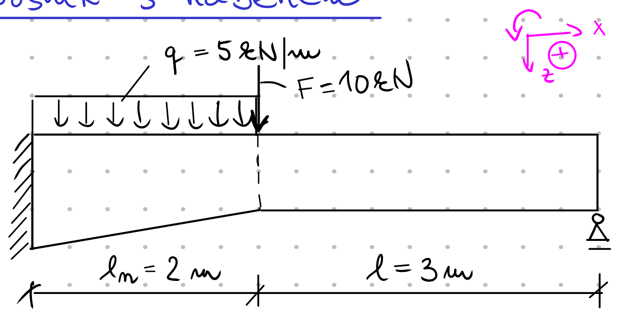
Nosník s náběhem

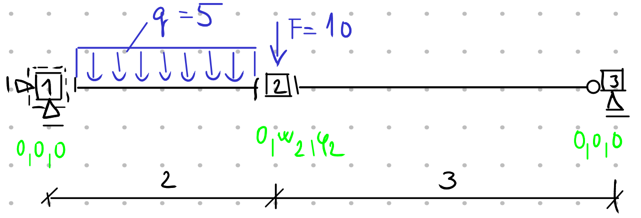


$E = 2\cdot10^{7}\,kPa$

$I = 5\cdot10^{-4}\,m^4$ } obdélníkový průřez

$I_0 = 10^{-4}\,m^4$

$A_0 = 0{,}03\,m^2$



- model $n_p = 2$

$u_2, u_3 = 0$ s ohledem na zatížení

$\varphi_3 = 0$ —o

$r = \begin{Bmatrix} w_2 \\ \varphi_2 \end{Bmatrix}$ $\{S\} = \begin{Bmatrix} 10 \\ 0 \end{Bmatrix}$

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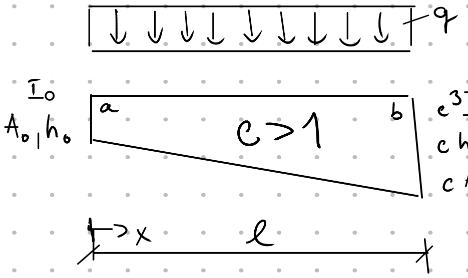
deformační součinitelé prutu (a–b) str. 265

$x = x^*$



$h(x) = \frac{h_0}{l}(l + cx - x)$

- b = konst. a $c > 1$
- osová tuhost

$EA(x) = \frac{EA_0}{l}(l + cx - x)$

- ohybová tuhost

$EI(x) = \frac{EI_0}{l^3}(l + cx - x)^3$

- z Maxwell-Mohrových vztahů

$$\delta_1 = \int_0^l \frac{dx}{EA(x)} = \frac{l}{EA_0}\,\frac{\ln(c)}{c-1}$$

$$\alpha_{ab} = \frac{1}{l^2}\int_0^l \frac{(l-x)^2}{EI(x)}\,dx = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\ln(c) + \frac{c^2}{2} - 2c + \frac{3}{2}\right]$$

$$\alpha_{ba} = \frac{1}{l^2}\int_0^l \frac{x^2}{EI(x)}\,dx = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\ln(c) + \frac{2}{c} - \frac{1}{2c^2} - \frac{3}{2}\right]$$

$$\beta = \frac{1}{l^2}\int_0^l \frac{x(l-x)}{EI(x)}\,dx = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\frac{c}{2} - \ln(c) - \frac{1}{2c}\right]$$

$$\varphi_{ab} = \frac{1}{l}\int_0^l \frac{M(x)(l-x)}{EI(x)}\,dx = \frac{ql^3}{EI_0}\,\frac{1}{2(c-1)^4}\left[\frac{c^2}{2} + 2c - (2c+1)\ln(c) - \frac{5}{2}\right]$$

$$\varphi_{ba} = \frac{1}{l}\int_0^l \frac{M(x)\cdot x}{EI(x)}\,dx = \frac{ql^3}{EI_0}\,\frac{1}{2(c-1)^4}\left[(c+2)\ln c - \frac{5}{2}c + \frac{1}{2c} + 2\right]$$

- pro náš prut - $a = 2, b = 1$

$I = c^3 I_0 \Rightarrow c^3 = \frac{I}{I_0} = \frac{5\cdot10^{-4}}{10^{-4}} = 5 \Rightarrow c = \sqrt[3]{5} \doteq 1{,}709976 \doteq 1{,}71$

$$\delta_1 = \frac{l}{EA_0}\,\frac{\ln(c)}{c-1} = \frac{2}{2\cdot10^7\cdot0{,}03}\cdot\frac{\ln(1{,}71)}{1{,}71-1} = 2{,}519\cdot10^{-6}$$

$$\alpha_{21} = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\ln(c) + \frac{c^2}{2} - 2c + \frac{3}{2}\right] = \frac{2}{2\cdot10^7\cdot10^{-4}(1{,}71-1)^3}\left[\ln(1{,}71) + \frac{1{,}71^2}{2} - 2\cdot1{,}71 + \frac{3}{2}\right] = 219{,}449\cdot10^{-6}$$

$$\alpha_{12} = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\ln(c) + \frac{2}{c} - \frac{1}{2c^2} - \frac{3}{2}\right] = \frac{2}{2\cdot10^7\cdot10^{-4}\cdot(1{,}71-1)^3}\left[\ln(1{,}71) + \frac{2}{1{,}71} - \frac{1}{2\cdot1{,}71^2} - \frac{3}{2}\right] = 98{,}045\cdot10^{-6}$$

$$\beta = \frac{l}{EI_0}\,\frac{1}{(c-1)^3}\left[\frac{c}{2} - \ln(c) - \frac{1}{2c}\right] = \frac{2}{2\cdot10^7\cdot10^{-4}\,(1{,}71-1)^3}\left[\frac{1{,}71}{2} - \ln(1{,}71) - \frac{1}{2\cdot1{,}71}\right] = 72{,}948\cdot10^{-6}$$

$$\varphi_{21}=\frac{ql^3}{EI_0}\,\frac{1}{2(c-1)^4}\left[\frac{c^2}{2}+2c-(2c+1)\ln(c)-\frac{5}{2}\right]=\frac{5\cdot2^3}{2\cdot10^7\cdot10^{-4}\cdot2\,(1,71-1)^4}\cdot\left[\frac{1,71^2}{2}+2\cdot1,71-(2\cdot1,71+1)\cdot\ln(1,71)-\frac{5}{2}\right]$$

$$\varphi_{21}=423,006\cdot10^{-6}$$

$$\varphi_{12}=\frac{ql^3}{EI_0}\,\frac{1}{2(c-1)^4}\left[(c+2)\ln c-\frac{5}{2}c+\frac{1}{2c}+2\right]=\frac{5\cdot2^3}{2\cdot10^7\cdot10^{-4}\cdot2\,(1,71-1)^4}\left[(1,71+2)\ln(1,71)-\frac{5}{2}1,71+\frac{1}{2\cdot1,71}+2\right]$$

$$\varphi_{12}=306,476\cdot10^{-6}$$

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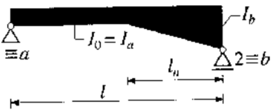
tabulka 14.6 -alternativní výpočet

Tabulka 14.6. Deformační úhly α, β, φ prostého nosníku s jednostranným přímkovým náběhem



$$c=\frac{I_0}{I_b},\quad \lambda=\frac{l_n}{l}$$

Absolutní hodnoty základních deformačních úhlů (obr. 4.8c, d)

$$\alpha_1=\varkappa_1\frac{l}{EI_0},\quad \alpha_2=\varkappa_2\frac{l}{EI_0},\quad \beta=\varkappa_3\frac{l}{EI_0}$$

a úhlů φ_1, φ_2 od spojitého rovnoměrného zatížení q po celé délce nosníku (obr. 4.8c)

$$\varphi_1=k_1^*\frac{ql^3}{EI_0},\quad \varphi_2=k_2^*\frac{ql^3}{EI_0}.$$

c \ λ		0,10	0,20	0,30	0,40	1,00
0,20	$\varkappa_1$	0,3332	0,3324	0,3303	0,3260	0,2195
	$\varkappa_2$	0,2837	0,2417	0,2066	0,1778	0,0980
	$\varkappa_3$	0,1647	0,1593	0,1511	0,1407	0,0730
	k_1^*	0,0416	0,0413	0,0404	0,0390	0,0212
	k_2^*	0,0407	0,0384	0,0351	0,0314	0,0153

$$c=\frac{I_0}{I_b}=\frac{10^{-4}}{5\cdot10^{-4}}=0,2\qquad \lambda=\frac{l_n}{l}=\frac{2}{2}=1$$

$$\alpha_1=\alpha_{21}=\varkappa_1\cdot\frac{l}{EI_0}=0,2195\cdot\frac{2}{2\cdot10^7\cdot10^{-4}}$$

$$\alpha_{21}=219,5\cdot10^{-6}$$

$$\alpha_2=\alpha_{12}=\varkappa_2\frac{l}{EI_0}=0,0980\cdot\frac{2}{2\cdot10^7\cdot10^{-4}}$$

$$\alpha_{12}=98\cdot10^{-6}$$

$$\beta=\varkappa_3\cdot\frac{l}{EI_0}=0,0730\cdot\frac{2}{2\cdot10^7\cdot10^{-4}}$$

$$\beta=73\cdot10^{-6}$$

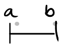
$$\varphi_1=\varphi_{21}=k_1^*\frac{ql^3}{EI_0}=0,0212\cdot\frac{5\cdot2^3}{2\cdot10^7\cdot10^{-4}}=424\cdot10^{-6}$$

$$\varphi_2=\varphi_{12}=k_2^*\frac{ql^3}{EI_0}=0,0153\cdot\frac{5\cdot2^3}{2\cdot10^7\cdot10^{-4}}=306\cdot10^{-6}$$

ⓐ(1–2)

$$D=\alpha_{21}\cdot\alpha_{12}-\beta^2=219,449\cdot10^{-6}\cdot98,045\cdot10^{-6}-(72,948\cdot10^{-6})^2$$

$$D=16,194\cdot10^{-9}$$



str. 255

$$\mathbf{k}^*_{ab}=\left[\begin{array}{ccc|ccc}
\frac{1}{\delta_1} & 0 & 0 & -\frac{1}{\delta_1} & 0 & 0\\
0 & \frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2} & -\frac{\alpha_{ba}+\beta}{Dl} & 0 & -\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2} & -\frac{\alpha_{ab}+\beta}{Dl}\\
0 & -\frac{\alpha_{ba}+\beta}{Dl} & \frac{\alpha_{ba}}{D} & 0 & \frac{\alpha_{ba}+\beta}{Dl} & \frac{\beta}{D}\\
\hline
-\frac{1}{\delta_1} & 0 & 0 & \frac{1}{\delta_1} & 0 & 0\\
0 & -\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2} & \frac{\alpha_{ba}+\beta}{Dl} & 0 & \frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2} & \frac{\alpha_{ab}+\beta}{Dl}\\
0 & -\frac{\alpha_{ab}+\beta}{Dl} & \frac{\beta}{D} & 0 & \frac{\alpha_{ab}+\beta}{Dl} & \frac{\alpha_{ab}}{D}
\end{array}\right], \tag{11.48}$$

kde D je hodnota determinantu soustavy (11.42) resp. (11.45)

$$D=\alpha_{ab}\alpha_{ba}-\beta^2. \tag{11.49}$$

$$[k^*_{12}]=\cancel{\mathbf{k}^*_{ab}}=\left[\begin{array}{ccc|ccc}
u_1 & w_1 & \varphi_1 & u_2 & w_2 & \varphi_2\\
\frac{1}{\delta_1} & 0 & 0 & -\frac{1}{\delta_1} & 0 & 0\\
0 & \overset{k_1}{\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2}} & \overset{-k_3}{-\frac{\alpha_{ba}+\beta}{Dl}} & 0 & \overset{k_1}{-\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2}} & \overset{-k_2}{-\frac{\alpha_{ab}+\beta}{Dl}}\\
0 & \overset{-k_3}{-\frac{\alpha_{ba}+\beta}{Dl}} & \frac{\alpha_{ba}}{D} & 0 & \overset{k_3}{\frac{\alpha_{ba}+\beta}{Dl}} & \overset{k_5}{\frac{\beta}{D}}\\
\hline
-\frac{1}{\delta_1} & 0 & 0 & \frac{1}{\delta_1} & 0 & 0\\
0 & \overset{-k_1}{-\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2}} & \overset{k_3}{\frac{\alpha_{ba}+\beta}{Dl}} & 0 & \overset{k_1}{\frac{\alpha_{ab}+\alpha_{ba}+2\beta}{Dl^2}} & \overset{k_2}{\frac{\alpha_{ab}+\beta}{Dl}}\\
0 & \overset{-k_2}{-\frac{\alpha_{ab}+\beta}{Dl}} & \overset{k_5}{\frac{\beta}{D}} & 0 & \overset{k_2}{\frac{\alpha_{ab}+\beta}{Dl}} & \overset{k_4}{\frac{\alpha_{ab}}{D}}
\end{array}\right], \tag{11.48}$$

kde D je hodnota determinantu soustavy (11.42) resp. (11.45)

$$D=\alpha_{ab}\alpha_{ba}-\beta^2. \tag{11.49}$$

$$k_1=\frac{\alpha_{12}+\alpha_{21}+2\beta}{Dl^2}=\frac{98,045\cdot10^{-6}+219,449\cdot10^{-6}+2\cdot72,948\cdot10^{-6}}{16,194\cdot10^{-9}\cdot2^2}=7153,730$$

$$k_2=\frac{\alpha_{12}+\beta}{Dl}=\frac{98,045\cdot10^{-6}+72,948\cdot10^{-6}}{16,194\cdot10^{-9}\cdot2}=5279,517$$

$$k_3=\frac{\alpha_{21}+\beta}{Dl}=\frac{219,449\cdot10^{-6}+72,948\cdot10^{-6}}{16,194\cdot10^{-9}\cdot2}=9027,942$$

$$k_4=\frac{\alpha_{12}}{D}=\frac{98,045\cdot10^{-6}}{16,194\cdot10^{-9}}=6054,403\qquad k_5=\frac{\beta}{D}=\frac{72,948\cdot10^{-6}}{16,194\cdot10^{-9}}=4504,631$$

$$[k^*_{12}]=\begin{array}{c}
\begin{array}{ccc} u_1\,w_1\,\varphi_1\,u_2 & w_2 & \varphi_2 \end{array}\\
\left[\begin{array}{ccc}
 & 0 & 0\\
 & -7153,73 & -5279,517\\
 & 9027,924 & 4504,631\\
 & 0 & 0\\
 & 7153,73 & 5279,517\\
 & 5279,517 & 6054,403
\end{array}\right]
\end{array}
\begin{array}{c} u_1\\ w_1\\ \varphi_1\\ u_2\\ w_2\\ \varphi_2 \end{array}$$

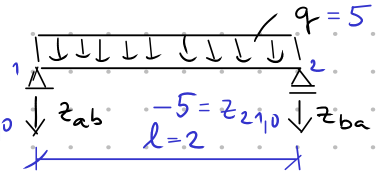
$$[k^*_{23}] = \begin{array}{c|c} \begin{array}{cccc} u_2 & w_2 & \varphi_2 & u_3\,w_3\,\varphi_3 \end{array} & \\ \left[\begin{array}{cccc} & 0 & 0 & \\ & 222,\overline{2} & -666,\overline{6} & \\ & -666,\overline{6} & 2000 & \\ & 0 & 0 & \\ & -222,\overline{2} & 666,\overline{6} & \\ & 0 & 0 & \end{array}\right] & \begin{array}{c} u_2 \\ w_2 \\ \varphi_2 \\ w_3 \\ w_3 \\ \varphi_3 \end{array} \end{array}$$

tab. 8.2 b

$s = 0$

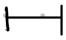
$c = 1$

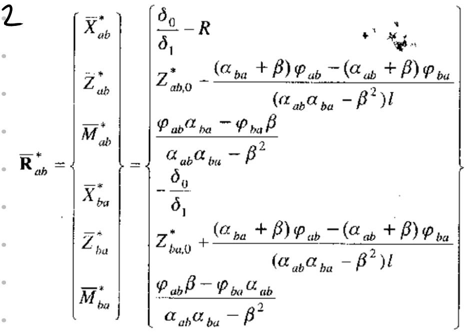
$$[k] = \begin{array}{cc} \begin{array}{cc} w_2 & \varphi_2 \end{array} & \\ \left[\begin{array}{c|c} 7153,73 + 222,\overline{2} & (5279,517 + (-666,\overline{6}) \\ \hline 5279,517 + (-666,\overline{6}) & 6054,403 + 2000 \end{array}\right] & \begin{array}{c} w_2 \\ \varphi_2 \end{array} \end{array} = \begin{bmatrix} 7375,95\overline{2} & 4612,850\overline{3} \\ 4612,850\overline{3} & 8054,403 \end{bmatrix}$$



$z_{12,0}$ $\| $ $-\frac{5\cdot 2}{2} = -5$

str. 252





$$\overline{\mathbf{R}}^*_{ab} = \begin{Bmatrix} \overline{X}^*_{ab} \\ \overline{Z}^*_{ab} \\ \overline{M}^*_{ab} \\ \overline{X}^*_{ba} \\ \overline{Z}^*_{ba} \\ \overline{M}^*_{ba} \end{Bmatrix} = \begin{Bmatrix} \frac{\delta_0}{\delta_1} - R \\ Z^*_{ab,0} - \frac{(\alpha_{ba}+\beta)\varphi_{ab} - (\alpha_{ab}+\beta)\varphi_{ba}}{(\alpha_{ab}\alpha_{ba}-\beta^2)l} \\ \frac{\varphi_{ab}\alpha_{ba} - \varphi_{ba}\beta}{\alpha_{ab}\alpha_{ba}-\beta^2} \\ -\frac{\delta_0}{\delta_1} \\ Z^*_{ba,0} + \frac{(\alpha_{ba}+\beta)\varphi_{ab} - (\alpha_{ab}+\beta)\varphi_{ba}}{(\alpha_{ab}\alpha_{ba}-\beta^2)l} \\ \frac{\varphi_{ab}\beta - \varphi_{ba}\alpha_{ab}}{\alpha_{ab}\alpha_{ba}-\beta^2} \end{Bmatrix}$$

$$\{\overline{R}^*_{12}\} = \begin{Bmatrix} -5 \overset{0}{-} \frac{(219,449+72,948)\cdot 306,476 - (98,045+72,948)\cdot 423,006}{(98,045\cdot 219,449 - 72,948^2)\cdot 2} \\ \frac{306,476\cdot 219,449 - 423,006\cdot 72,948}{98,045\cdot 219,449 - 72,948^2} \\ -5 \overset{0}{+} \frac{(219,449+72,948)\cdot 306,476 - (98,045+72,948)\cdot 423,006}{(98,045\cdot 219,449 - 72,948^2)\cdot 2} \\ \frac{306,476\cdot 72,948 - 423,006\cdot 98,045}{98,045\cdot 219,449 - 72,948^2} \end{Bmatrix}$$

$$\{\overline{R}^*_{12}\} = \{ \overset{u_1}{0} ; \overset{w_1}{-5,533} ; \overset{\varphi_1}{2,248} ; \overset{u_2}{0} ; \boxed{\overset{w_2}{-4,466} ; \overset{\varphi_2}{-1,180}} \}^T$$

$$\{F\} = \{S\} - \{\overline{R}\} = \begin{Bmatrix} 10 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -4,466 \\ -1,180 \end{Bmatrix} = \begin{Bmatrix} 14,466 \\ 1,180 \end{Bmatrix}$$

$$[k]\cdot\{r\} = \{F\}$$

$$\begin{bmatrix} 7375,95\overline{2} & 4612,850\overline{3} \\ 4612,850\overline{3} & 8054,403 \end{bmatrix} \begin{Bmatrix} w_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} 14,466 \\ 1,180 \end{Bmatrix}$$

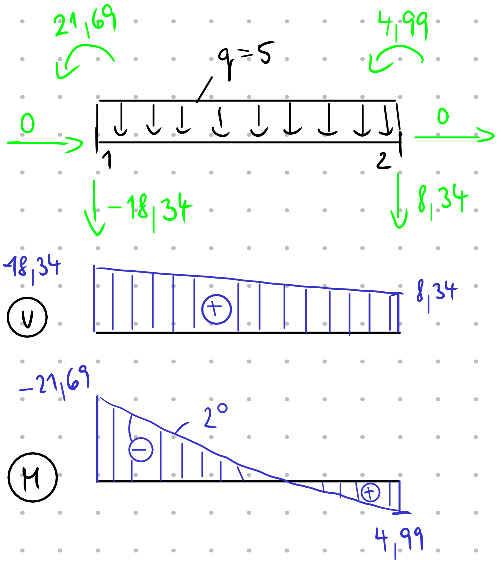
$$\begin{Bmatrix} w_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} 2,91294 \\ -1,52177 \end{Bmatrix} \cdot 10^{-3} \quad \begin{matrix} [m] \\ [rad] \end{matrix}$$

$$\{R_{12}\} = \{\overline{R}_{12}\} + \{\hat{R}_{12}\} = \{\overline{R}_{12}\} + [k_{12}]\cdot\{r_{12}\}$$

$$\{R_{12}\} = \begin{Bmatrix} 0 \\ -5,533 \\ 2,248 \\ 0 \\ -4,466 \\ -1,18 \end{Bmatrix} + \begin{array}{cc} \begin{array}{ccc} u_1\,w_1\,\varphi_1\,u_2 & w_2 & \varphi_2 \end{array} & \\ \left[\begin{array}{ccc} & 0 & 0 \\ & -7153,73 & -5279,517 \\ & 9027,924 & 4504,631 \\ & 0 & 0 \\ & 7153,73 & 5279,517 \\ & 5279,517 & 6054,403 \end{array}\right] & \begin{array}{c} u_1 \\ w_1 \\ \varphi_1 \\ u_2 \\ w_2 \\ \varphi_2 \end{array} \end{array} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2,91294 \\ -1,52117 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 0 \\ -18,34 \\ 21,69 \\ 0 \\ 8,34 \\ 4,99 \end{Bmatrix}$$

$$\{R_{23}\} = \{\bar{R}_{23}\} + \{\hat{R}_{23}\} = \{\underset{=\emptyset}{\bar{R}_{23}}\} + [k_{23}]\{r_{23}\}$$

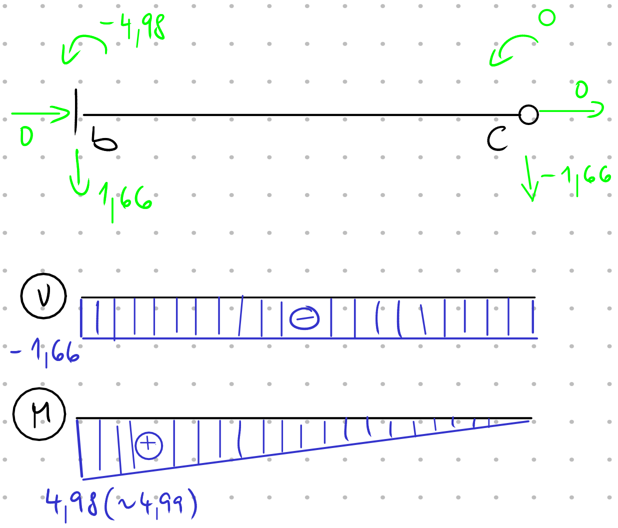
$$\{R_{23}\} = \left[\begin{array}{cccccc|c} u_2 & w_2 & \varphi_2 & u_3 & w_3 & \varphi_3 & \\ & 0 & 0 & & & & u_2 \\ & 222,\bar{2} & -666,\bar{6} & & & & w_2 \\ & -666,\bar{6} & 2000 & & & & \varphi_2 \\ & 0 & 0 & & & & u_3 \\ & -222,\bar{2} & 666,\bar{6} & & & & w_3 \\ & 0 & 0 & & & & \varphi_3 \end{array}\right] \cdot \begin{Bmatrix} 0 \\ 2,91294 \\ -1,52117 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 0 \\ 1,66 \\ -4,98 \\ 0 \\ -1,66 \\ 0 \end{Bmatrix}$$



$\Sigma F_{xi} = 0$ ✓

$\Sigma F_{zi} = 0 \oplus\downarrow \quad -18,34 + 5\cdot 2 + 8,34 = 0$ ✓

$\Sigma M_{ai} = 0 \oplus\circlearrowleft \quad 21,69 - 5\cdot 2\cdot 1 + 4,99 - 8,34\cdot 2 = 0$ ✓

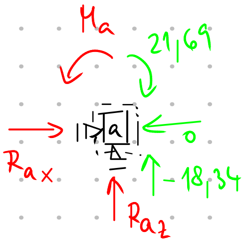


$\Sigma F_{xi} = 0$ ✓

$\Sigma F_{zi} = 0$ ✓

$\Sigma M_{bi} = 0 \quad \oplus\circlearrowleft$

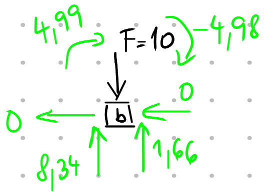
$-4,98 - (-1,66)\cdot 3 = 0$ ✓



$\Sigma F_{xi} = 0 \Rightarrow R_{ax} = 0$

$\Sigma F_{zi} = 0 \Rightarrow R_{az} = 18,34\,kN$

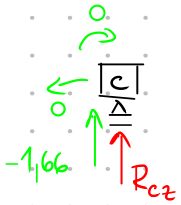
$\Sigma M_{ai} = 0 \Rightarrow M_a = 21,69\,kNm$



$\Sigma F_{xi} = 0$ ✓

$\Sigma F_{zi} = 0$ ✓

$\Sigma M_{bi} = 0$ ✓



$\Sigma F_{xi} = 0$ ✓

$\Sigma M_{ci} = 0$ ✓

$\Sigma F_{zi} = 0$

$\Downarrow$

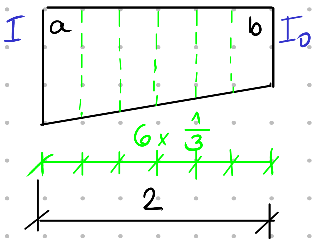
$R_{cz} = 1,66\,kN$

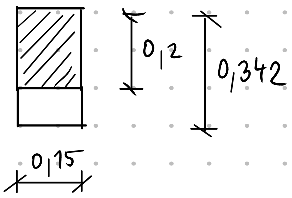
- numerická integrace

$$\int_a^b y\,dx \approx \frac{b-a}{3n}\left(y_a + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 2y_{n-2} + 4y_{n-1} + y_n\right)$$

n - sudé

↳ Simpson





$$\delta_1 = \int_0^l \frac{dx}{EA(x)} \qquad h(x) = 0{,}342 - \frac{(0{,}342-0{,}2)}{2}\cdot x = 0{,}342 - 0{,}071x$$

i	x_i	h_i	A_i	$1/A_i$
0	0	0,342	0,0513	
1	1/3	$0{,}318\overline{3}$	0,04775	
2	2/3	$0{,}294\overline{6}$	0,0442	
3	1	0,271	0,04065	
4	4/3	$0{,}247\overline{3}$	0,0371	
5	5/3	$0{,}223\overline{6}$	0,03355	
$n=6$	2	0,2	0,03	

$$\delta_1 = \frac{1}{E}\int_0^l \frac{1}{A(x)}\,dx \approx \frac{1}{E}\,\frac{\overset{=2}{l}-0}{3\cdot 6}\left[\frac{1}{0{,}0513} + \frac{4}{0{,}04775} + \frac{2}{0{,}0442} + \frac{4}{0{,}04065} + \frac{2}{0{,}0371} + \frac{4}{0{,}03355} + \frac{1}{0{,}03}\right]$$

$$\delta_1 = \underline{2{,}51877\cdot 10^{-6}}$$