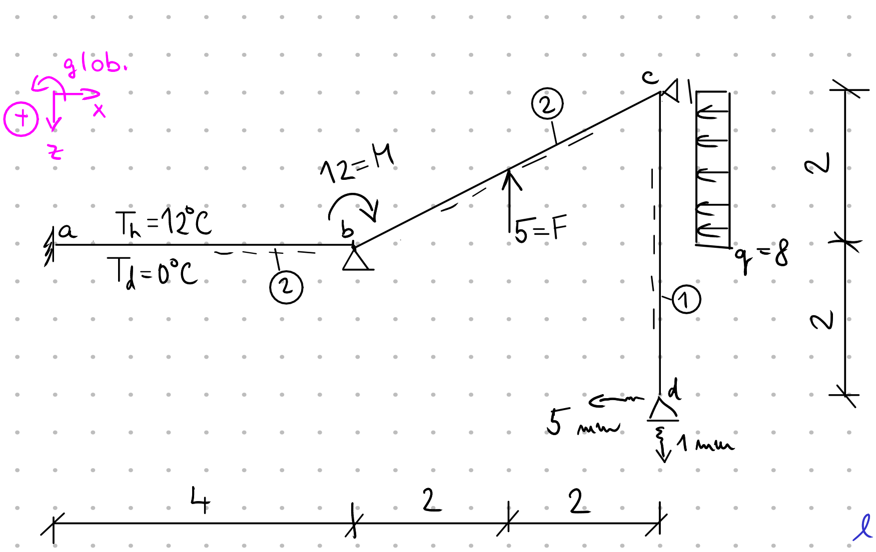
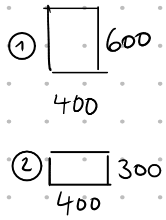


[mm]



$A = 0{,}4 \cdot 0{,}6 = 0{,}24\ m^2$

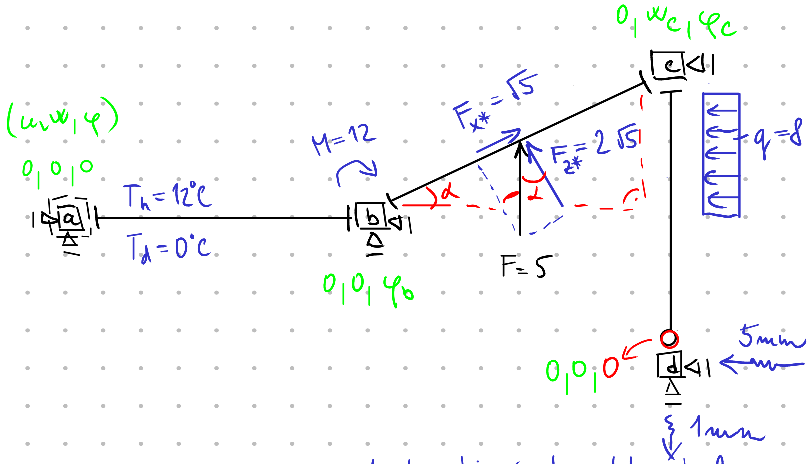
$I_y = \frac{1}{12} b h^3 = \frac{1}{12} \cdot 0{,}4 \cdot 0{,}6^3 = 7{,}2 \cdot 10^{-3}\ m^4$

$A = 0{,}4 \cdot 0{,}3 = 0{,}12\ m^2$

$I_y = \frac{1}{12}\, 0{,}4 \cdot 0{,}3^3 = 0{,}9 \cdot 10^{-3}\ m^4$

$E = 24\ GPa = 24 \cdot 10^6\ kPa$

$\alpha_T = 10^{-5}\ °C^{-1}$



$l_{bc} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}\ m$

$\sin\alpha = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5} \qquad \cos\alpha = \frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$F_{x*} = F \sin\alpha = 5\,\frac{\sqrt{5}}{5} = \sqrt{5}$

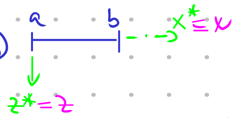
$F_{z*} = F \cos\alpha = 5 \cdot \frac{2\sqrt{5}}{5} = 2\sqrt{5}$

1) model, $n_p = 3$

$\{r\} = \begin{Bmatrix} \varphi_b \\ w_c \\ \varphi_c \end{Bmatrix} \qquad \{S\} = \begin{Bmatrix} -12 \\ 0 \\ 0 \end{Bmatrix}$

2) globální matice tuhosti jednotlivých prutů

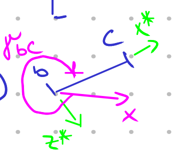
a-b



; $\gamma_{ab} = 0$; $s_{ab} = 0$, $c_{ab} = 1$; tab 8.2a ; $E_{ab} = 24 \cdot 10^6\ kPa$; $A_{ab} = 0{,}12\ m^2$; $I_{ab} = 0{,}9 \cdot 10^{-3}\ m^4$
$l_{ab} = 4\ m$

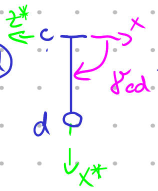
$$[k_{ab}] = \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \begin{bmatrix} & & & & & 0 \\ & & & & & -8{,}1 \\ & & & & & 10{,}8 \\ & & & & & 0 \\ & & & & & 8{,}1 \\ & & & & & \boxed{21{,}6} \end{bmatrix} \end{array} \begin{array}{c} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{array} \cdot 10^3$$

b-c



$\gamma_{bc} = 270°$; $s_{bc} = -\frac{\sqrt{5}}{5}$; $c_{bc} = \frac{2\sqrt{5}}{5}$; tab 8.2a ; $E_{bc} = 24 \cdot 10^6\ kPa$; $A_{bc} = 0{,}12\ m^2$; $I_{bc} = 0{,}9 \cdot 10^{-3}\ m^4$
$l_{bc} = 2\sqrt{5}\ m$

$$[k_{bc}] = \begin{array}{c} \begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \end{array} \\ \begin{bmatrix} & & -2{,}897944 & & 256{,}435853 & -2{,}897944 \\ & & -5{,}795888 & & -131{,}115871 & -5{,}795888 \\ & & \boxed{19{,}319627} & & \boxed{5{,}795888} & \boxed{9{,}659814} \\ & & 2{,}897944 & & -256{,}435853 & 2{,}897944 \\ & & \boxed{5{,}795888} & & \boxed{131{,}115871} & \boxed{5{,}795888} \\ & & \boxed{9{,}659814} & & \boxed{5{,}795888} & \boxed{19{,}319627} \end{bmatrix} \end{array} \begin{array}{c} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{array} \cdot 10^3$$

(c-d) 

$\gamma_{cd} = 90°$; $s_{cd} = 1$; $c_{cd} = 0$; tab 8.2b ; $E_{cd} = 24\cdot10^{6}$ kPa ; $A_{cd} = 0{,}24\,m^2$; $I_{cd} = 7{,}2\cdot10^{-3}\,m^4$

$l_{cd} = 4\,m$

pro výpočet $\tilde{R}_{cd}$

$$[k_{cd}] = \begin{array}{cccccc} u_c & w_c & \varphi_c & u_d & w_d & \varphi_d \end{array}$$

$$[k_{cd}] = \left[\begin{array}{cccc} 0 & 32{,}4 & -8{,}1 & 0 \\ \boxed{1440} & \boxed{0} & 0 & -1440 \\ \boxed{0} & \boxed{129{,}6} & -32{,}4 & 0 \\ 0 & -32{,}4 & 8{,}1 & 0 \\ -1440 & 0 & 0 & 1440 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{c} u_c \\ w_c \\ \varphi_c \\ u_d \\ w_d \\ \varphi_d \end{array} \cdot 10^3$$

3) globální matice tuhosti soustavy - pro sestavení této matice a výpočet neznámých deformací potřebujeme pouze čísla $\boxed{}$

$$[k] = \begin{array}{c} \begin{array}{ccc} \varphi_b & w_c & \varphi_c \end{array} \\ \left[\begin{array}{ccc} 19{,}319627 + 21{,}6 & 5{,}795888 & 9{,}659814 \\ 5{,}795888 & 131{,}115871 + 1440 & 5{,}795888 + 0 \\ 9{,}659814 & 5{,}795888 + 0 & 19{,}319627 + 129{,}6 \end{array}\right] \end{array} \begin{array}{c} \varphi_b \\ w_c \\ \varphi_c \end{array} \cdot 10^3$$

$$[k] = \left[\begin{array}{ccc} 40{,}919627 & 5{,}795888 & 9{,}659814 \\ 5{,}795888 & 1571{,}115871 & 5{,}795888 \\ 9{,}659814 & 5{,}795888 & 148{,}919627 \end{array}\right] \cdot 10^3$$

4) primární vektory koncových sil

a $|$ $T_h = 12°C$ / $T_d = 0°C$ $|$ b

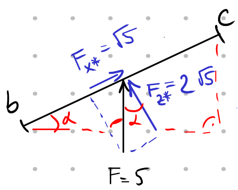
$\Delta T_0 = (T_d + T_h)/2 = 6°C$ $\quad h_{ab} = 0{,}3\,m$

$\Delta T_1 = T_d - T_h = -12°C$

(a-b) tab. 8.1a/10 $\quad E_{ab} = 24\cdot10^6$ kPa; $A_{ab} = 0{,}12\,m^2$; $I_{ab} = 0{,}9\cdot10^{-3}\,m^4$; $l_{ab} = 4\,m$

$$\{\bar{R}^*_{ab}\} = \{\bar{R}_{ab}\} = \{\underset{u_a}{172{,}8};\ \underset{w_a}{0};\ \underset{\varphi_a}{-8{,}64};\ \underset{u_b}{-172{,}8};\ \underset{w_b}{0};\ \underset{\varphi_b}{\boxed{8{,}64}}\}^T$$

(b-c) tab. 8.1a/2 $\quad s_{bc} = -\frac{\sqrt{5}}{5}$; $c_{bc} = \frac{2\sqrt{5}}{5}$; tab 8.2a ; $E_{bc} = 24\cdot10^6$ kPa ; $A_{bc} = 0{,}12\,m^2$; $I_{bc} = 0{,}9\cdot10^{-3}\,m^4$, $l_{bc} = 2\sqrt{5}\,m$



$$\{\bar{R}^*_{bc}\} = \{\underset{u_b}{-\frac{\sqrt{5}}{2}};\ \underset{w_b}{-\frac{(-2\sqrt{5})}{2}};\ \underset{\varphi_b}{\boxed{\frac{(-2\sqrt{5})\cdot2\sqrt{5}}{8}}};\ \underset{u_c}{-\frac{\sqrt{5}}{2}};\ \underset{w_c}{\boxed{+\frac{2\sqrt{5}}{2}}};\ \underset{\varphi_c}{\boxed{\frac{2\sqrt{5}\cdot2\sqrt{5}}{8}}}\}^T$$

$$\{\bar{R}^*_{bc}\} = \{-1{,}118034;\ 2{,}236068;\ \boxed{-2{,}5};\ -1{,}118034;\ \boxed{2{,}236068};\ \boxed{2{,}5}\}^T$$

$$\{\bar{R}_{bc}\} = [T_{bc}]^T\{\bar{R}^*_{bc}\} = \left[\begin{array}{cccccc} 0{,}894427 & +0{,}447214 & 0 & & & \\ -0{,}447214 & 0{,}894427 & 0 & & & \\ 0 & 0 & 1 & & & \\ & & & 0{,}894427 & -0{,}447214 & 0 \\ & & & 0{,}447214 & 0{,}894427 & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \left\{\begin{array}{c} -1{,}118034 \\ 2{,}236068 \\ -2{,}5 \\ -1{,}118034 \\ 2{,}236068 \\ 2{,}5 \end{array}\right\} = \left\{\begin{array}{c} 0 \\ 2{,}5 \\ \boxed{-2{,}5} \\ 0 \\ \boxed{2{,}5} \\ \boxed{2{,}5} \end{array}\right\}$$

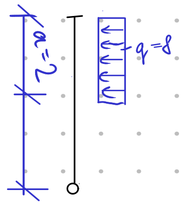
$$[T] = \left[\begin{array}{cccc} c & s & 0 & \\ -s & c & 0 & \\ 0 & 0 & 1 & \\ & & & 1 \end{array}\right]$$

(c-d) ⊢○ tab. 8.1b|7 $s_{cd}=1$; $c_{cd}=0$; tab 8.2b; $E_{cd}=24\cdot10^6\,kPa$; $A_{cd}=0{,}24\,m^2$; $I_{cd}=7{,}2\cdot10^{-3}\,m^4$; $\ell_{cd}=4\,m$

$$\{\bar R_{cd}\}=\{\bar R_{cd}\}_q+\{\tilde R_{cd}\}=\{\bar R_{cd}\}_q+[k_{cd}]\cdot\{\tilde r_{cd}\}$$

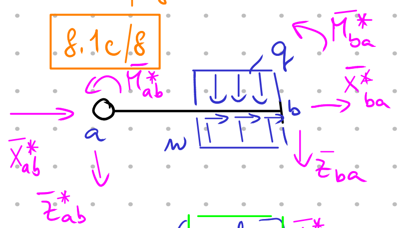
$$\begin{array}{cccccccc} & & u_c & w_c & \varphi_c & u_d & w_d & \varphi_d \\ \{\bar R^*_{cd}\}_q = & \{ & 0; & -14{,}25; & ; & 0; & ; & 0\}^T\end{array}$$

$$-\frac{8\cdot2\cdot4^3-4\cdot2^3\cdot4+2^4}{8\cdot4^3}\cdot8$$

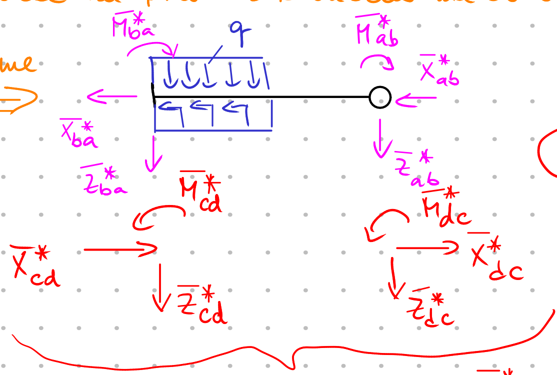


Pozn. spojité zatížení u tuhého konce na prutu s kloubem nalezneme v tabulce

8.1c/8



překlopíme



My chceme

$$\{\bar R^*_{ab}\}=\begin{Bmatrix}-\frac{ml}{8}\\ -\frac{7}{128}ql\\ 0\\ -\frac{3}{8}ml\\ -\frac{57}{128}ql\\ -\frac{9}{128}ql^2\end{Bmatrix}\begin{matrix}\bar X^*_{ab}\\ \bar Z^*_{ab}\\ \bar M^*_{ab}\\ \bar X^*_{ba}\\ \bar Z^*_{ba}\\ \bar M^*_{ba}\end{matrix}$$

$\bar X^*_{cd}=-\bar X^*_{ba}$ $\qquad$ $\bar X^*_{dc}=-\bar X^*_{ab}$

$\bar Z^*_{cd}=\bar Z^*_{ba}$ $\qquad$ $\bar Z^*_{dc}=\bar Z^*_{ab}$

$\bar M^*_{cd}=-\bar M^*_{ba}$ $\qquad$ $\bar M^*_{dc}=-\bar M^*_{ab}$

$$\{\bar R^*_{cd}\}=\begin{Bmatrix}\ominus\\ \oplus\\ \ominus\\ \ominus\\ \oplus\\ \ominus\end{Bmatrix}=\begin{Bmatrix}\frac{3}{8}ml\\ -\frac{57}{128}ql\\ \frac{9}{128}ql^2\\ ml/8\\ -\frac{7}{128}ql\\ 0\end{Bmatrix}$$

pro q↓ m←

další možnost tab 8.1b/6 − 8.1b/8

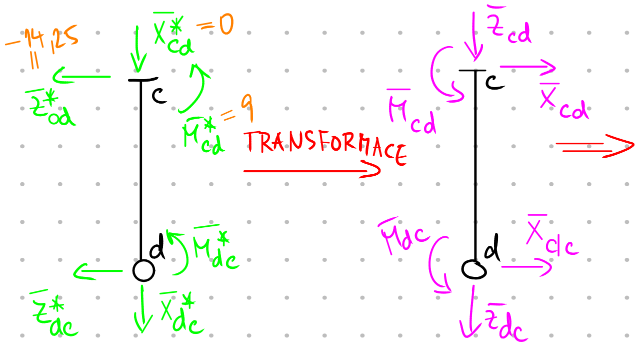
$$\{\bar R^*_{cd}\}=\begin{Bmatrix}-\frac{1}{2}ml+\frac{1}{8}ml\\ -\frac{5}{8}ql+\frac{23}{128}ql\\ \frac{1}{8}ql^2-\frac{7}{128}ql^2\\ \vdots\end{Bmatrix}\begin{matrix}\\ =-\frac{5}{8}\cdot8\cdot4+\frac{23}{128}\cdot8\cdot4=-14{,}25\ ✓\\ =\frac{1}{8}8\cdot4^2-\frac{7}{128}8\cdot4^2=9\ ✓\\ \\ \end{matrix}$$

$$\begin{array}{cccccccc} & & u_c & w_c & \varphi_c & u_d & w_d & \varphi_d \\ \{\bar R^*_{cd}\}_q = & \{ & 0; & \boxed{-14{,}25; & 9}; & 0; & -1{,}75; & 0\}^T\end{array}$$

$$-\frac{57}{128}\cdot8\cdot4 \qquad \frac{9}{128}\cdot8\cdot4^2$$

$$\{\bar R_{cd}\}_q=[T_{cd}]^T\{\bar R^*_{cd}\}_q=\begin{bmatrix}0&-1&0&&&\\1&0&0&&&\\0&0&1&&&\\&&&0&-1&0\\&&&1&0&0\\&&&0&0&1\end{bmatrix}\begin{Bmatrix}0\\-14{,}25\\16\\0\\-1{,}75\\0\end{Bmatrix}\begin{matrix}u_c\\w_c\\\varphi_c\\u_d\\w_d\\\varphi_d\end{matrix}=\begin{Bmatrix}+14{,}25\\0\\9\\+1{,}75\\0\\0\end{Bmatrix}$$

Pozn. u prutů s $\gamma=90°,180°,270°$ můžeme provést transformaci pomocí náčrtku



$\bar X_{cd}=-\bar Z^*_{cd}=-(-14{,}25)=14{,}25$

$\bar Z_{cd}=\bar X^*_{cd}=0$

$\bar M_{cd}=\bar M^*_{cd}=9$

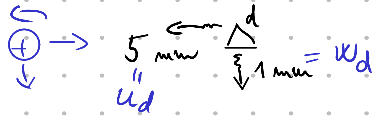
$\bar X_{dc}=-\bar Z^*_{dc}$

$\bar Z_{dc}=\bar X^*_{dc}$

$\bar M_{dc}=\bar M^*_{dc}$

$\{\tilde{R}_{cd}\}$ – vyvolaný globální primární vektor od zadaných popuštění podpor
– určíme přímo globální vektor (matici máme, popuštění podpor je v globálních směrech)
– lokální vektor nepotřebujeme (nemáme lokální matici k^*_{cd} ; museli bychom provést transformaci jak k_{cd} tak $\tilde{r}_{cd}$)

vektor daných přemístění $\{\tilde{r}_{cd}\} = \{u_c \mid w_c \mid \varphi_c \mid u_d \mid w_d \mid \varphi_d\}^T$



$$\{\tilde{r}_{cd}\} = \{0 \mid 0 \mid 0 \mid -5\cdot10^{-3} \,;\, 10^{-3} \,;\, 0\}^T$$

$$\{\tilde{R}_{cd}\} = [k_{cd}]\cdot\{\tilde{r}_{cd}\} = \begin{array}{c} \begin{array}{cccccc} u_c & w_c & \varphi_c & u_d & w_d & \varphi_d \end{array} \\ \begin{bmatrix} & & & -8{,}1 & 0 & \\ & & & 0 & -1440 & \\ & & & -32{,}4 & 0 & \\ & & & 8{,}1 & 0 & \\ & & & 0 & 1440 & \\ & & & 0 & 0 & \end{bmatrix} \end{array} \cdot 10^3 \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -5\cdot10^{-3} \\ 10^{-3} \\ 0 \end{Bmatrix} \begin{array}{c} u_c \\ w_c \\ \varphi_c \\ u_d \\ w_d \\ \varphi_d \end{array} = \begin{Bmatrix} 40{,}5 \\ \boxed{-1440} \\ \boxed{162} \\ -40{,}5 \\ 1440 \\ 0 \end{Bmatrix}$$

$$\{\bar{R}_{cd}\} = \{\bar{R}_{cd}\}_q + \{\tilde{R}_{cd}\} = \{\overset{u_c}{54{,}75} \mid \boxed{\overset{w_c}{-1440} ;\ \overset{\varphi_c}{171}} \mid \overset{u_d}{-38{,}75} \,;\, \overset{w_d}{1440} \,;\, \overset{\varphi_d}{0}\}^T$$

5) vektor pravé strany $\{F\} = \{S\} - \{\bar{R}\}$

$$\{F\} = \begin{array}{c} \varphi_b \\ w_c \\ \varphi_c \end{array} \begin{Bmatrix} -12 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} \overset{\bar{R}_{ab}}{8{,}64} + \overset{\bar{R}_{bc}}{(-2{,}5)} + \overset{\bar{R}_{cd}}{0} \\ 0 + 2{,}5 + (-1440) \\ 0 + 2{,}5 + 171 \end{Bmatrix} = \begin{Bmatrix} -18{,}14 \\ 1437{,}5 \\ -173{,}5 \end{Bmatrix}$$

6) řešení soustavy $[K]\cdot\{r\} = \{F\}$

$$\begin{bmatrix} 40{,}919627 & 5{,}795888 & 9{,}659814 \\ 5{,}795888 & 1571{,}115871 & 5{,}795888 \\ 9{,}659814 & 5{,}795888 & 148{,}919627 \end{bmatrix} \cdot 10^3 \begin{Bmatrix} \varphi_b \\ w_c \\ \varphi_c \end{Bmatrix} = \begin{Bmatrix} -18{,}14 \\ 1437{,}5 \\ -173{,}5 \end{Bmatrix}$$

$$\begin{Bmatrix} \varphi_b \\ w_c \\ \varphi_c \end{Bmatrix} = \begin{Bmatrix} -0{,}2947 \\ 0{,}9204 \\ -1{,}18176 \end{Bmatrix} \cdot 10^{-3} \begin{array}{l} \text{rad} \\ \text{m} \\ \text{rad} \end{array}$$

7) výpočet koncových sil (primární + sekundární)

$$\{R_{ab}\} = \{\bar{R}_{ab}\} + \{\hat{R}_{ab}\} = \{\bar{R}_{ab}\} + [k_{ab}]\cdot\{r_{ab}\}$$

$$\{R_{ab}\} = \begin{Bmatrix} 172{,}8 \\ 0 \\ -8{,}64 \\ -172{,}8 \\ 0 \\ 8{,}64 \end{Bmatrix} + \begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \begin{bmatrix} & & & & & 0 \\ & & & & & -8{,}1 \\ & & & & & 10{,}8 \\ & & & & & 0 \\ & & & & & 8{,}1 \\ & & & & & 21{,}6 \end{bmatrix} \end{array} \begin{array}{c} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{array} \cdot 10^3 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0{,}2947 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 172{,}8 \\ 2{,}38707 \\ -11{,}82276 \\ -172{,}8 \\ -2{,}38707 \\ 2{,}27448 \end{Bmatrix}$$

$$\{R_{ab}\} = \{R^*_{ab}\}$$

$$\{R_{bc}\} = \{\bar{R}_{bc}\} + \{\hat{R}_{bc}\} = \{\bar{R}_{bc}\} + [k_{bc}] \cdot \{r_{bc}\}$$

$$\{R_{bc}\} = \begin{Bmatrix} 0 \\ 2{,}5 \\ -2{,}5 \\ 0 \\ 2{,}5 \\ 2{,}5 \end{Bmatrix} + \begin{array}{c} \begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \end{array} \\ \begin{bmatrix} & & -2{,}897944 & & 256{,}435853 & -2{,}897944 \\ & & -5{,}795888 & & -131{,}115871 & -5{,}795888 \\ & & 19{,}319627 & & 5{,}795888 & 9{,}659814 \\ & & 2{,}897944 & & -256{,}435853 & 2{,}897944 \\ & & 5{,}795888 & & 131{,}115871 & 5{,}795888 \\ & & 9{,}659814 & & 5{,}795888 & 19{,}319627 \end{bmatrix} \end{array} \begin{array}{c} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{array} \cdot 10^3 \cdot \begin{Bmatrix} 0 \\ 0 \\ -0{,}2947 \\ 0 \\ 0{,}9204 \\ -1{,}18176 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 240{,}302 \\ -109{,}622 \\ -14{,}275 \\ -240{,}302 \\ 114{,}622 \\ -17{,}843 \end{Bmatrix}$$

$$\{R^*_{bc}\} = [T_{bc}] \cdot \{\bar{R}_{bc}\} = \left[\begin{array}{ccc|ccc} 0{,}894427 & -0{,}447214 & 0 & & & \\ +0{,}447214 & 0{,}894427 & 0 & & & \\ 0 & 0 & 1 & & & \\ \hline & & & & & \\ & & & & & \\ & & & & & \end{array}\right] \begin{Bmatrix} 240{,}302 \\ -109{,}622 \\ -14{,}275 \\ -240{,}302 \\ 114{,}622 \\ -17{,}843 \end{Bmatrix} = \begin{Bmatrix} 263{,}957 \\ 9{,}418 \\ -14{,}275 \\ -266{,}194 \\ -4{,}946 \\ -17{,}843 \end{Bmatrix}$$

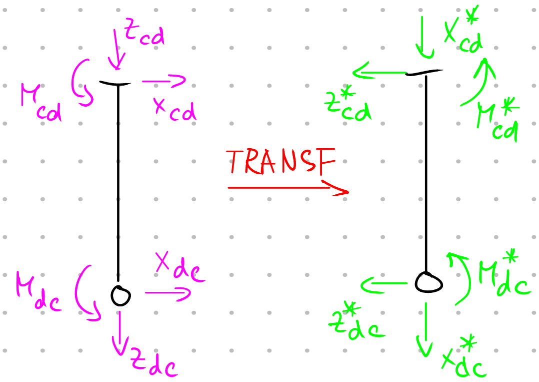
$s_{bc} = -\frac{\sqrt{5}}{5}$; $c_{bc} = \frac{2\sqrt{5}}{5}$

$$T_{sub} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

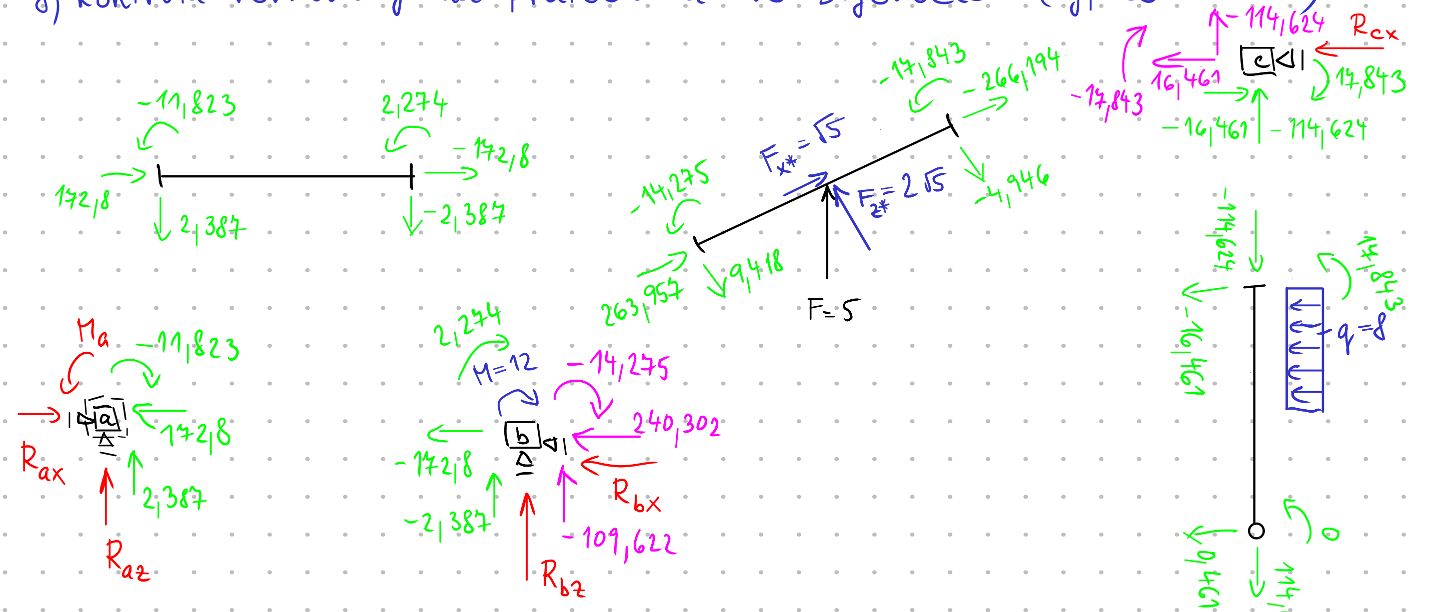
$$\{R_{cd}\} = \{\bar{R}_{cd}\} + \{\hat{R}_{cd}\} = \{\bar{R}_{cd}\} + [k_{cd}] \cdot \{r_{cd}\}$$

$$\{R_{cd}\} = \begin{Bmatrix} 54{,}75 \\ -1440 \\ 141 \\ -38{,}75 \\ 1440 \\ 0 \end{Bmatrix} + \begin{array}{c} \begin{array}{cccccc} u_c & w_c & \varphi_c & u_d & w_d & \varphi_d \end{array} \\ \begin{bmatrix} & 0 & 32{,}4 & & & \\ & 1440 & 0 & & & \\ & 0 & 129{,}6 & & & \\ & 0 & -32{,}4 & & & \\ & -1440 & 0 & & & \\ & 0 & 0 & & & \end{bmatrix} \end{array} \begin{array}{c} u_c \\ w_c \\ \varphi_c \\ u_d \\ w_d \\ \varphi_d \end{array} \cdot 10^3 \begin{Bmatrix} 0 \\ 0{,}9204 \\ -1{,}18176 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 16{,}461 \\ -114{,}624 \\ 17{,}843 \\ -0{,}461 \\ 114{,}624 \\ 0 \end{Bmatrix}$$

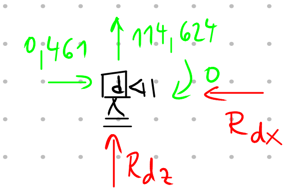
$$\{R^*_{cd}\} = \begin{Bmatrix} -114{,}624 \\ -16{,}461 \\ 17{,}843 \\ 114{,}624 \\ +0{,}461 \\ 0 \end{Bmatrix}$$



8) kontrola rovnováhy na prutech a ve styčnících (výpočet reakcí)



– provedeme kontrolu a výpočet reakcí pomocí silových a momentových podmínek rovnováhy $\Sigma F = 0$ $\Sigma M = 0$



9) vykreslení vnitřních sil