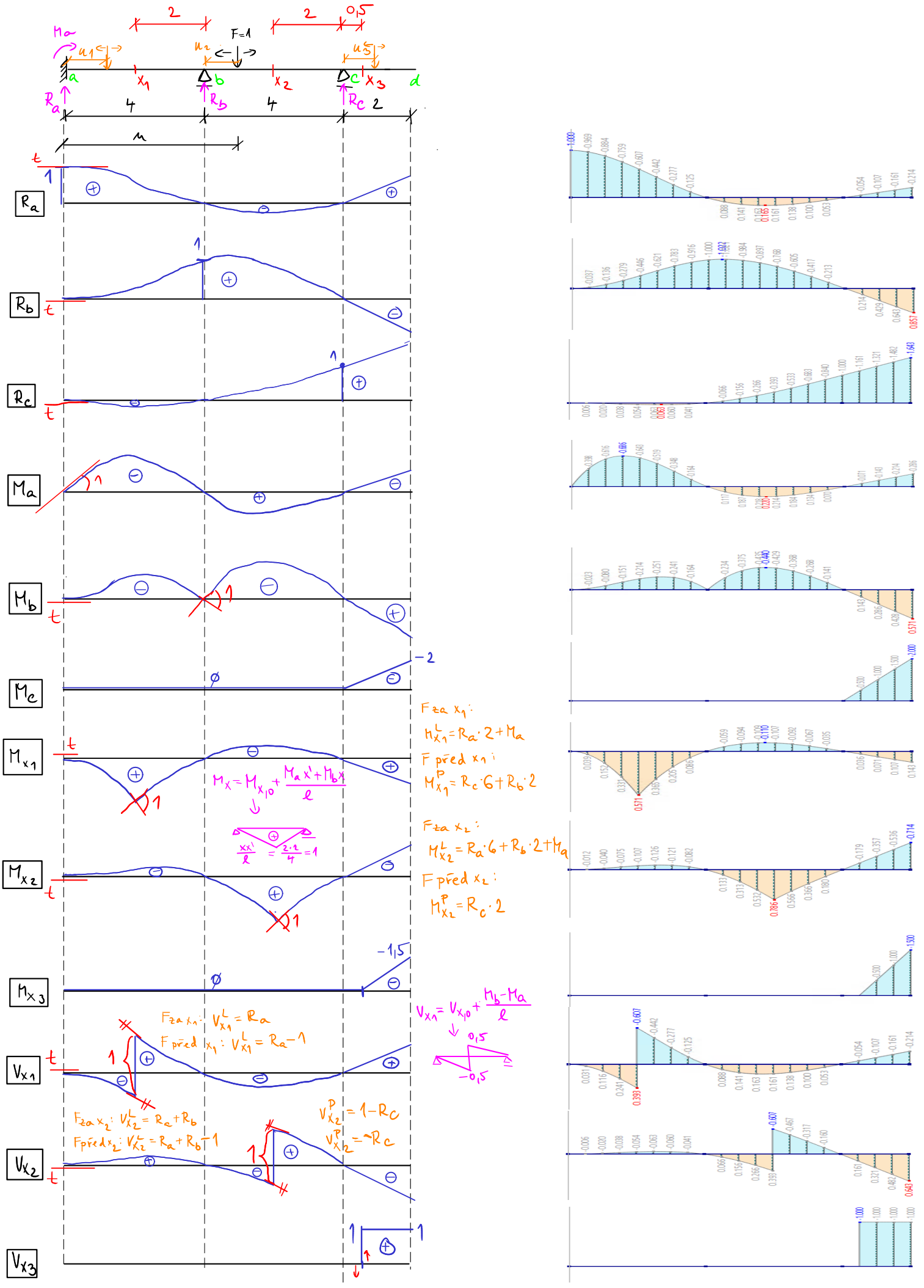
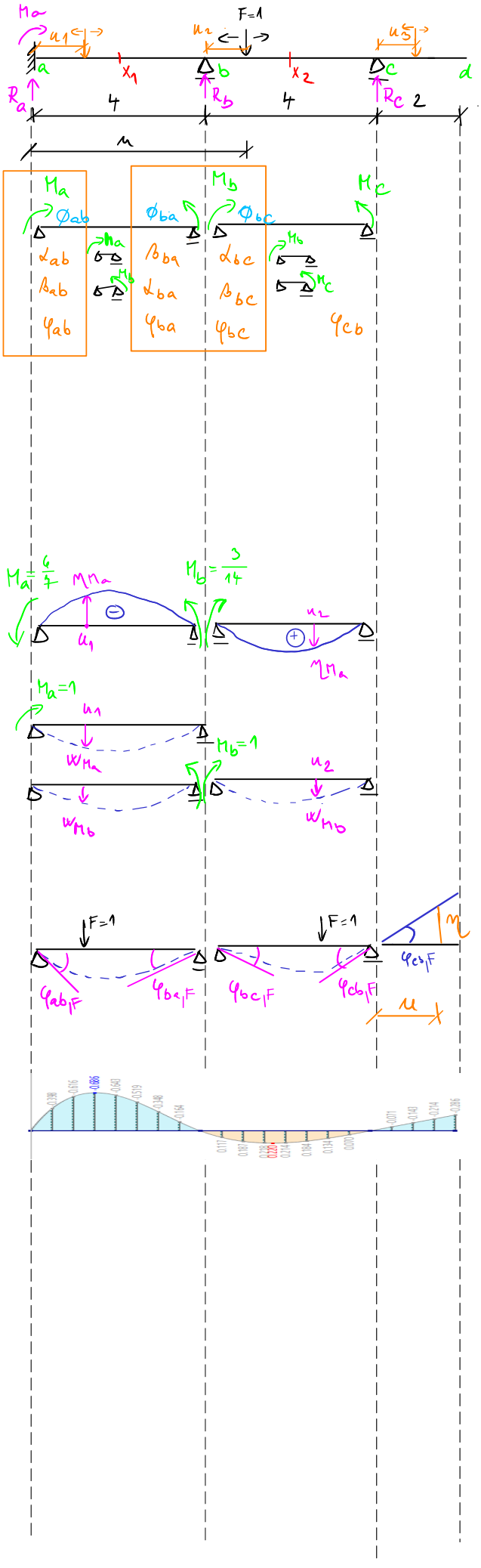
Příčinkové čáry (PČ) na složitějších staticky neurčitých (SN) kcích



Výpočet spojitého nosníku - M_a



$EI = konst.$

$$\varphi_{ab} = -1$$

$$\alpha_{ab} \cdot M_a + \beta_{ab} M_b = -1$$

$$\varphi_{ba} = -\varphi_{bc}$$

$$\beta_{ba} M_a + (\alpha_{bc} + \alpha_{ba}) M_b + \beta_{bc} M_c = 0$$

$$\alpha_{ab} = \frac{l}{3EI} = \frac{4}{3EI} = \alpha_{ba} = \alpha_{bc} = \alpha_{cb}$$

$$\beta_{ab} = \frac{l}{6EI} = \frac{2}{3EI} = \beta_{ba} = \beta_{bc} = \beta_{cb}$$

$$\left.\begin{aligned} \frac{4}{3} M_a + \frac{2}{3} M_b &= -1 \\ \frac{2}{3} M_a + \left(\frac{4}{3} + \frac{4}{3}\right) M_b &= 0 \end{aligned}\right\} \quad \begin{aligned} M_a &= -\frac{6}{7} \\ M_b &= \frac{3}{14} \end{aligned}$$

Tabulka 5.2 Pootočení u prostého nosníku konstantní ohybové tuhosti

Pootočení φ_{ab} podporového průřezu a		Pootočení φ_{ba} podporového průřezu b
$\varphi_{ab}^F = \frac{1}{6}\frac{F}{EI}\frac{ab}{l}(l+b)$ (5.8a)		$\varphi_{ba}^F = \frac{1}{6}\frac{F}{EI}\frac{ab}{l}(l+a)$ (5.8b)

Maxwellova věta

$$w_{M_a} = \varphi_{ab|F} \qquad w_{M_b} = \varphi_{ba|F} = \varphi_{bc|F}$$

pole a-b

$$\eta_{M_a} = w_{M_a} \cdot M_a + w_{M_b} \cdot M_b = \varphi_{ab|F} \cdot M_a + \varphi_{ba|F} M_b$$

$$\eta_{M_a} = \frac{Fab(l+b)}{6EIl} \cdot M_a + \frac{Fab(l+a)}{6EIl} M_b$$

$$\eta_{M_a} = \frac{1 \cdot u \cdot (4-u)\cdot(4+(4-u))}{24} \cdot \left(-\frac{6}{7}\right) + \frac{1 \cdot u(4-u)\cdot(4+u)}{24} \cdot \frac{3}{14}$$

$$\eta_{M_a} = \frac{-(u^3 - 12u^2 + 32u)\cdot 4}{8 \cdot 14} + \frac{(-u^3 + 16u)}{8 \cdot 14}$$

$$\eta_{M_a} = \frac{-5u^3 + 48u^2 - 112u}{112}$$

pole b-c

$$\eta_{M_a} = w_{M_b} \cdot M_b = \varphi_{bc|F} \cdot M_b$$

$$\eta_{M_a} = \frac{Fab(l+b)}{6EIl} M_b$$

$$\eta_{M_a} = \frac{1 \cdot u \cdot (4-u)(4+(4-u))}{24} \cdot \frac{3}{14}$$

$$\eta_{M_a} = \frac{(u^3 - 12u^2 + 32u)}{8 \cdot 14}$$

pole c-d

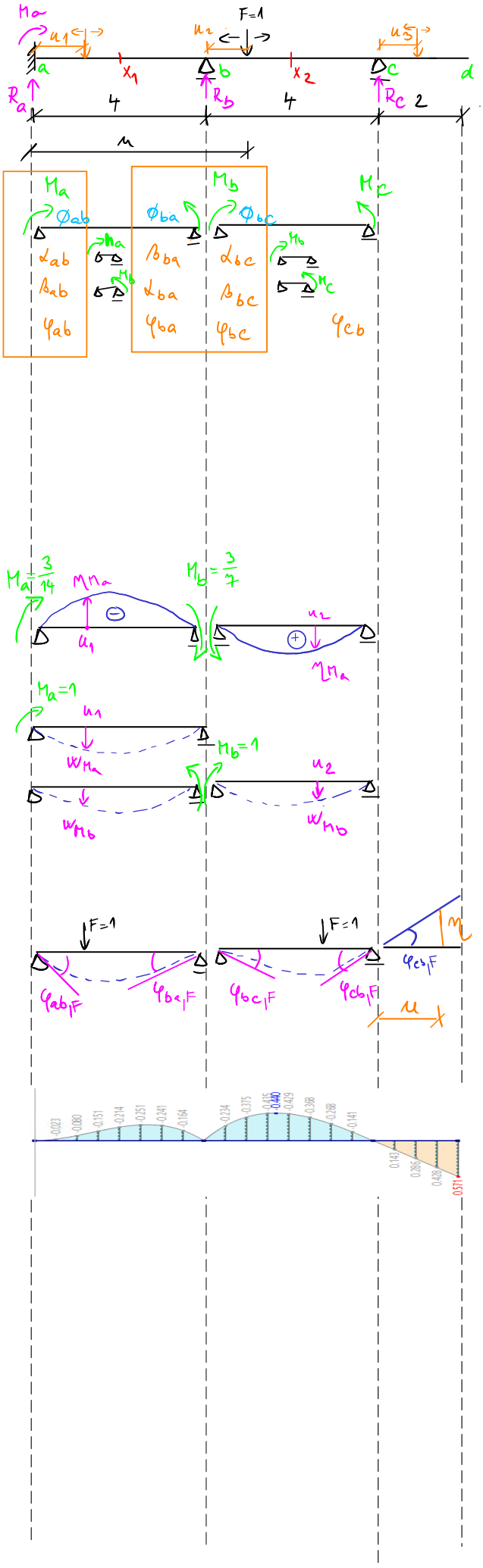
- natočení $\varphi_{cb|F}$ od M_b

$$\frac{\eta_{M_a}}{u} = -\varphi_{cb|F} \cdot M_b$$

$$\eta_{M_a} = -\varphi_{cb|F} \cdot M_b \cdot u = \frac{-l}{6EI} \cdot M_b \cdot u = \frac{-4}{6} \cdot \frac{3}{14} u$$

$$\eta_{M_a} = \frac{-u}{7}$$

Výpočet spojitého nosníku – M_b

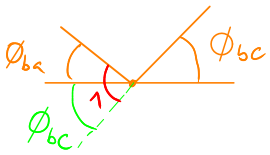


$EI = konst.$

$\varphi_{ab} = 0$

$\alpha_{ab} \cdot M_a + \beta_{ab} M_b = 0$

$\varphi_{ba} + \varphi_{bc} = -1$



$\beta_{ba} M_a + (\alpha_{bc} + \alpha_{ba}) M_b + \beta_{bc} M_c = 0$

$\alpha_{ab} = \frac{l}{3EI} = \frac{4}{3EI} = \alpha_{ba} = \alpha_{bc} = \alpha_{cb}$

$\beta_{ab} = \frac{l}{6EI} = \frac{2}{3EI} = \beta_{ba} = \beta_{bc} = \beta_{cb}$

$$\left.\begin{aligned} \tfrac{4}{3} M_a + \tfrac{2}{3} M_b &= 0 \\ \tfrac{2}{3} M_a + \left(\tfrac{4}{3} + \tfrac{4}{3}\right) M_b &= -1 \end{aligned}\right\} \quad \begin{aligned} M_a &= \tfrac{3}{14} \\ M_b &= -\tfrac{3}{7} \end{aligned}$$

Tabulka 5.2 Pootočení u prostého nosníku konstantní ohybové tuhosti

Pootočení φ_{ab} podporového průřezu a		Pootočení φ_{ba} podporového průřezu b
$\varphi_{ab}^F = \frac{1}{6}\frac{F}{EI}\frac{ab}{l}(l+b)$ (5.8a)		$\varphi_{ba}^F = \frac{1}{6}\frac{F}{EI}\frac{ab}{l}(l+a)$ (5.8b)

Maxwellova věta

$w_{Ma} = \varphi_{ab|F} \qquad w_{Mb} = \varphi_{ba|F} = \varphi_{bc|F}$

pole a–b

$$\eta_{M_b} = w_{Ma} \cdot M_a + w_{Mb} \cdot M_b = \varphi_{ab|F} \cdot M_a + \varphi_{ba|F} M_b$$

$$\eta_{M_b} = \frac{Fab(l+b)}{6EIl} \cdot M_a + \frac{Fab(l+a)}{6EIl} M_b$$

$$\eta_{M_b} = \frac{1 \cdot u \cdot (4-u) \cdot (4+(4-u))}{24} \cdot \frac{3}{14} + \frac{1u(4-u)\cdot(4+u)}{24}\left(-\frac{3}{7}\right)$$

$$\eta_{M_b} = \frac{-(u^3 - 12u^2 + 32u)\cdot 4}{112} + \frac{-(-u^3 + 16u)}{56}$$

$$\eta_{M_b} = \frac{3u^3 - 12u^2}{112}$$

pole b–c

$$\eta_{M_b} = w_{M_b} \cdot M_b = \varphi_{bc|F} \cdot M_b$$

$$\eta_{M_b} = \frac{Fab(l+b)}{6EIl} M_b$$

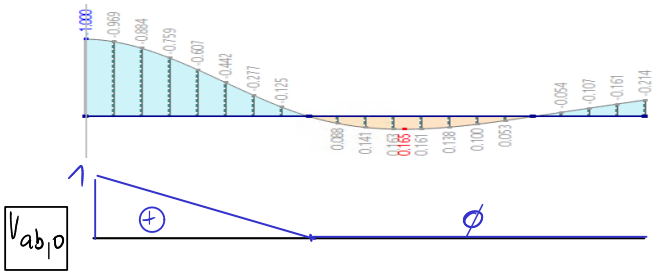
$$\eta_{M_b} = \frac{1 \cdot u \cdot (4-u)(4+(4-u))}{24} \cdot \left(-\frac{3}{7}\right)$$

$$\eta_{M_b} = \frac{-u^3 + 12u^2 - 32u}{56}$$

pole c–d – natočení $\varphi_{cb|F}$ od M_b $\qquad \frac{\eta_{Ma}}{u} = -\varphi_{cb|F} \cdot M_b$

$$\eta_{Ma} = -\varphi_{cb|F} \cdot M_b \cdot u = \frac{-l}{6EI} \cdot M_b \cdot u = -\frac{4}{6} \cdot \left(-\frac{3}{7}\right) \cdot u$$

$$\eta_{M_b} = \frac{2u}{7}$$



$$R_a = V_{ab} = V_{ab|0} + \frac{M_b - M_a}{l_{ab}}$$

pole a-b

$$V_{ab|0} = \frac{4-u}{4}$$

$$R_a = \frac{4-u}{4} + \frac{3u^3 - 12u^2}{112 \cdot 4} - \frac{-5u^3 + 48u^2 - 112u}{112 \cdot 4}$$

$$R_a = \frac{2u^3 - 15u + 112}{112}$$

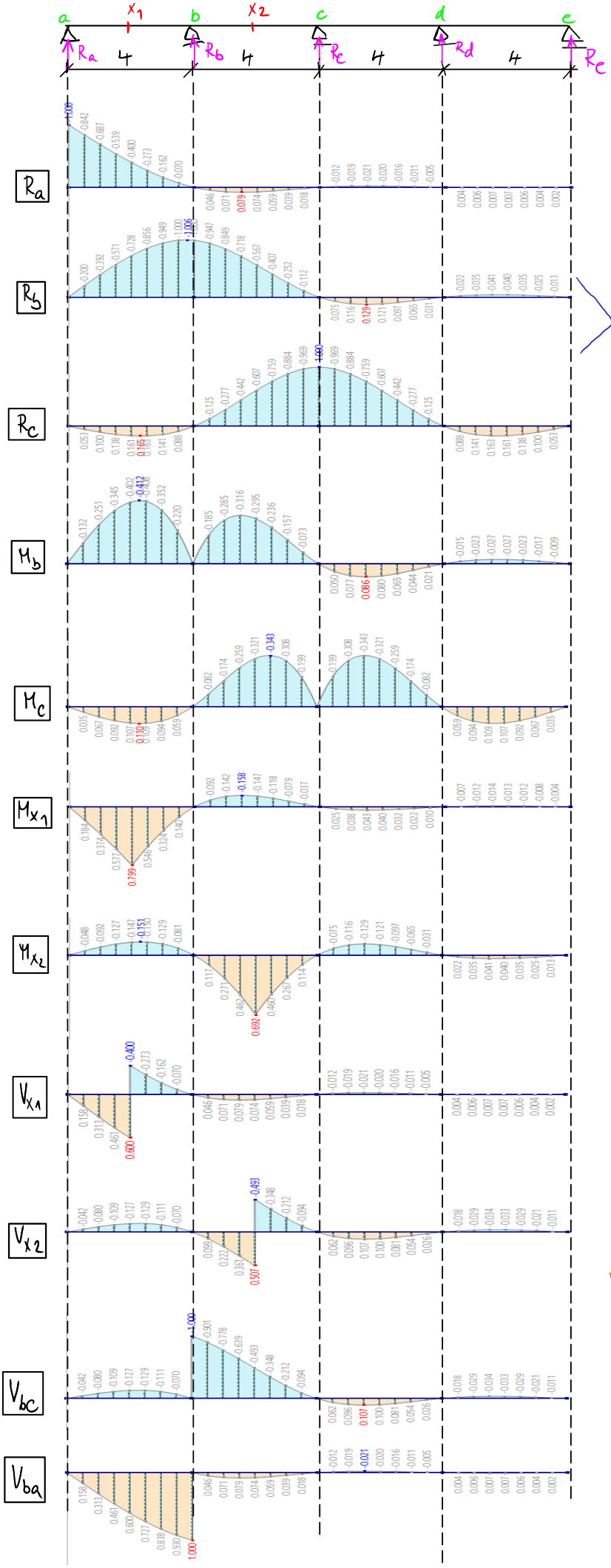
pole b-c

$$R_a = \left[\frac{-u^3 + 12u^2 - 32u}{56} - \frac{(u^3 - 12u^2 + 32u)}{112}\right] \cdot \frac{1}{4}$$

$$R_a = \frac{-3u^3 + 36u^2 - 96u}{448}$$

pole c-d

$$R_a = \left[\frac{2u}{7} - \left(-\frac{u}{7}\right)\right] \cdot \frac{1}{4} = \frac{3u}{28}$$



jsou-li pole symetrická, pak je 1 maximum
- jinak vždy max (>1) v měkčím poli

$M_{x_1} = M_{x_1,0} + \frac{M_a x' + M_b x}{l_{ab}}$

$M_{x_1} = M_{x_1,0} + \frac{M_b \cdot 2}{4 \cdot 2}$

$\frac{x x'}{l} = \frac{2 \cdot 2}{4} = 1$

$M_{x_2} = M_{x_2,0} + \frac{M_a x' + M_b x}{l_{ab}}$

$\frac{x x'}{l} = \frac{2 \cdot 2}{4} = 1$

$M_{x_2} = M_{x_2,0} + \frac{M_a \cdot 2}{4 \cdot 2} + \frac{M_b \cdot 2}{4 \cdot 2}$

V_{x_1} od x_1 vpravo - síla F za x_1 (vpravo)

$V_{x_1} = R_a$

V_{x_1} vlevo - síla F před x_1 (vlevo)

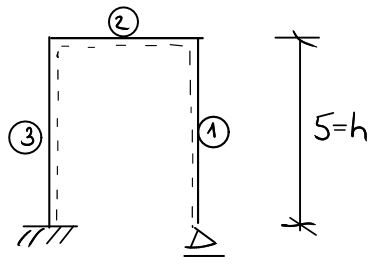
$V_{x_1} = R_a - 1$

V_{x_2} vpravo : $V_{x_2} = R_a + R_b$

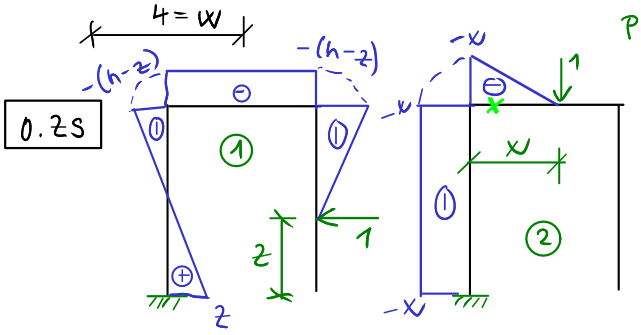
V_{x_2} vlevo : $V_{x_2} = R_a + R_b - 1$

PČ SN RÁM

$n_s = 1$

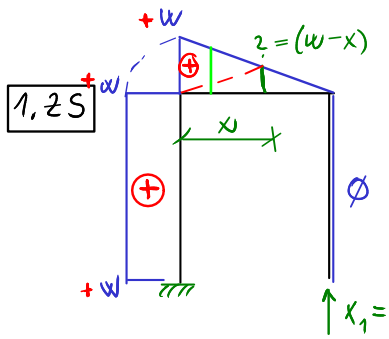


výpočet $|Ra|$



①②③ jednotková síla se pohybuje po prutu ①

$\frac{z}{w-x} = \frac{w}{w}$



$$ {}^1\delta_{10} = \frac{1}{EI}\left[\frac{1}{2}w\cdot w\cdot(z-h) + \frac{(z-h)+z}{2}\cdot h\cdot w\right] = -\frac{90-28z}{EI} $$

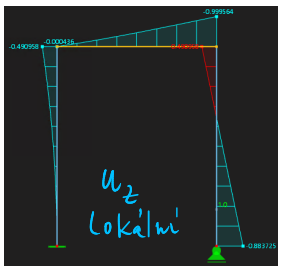
$$ {}^3\delta_{10} = -\frac{1}{EI}\left[\frac{1}{2}z\cdot z\cdot w\right] = -\frac{2z^2}{EI} $$

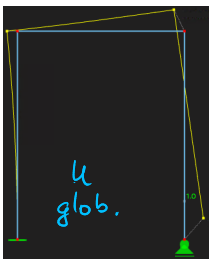
$$ {}^2\delta_{10} = -\frac{1}{EI}\left[h\cdot x\cdot w + \frac{1}{2}x\cdot x\cdot\left(\frac{2}{3}w + \frac{1}{3}(w-x)\right)\right] = -\frac{1}{EI}\left[-\frac{x^3}{6} + 2x^2 + 20x\right] $$

$$ \delta_{11} = \frac{1}{EI}\left[\frac{1}{2}w\cdot w\cdot\frac{2}{3}w + h\cdot w\cdot w\right] $$

$$ \delta_{11} = \frac{304}{3EI} $$

$$ \delta_{11}X_1 + \delta_{10} = 0 \Rightarrow X_1 = R_a = -\frac{\delta_{10}}{\delta_{11}} $$





$$ R_{a1} = \frac{135}{152} - \frac{21z}{76} $$

$$ R_{a2} = -\frac{x^3}{608} + \frac{3x^2}{152} + \frac{15x}{76} $$

$$ R_{a3} = \frac{3z^2}{152} $$

$R_{a1}(z=0) = 0{,}888$ $\quad R_{a1}(z=h=5) = -0{,}493$

$R_{a2}(x=0) = 0$ $\quad R_{a2}(x=w=4) = 1$ ✓

$R_{a3}(z=0) = 0$ $\quad R_{a3}(z=h=5) = 0{,}493$

Tuhý rámový styčník 90°

– sklon před i za by měl být stejný

$$ R'_{a1} = -\frac{21}{76} = -0{,}276 $$

$$ R'_{a2} = -\frac{3x^2}{608} + \frac{6x}{152} + \frac{15}{76} $$

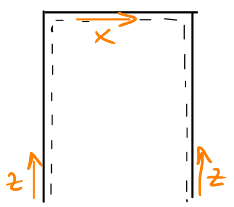
$$ R'_{a3} = \frac{6z}{152} $$

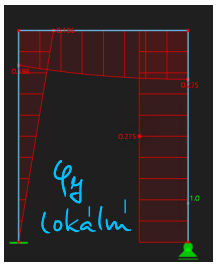
$R'_{a2}(x=w=4) = 0{,}276$ ✓

$R'_{a2}(x=0) = 0{,}197$

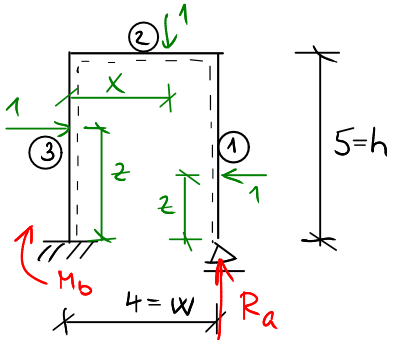
$R'_{a3}(z=h=5) = 0{,}197$ ✓

jiné znaménko kvůli použitému souřadnému sys.





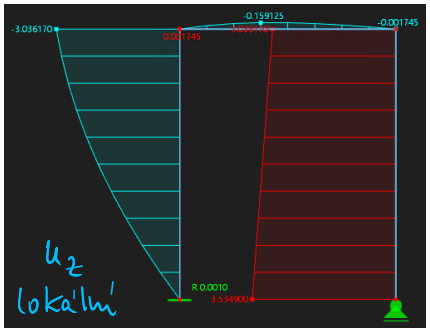
výpočet M_b – silovka nebo podmínky rovnováhy

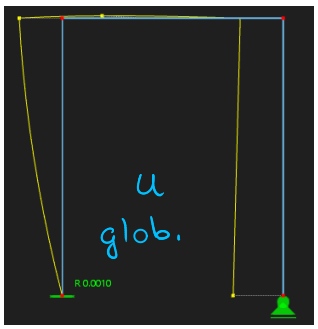


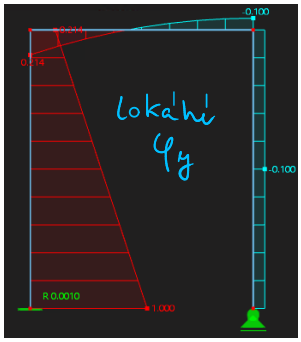
$$M_{b1} = R_{a1} \cdot w + z = \frac{135}{38} - \frac{2z}{19}$$

$$M_{b2} = R_{a2} w - x = -\frac{x^3}{152} + \frac{3x^2}{38} - \frac{4x}{19}$$

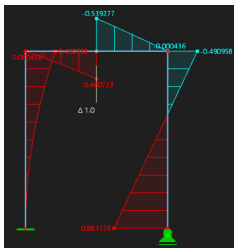
$$M_{b3} = R_{a3} w - z = \frac{3z^2}{38} - z$$

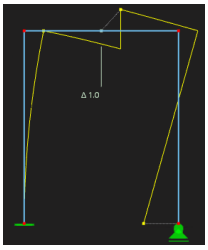


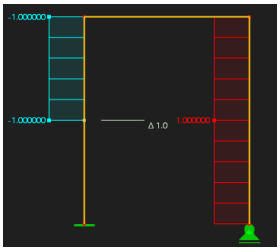


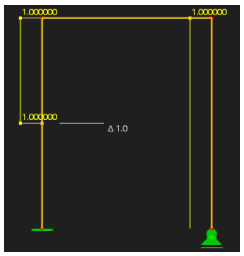


V_x









M_x