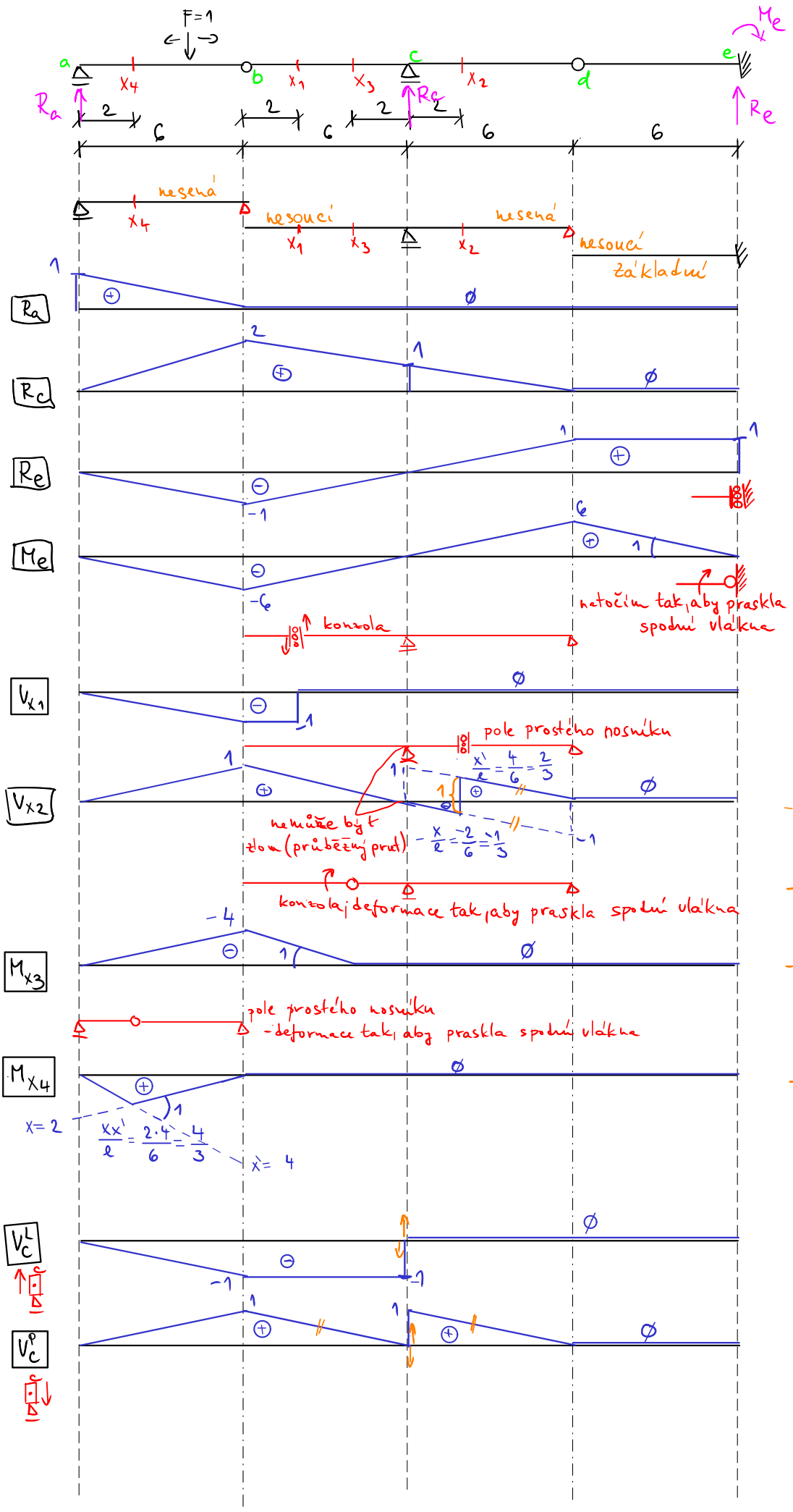
OPAKOVACÍ - PŘÍČINKOVÉ ČÁRY (PČ) NA STATICKY URČITÝCH (SU) KCÍCH



1) PČ - SU části jsou složené z úseček !!

2) zlomy PČ pouze
- ve vnitřních kloubech —o—
- v místě vnášené deformace

3) body nespojitosti pouze v místě vnášené deformace

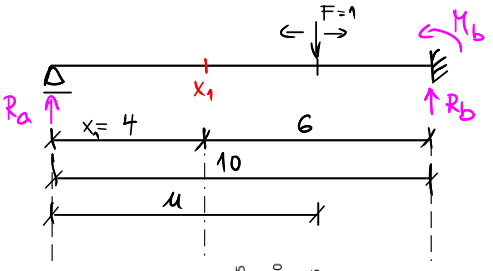
4) PČ respektují podepření (prochází podporami)

- $\frac{\oplus}{\ominus}$ R, M_R, V_x momentová reakce
- $\frac{\ominus}{\oplus}$ M_x
- pořadnice PČ jsou nenulové (projevuje se) pouze na částech nesených vyšetřovaným částí

KINEMATICKÁ METODA

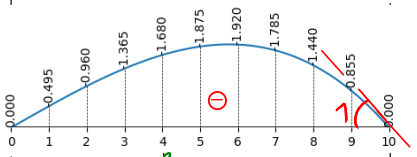
- odebereme vazbu příslušející řešené statické veličině
- vznikne mechanismus (staticky přeurčitá kce)
- v místě odebrané vazby vneseme jednotkovou deformaci (posun, pootočení)
- deformace mechanismu vnesením jednotkové deformace vytvoří tvar PČ
- pořadnice dopočítáme z podobnosti trojúhelníků

PŘÍČINKOVÉ ČÁRY (PČ) STATICKY NEURČITÝCH (SN) KCÍ



KINEMATICKÁ

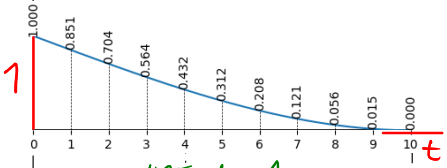
M_b



$M_{b1} = \frac{1^3 - 100\cdot 1}{200} = -0{,}495$ ✓

$M_{b4} = \frac{4^3 - 100\cdot 4}{200} = -1{,}68$ ✓

R_a

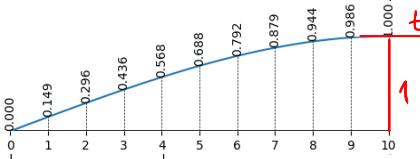


$R_{a1} = \frac{-0{,}495 + 10 - 1}{10} = 0{,}8505$ ✓ nebo

$R_{a1} = \frac{1^3 - 300\cdot 1 + 2000}{2000} = 0{,}8505$ ✓

$R_{a4} = \frac{-1{,}68 + 10 - 4}{10} = 0{,}432$ ✓

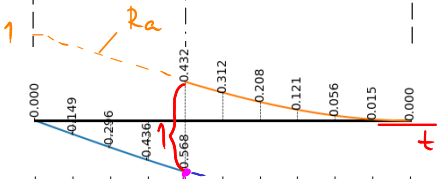
R_b



$R_{b1} = 1 - R_{a1} = 1 - 0{,}8505 = 0{,}1495$ ✓

$R_{b4} = 1 - R_{a4} = 1 - 0{,}432 = 0{,}568$ ✓

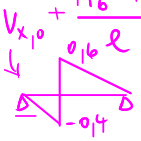
V_{x1}



$R_a \cdot 1 = -R_b$

$-0{,}4 + \frac{-1{,}68 - 0}{10} = -0{,}568$ ✓

$V_x = V_{x_{1,0}} + \frac{M_b - M_a}{0{,}16\,\ell}$



M_{x1}

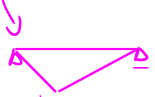
$R_a \cdot 4$

$R_b \cdot 6 + M_b$

$2{,}4 + \frac{0 + (-1{,}68)\cdot 4}{10} = 1{,}728$

$R_a \cdot 4 = 0{,}432 \cdot 4 = 1{,}728$ ✓ $\quad R_b \cdot 6 + M_b = 0{,}568 \cdot 6 - 1{,}68 = 1{,}728$ ✓

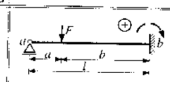
$M_x = M_{x_{1,0}} + \frac{M_a x' + M_b x}{\ell}$



Analytické řešení

výpočet např. M_b silovou, 3momentovou metodou, tabulky

Tabulka 14.11. Primární moment a reakce jednostranně dokonale vetknutého nosníku konstantního průřezu

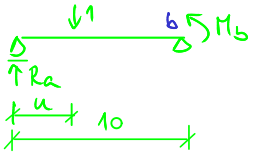
Zatěž. případ	Schéma zatížení	$\overline{M}_{ba}$	$\overline{V}_{ab} = \overline{R}_a$	$\overline{V}_{ba} = -\overline{R}_b$
1		$\frac{Fab}{2l^2}(l+a)$	$\frac{Fb^2}{2l^3}(2l+a)$	$-\frac{Fa}{2l^3}(3l^2 - a^2)$

M_b: $M_b = -\frac{Fu(\ell - u)}{2\ell^2}(\ell + u)$ $\quad F=1,\ \ell = 10$

$M_b = -\frac{u(10-u)}{200}\cdot(10+u) = -\frac{u(100-u^2)}{200}$

$M_b = \frac{u^3 - 100u}{200}$ — křivky 3^0

R_a — podobně jako M_b nebo ze silové/momentové podmínky rovnováhy



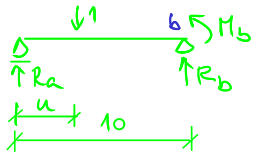
$\oplus\circlearrowright \quad \Sigma M_{bi} = 0: \quad M_b + 1\cdot(10-u) - R_a\cdot 10 = 0$

$R_a = \frac{M_b + 10 - u}{10}$

$R_a = \frac{1}{10}\left[\frac{u^3 - 100u}{200} + 10 - u\right]$

$R_a = \frac{1}{10}\left[\frac{u^3 - 100u + 2000 - 200u}{200}\right] = \frac{u^3 - 300u + 2000}{2000}$

R_b — podobně jako M_b nebo ze silové/momentové podmínky rovnováhy

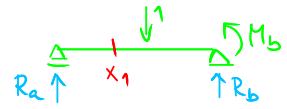


$\Sigma F_{zi} = 0 \quad \oplus\uparrow$

$R_a + R_b - 1 = 0$

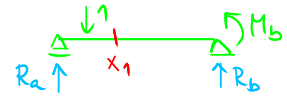
$R_b = 1 - R_a$

V_{x1} F za x_1



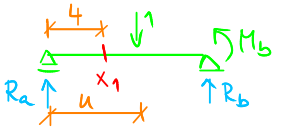
$V_{x1}^L = R_a$ nebo $V_{x1}^P = -R_b + 1$

F před x_1



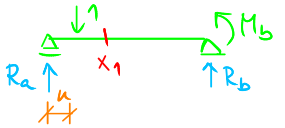
$V_{x1}^L = R_a - 1$ nebo $V_{x2}^P = -R_b$

M_{x1} F za x_1

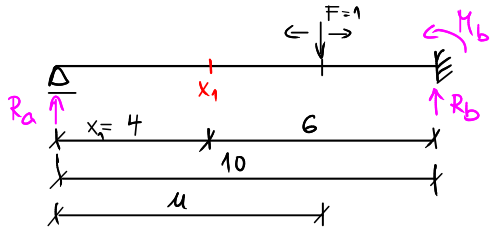


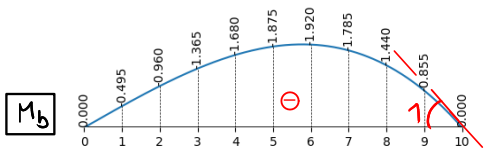
$M_{x1}^L = R_a \cdot 4$ nebo $M_{x1}^P = M_b + R_b\cdot 6 - 1\cdot(u-4)$

F před x_1

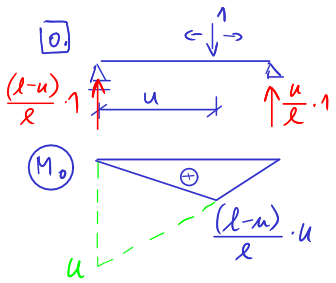


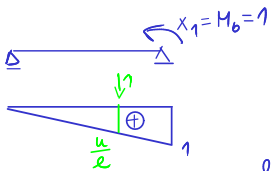
$M_{x1}^L = R_a\cdot 4 - 1(4-u)$ nebo $M_{x1}^P = M_b + R_b\cdot 6$





M_b silovou metodou – $n_s = 1\times$





$$\delta_{11} = \frac{1}{2} l \cdot 1 \cdot \left(\frac{2}{3}1\right)\frac{1}{EI} = \frac{l}{3EI}$$

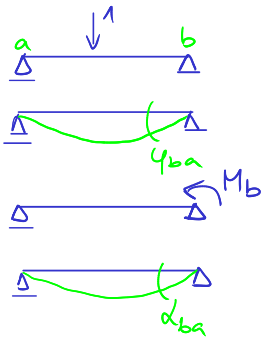
$$\delta_{10} = \frac{1}{EI}\left[\frac{1}{2}u\cdot l\left(\frac{1}{3}1\right) - \frac{1}{2}u\cdot u\cdot\left(\frac{1}{3}\frac{u}{l}\right)\right] =$$

$$= \frac{1}{EI}\left[\frac{1}{6}ul - \frac{1}{6}\frac{u^3}{l}\right] = \frac{ul^2 - u^3}{6EIl}$$

$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = M_b = -\frac{\delta_{10}}{\delta_{11}}$$

$$M_b = \frac{u^3 - 100u}{200}$$

Postup jako v metodě třímomentových rovnic



$$\varphi_{ba} + \alpha_{ba}\cdot M_b = 0$$

$$\alpha_{ba} = \left|\begin{matrix}\text{tab 5.2}\\ 13a\end{matrix}\right| = \frac{l}{3EI}$$

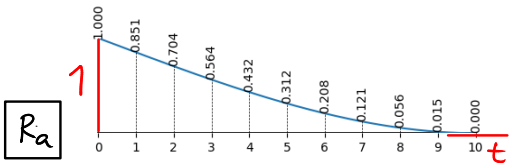
$$\varphi_{ba} = \left|\begin{matrix}\text{tab. 5.2}\\ 8b\end{matrix}\right| = \frac{Fab(l+a)}{6EIl}$$

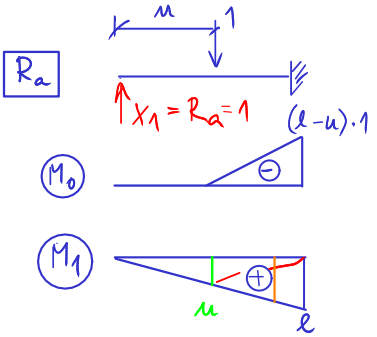
$F=1 \quad l=10 \quad a=u \quad b=l-u$

$$M_b = -\frac{\varphi_{ba}}{\alpha_{ba}} = -\frac{Fab(l+a)}{2l^2} = -\frac{1\cdot u\cdot(10-u)(10+u)}{2\cdot 10^2}$$

$$M_b = \frac{u^3 - 100u}{200}$$

R_a silová metoda





$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}l\cdot l\cdot\left(\frac{2}{3}l\right)\right] = \frac{l^3}{3EI} \quad (l=10,\ l^3=10^3)$$

$$\delta_{10} = -\frac{1}{EI}\left[\frac{1}{2}(l-u)^2\cdot 1\cdot\left(\frac{1}{3}u + \frac{2}{3}l\right)\right] \quad (l=10)$$

$$\delta_{11} X_1 + \delta_{10} = 0$$

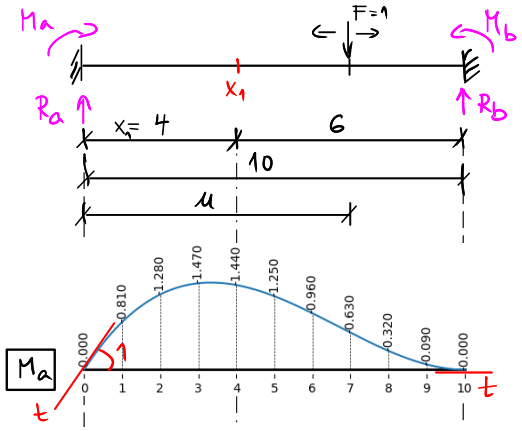
$$\delta_{10} = \frac{-1}{EI}\left[\frac{1}{2}\left(10^2 - 20u + u^2\right)\cdot 1\cdot\left(\frac{1}{3}u + \frac{20}{3}\right)\right] =$$

$$= \frac{-1}{EI}\left[\frac{1}{2}\left(\frac{100}{3}u + \frac{2000}{3} - \frac{20}{3}u^2 - \frac{400}{3}u + \frac{1}{3}u^3 + \frac{20}{3}u^2\right)\right] =$$

$$= \frac{-1}{6EI}\left[u^3 - 300u + 2000\right]$$

$$R_a = X_1 = -\frac{\delta_{10}}{\delta_{11}} = +\frac{\frac{1}{6EI_2}\left(u^3 - 300u + 2000\right)}{\frac{1}{3EI}\,1000}$$

$$R_a = \frac{u^3 - 300u + 2000}{2000}$$



Analytické řešení

výpočet silovou, 3 momentovou metodou, tabulky

Tabulka 14.10. Primární momenty a reakce oboustranně dokonale vetknutého nosníku konstantního průřezu

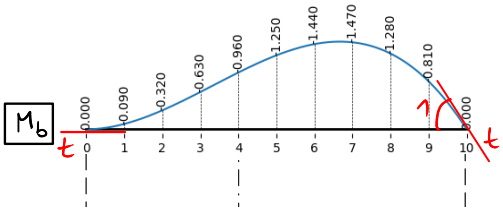
Zatěž. případ	Schéma zatížení	$\bar{M}_{ab}$	$\bar{M}_{ba}$	$\bar{V}_{ab}=\bar{R}_a$	$\bar{V}_{ba}=-\bar{R}_b$
1		$-\frac{Fab^2}{l^2}$	$\frac{Fa^2b}{l^2}$	$\frac{Fb^2}{l^3}(l+2a)$	$-\frac{Fa^2}{l^3}(l+2b)$

KINEMATICKÁ

Ma

$$M_a = -\frac{F u (l-u)^2}{l^2} \qquad F=1 \quad l=10$$

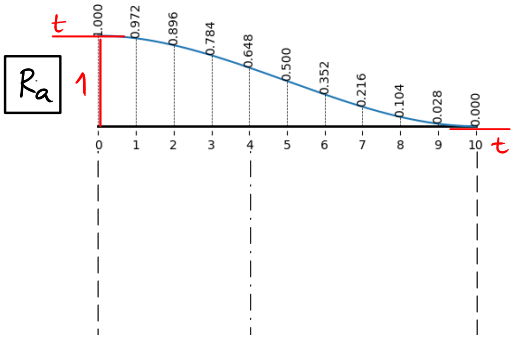
$$M_a = -\frac{u(10-u)^2}{100} = -\frac{u^3-20u^2+100u}{100}$$



Mb

$$M_b = -\frac{Fa^2b}{l^2}$$

$$M_b = -\frac{u^2(10-u)}{100} = \frac{u^3-10u^2}{100}$$



Ra

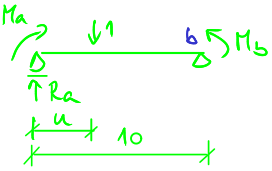
podobně jako Mb nebo ze silové/momentové podmínky rovnováhy

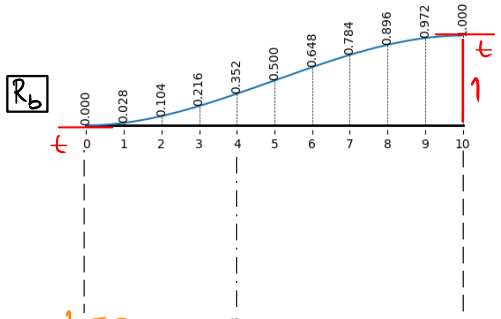
$\Sigma M_{bi}=0 \oplus \circlearrowleft$

$$M_a + R_a \cdot 10 - 1\cdot(10-u) - M_b = 0$$

$$10R_a = (10-u) + \frac{u^3-20u^2+100u}{100} + \frac{u^3-10u^2}{100}$$

$$R_a = \frac{2u^3-30u^2+1000}{1000}$$





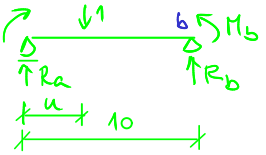
Rb

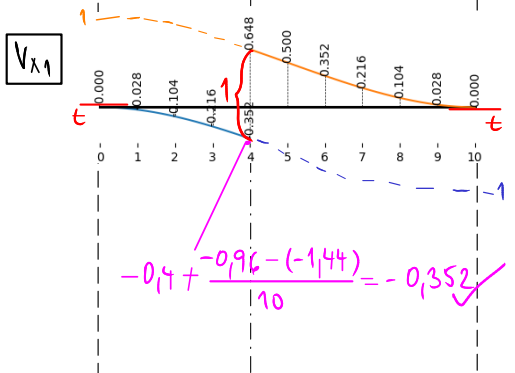
podobně jako Mb nebo ze silové/momentové podmínky rovnováhy

$\Sigma F_{zi}=0 \oplus \uparrow$

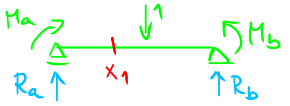
$$R_a + R_b - 1 = 0$$

$$R_b = 1 - R_a$$



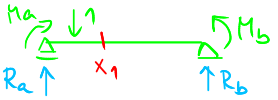


Vx1 F za x_1



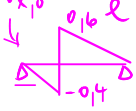
$$V_{x1}^L = R_a \quad \text{nebo} \quad V_{x1}^P = -R_b + 1$$

F před x_1

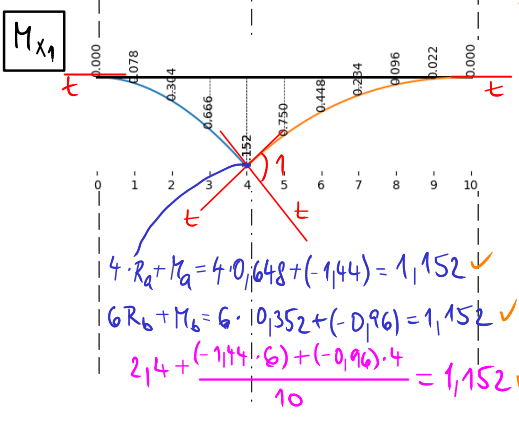


$$V_{x1}^L = R_a - 1 \quad \text{nebo} \quad V_{x1}^P = -R_b$$

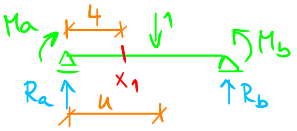
$$V_x = V_{x,0} + \frac{M_b - M_a}{0,16\,l}$$



$$-0,4 + \frac{-0,96-(-1,44)}{10} = -0,352$$

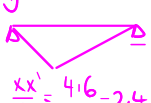


Mx1 F za x_1

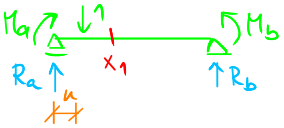


$$M_{x1}^L = R_a \cdot 4 + M_a \quad \text{nebo} \quad M_{x1}^P = M_b + R_b \cdot 6 - 1\cdot(u-4)$$

$$M_x = M_{x,0} + \frac{M_a x' + M_b x}{l}$$



F před x_1



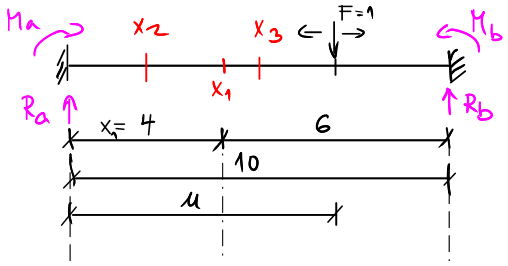
$$M_x^L = R_a \cdot 4 - 1(4-u) + M_a$$

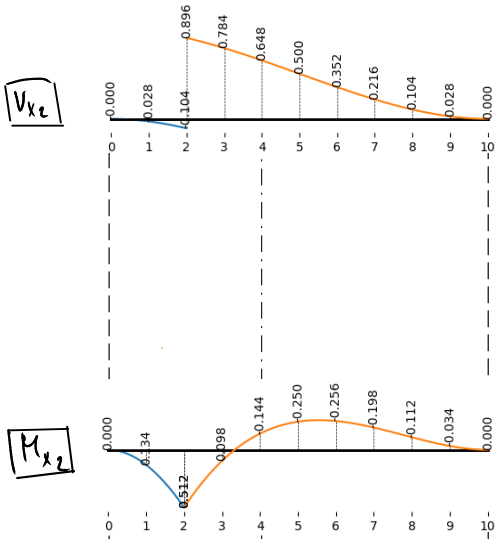
nebo $M_{x1}^P = M_b + R_b \cdot 6$

$4\cdot R_a + M_a = 4\cdot 0,648 + (-1,44) = 1,152$ ✓

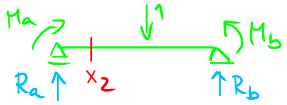
$6R_b + M_b = 6\cdot 0,352 + (-0,96) = 1,152$ ✓

$$2,4 + \frac{(-1,44)\cdot 6 + (-0,96)\cdot 4}{10} = 1,152$$ ✓



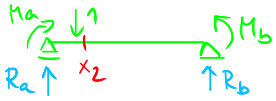


$V_{x2} = V_{x1}$ F za x_2



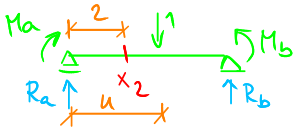
$V^L_{x2} = R_a$ nebo $V^P_{x2} = -R_b + 1$

F před x_2



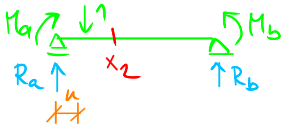
$V^L_{x2} = R_a - 1$ nebo $V^P_{x2} = -R_b$

M_{x2} F za x_2



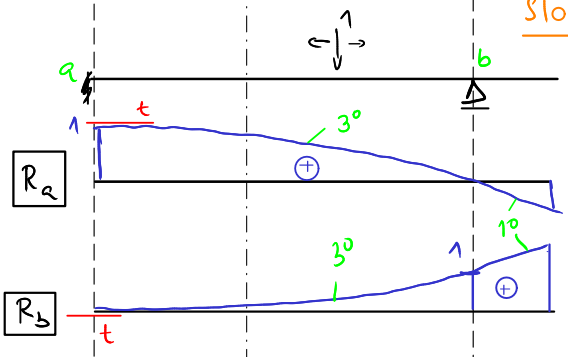
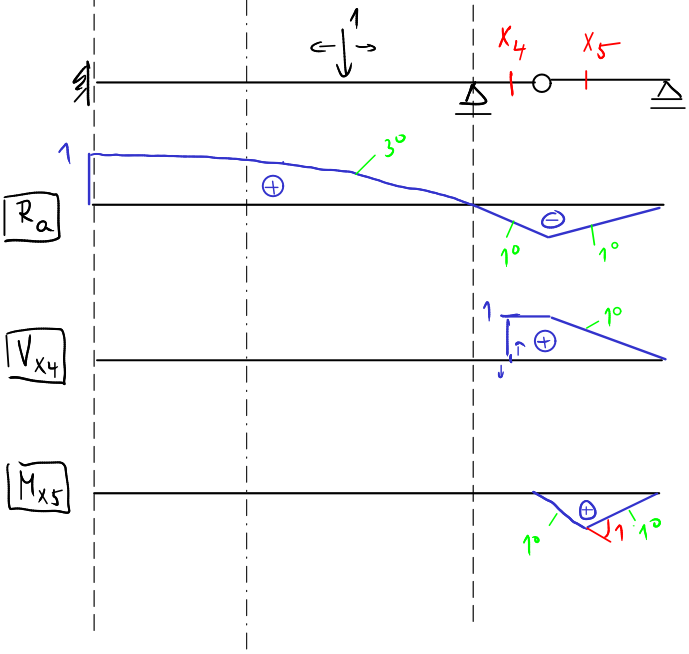
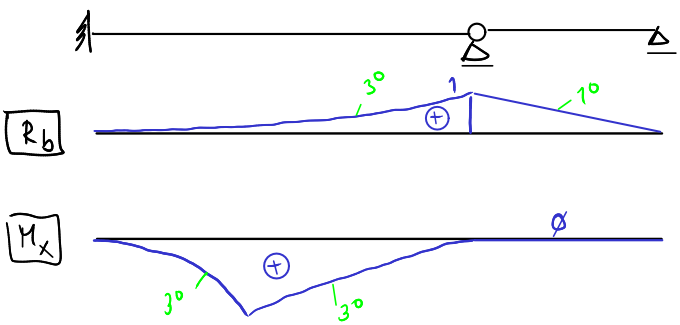
$M^L_{x_2} = R_a \cdot 2 + M_a$ nebo $M^P_{x_2} = M_b + R_b \cdot 2 - 1 \cdot (u - 8)$

F před x_2



$M^L_{x_2} = R_a \cdot 2 - 1(2 - u) + M_a$

nebo $M^P_{x_2} = M_b + R_b \cdot 8$

Složený SN nosník s SU částí

křivka 3° — lineární (1°)