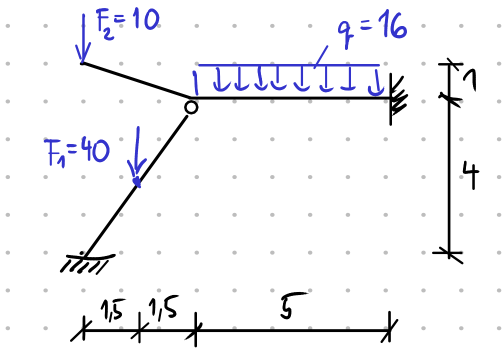
ROVINNÝ RÁM – ODM



$A = 0{,}1\ m^2$

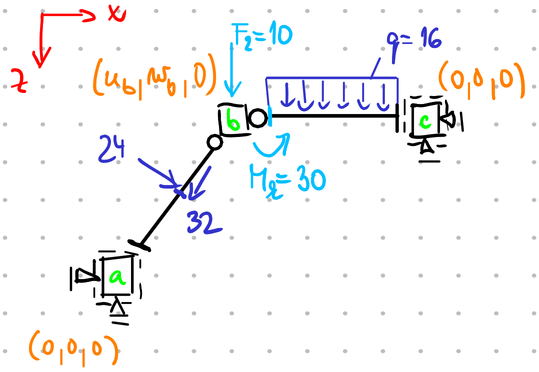
$I = 1{,}5\cdot 10^{-3}\ m^4$

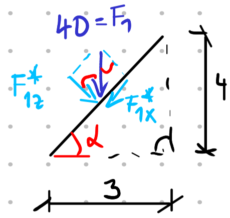
$E = 25\ GPa$

1) model a $n_p = 2$

2) vektor neznámých deformací a styčníkových uzlových zatížení

$$\{r\} = \begin{Bmatrix} u_b \\ w_b \end{Bmatrix} \qquad \{S\} = \begin{Bmatrix} 0 \\ 10 \end{Bmatrix}$$





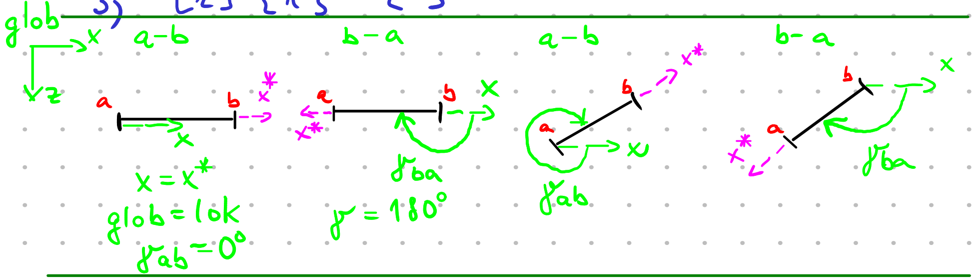
$\cos\alpha = \frac{3}{5}$

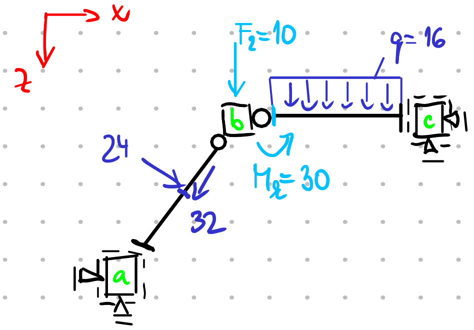
$\sin\alpha = \frac{4}{5}$

$F^*_{1z} = F_1\cdot\cos\alpha = 40\cdot\frac{3}{5} = 24\ kN$

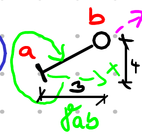
$F^*_{1x} = F_1\ \sin\alpha = 40\ \frac{4}{5} = 32\ kN$

3) $[k]\cdot\{r\} = \{F\}$





ab



$l_{ab} = 5\ m$ $\left(\begin{matrix} A_{ab} = \\ I_{ab} = \end{matrix}\ E_{ab} = \right)$

$c_{ab} = \cos\gamma_{ab} = +\frac{3}{5} = 0{,}6$

$s_{ab} = \sin\gamma_{ab} = -\frac{4}{5} = -0{,}8$

$A = 0{,}1\ m^2$

$I = 1{,}5\cdot 10^{-3}\ m^4$

$E = 25\ GPa$

$EA = 2{,}5\cdot 10^6\ kPa\ m^2$

$EI = 37500\ kPa\ m^4$

(8.2b)

$$[k_{a,b}] = \begin{array}{c|cccccc} & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \hline u_a & \frac{EA}{l}c^2+\frac{3EI}{l^3}s^2 & \left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & \frac{3EI}{l^2}s & -\left(\frac{EA}{l}c^2+\frac{3EI}{l^3}s^2\right) & -\left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & 0 \\ w_a & \left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & \frac{EA}{l}s^2+\frac{3EI}{l^3}c^2 & -\frac{3EI}{l^2}c & -\left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & -\left(\frac{EA}{l}s^2+\frac{3EI}{l^3}c^2\right) & 0 \\ \varphi_a & \frac{3EI}{l^2}s & -\frac{3EI}{l^2}c & \frac{3EI}{l} & -\frac{3EI}{l^2}s & \frac{3EI}{l^2}c & 0 \\ u_b & -\left(\frac{EA}{l}c^2+\frac{3EI}{l^3}s^2\right) & -\left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & -\frac{3EI}{l^2}s & \frac{EA}{l}c^2+\frac{3EI}{l^3}s^2 & \left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & 0 \\ w_b & -\left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & -\left(\frac{EA}{l}s^2+\frac{3EI}{l^3}c^2\right) & \frac{3EI}{l^2}c & \left(\frac{EA}{l}-\frac{3EI}{l^3}\right)cs & \frac{EA}{l}s^2+\frac{3EI}{l^3}c^2 & 0 \\ \varphi_b & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

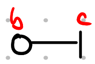
globální

$$[k_{ab}] = \begin{array}{cccccc|c} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b & \\ \hline & & & -180{,}576 & 239{,}568 & & u_a \\ & & & 239{,}568 & -320{,}324 & & w_a \\ & & & 3{,}6 & 2{,}7 & & \varphi_a \\ & & & 180{,}576 & -239{,}568 & & u_b \\ & & & -239{,}568 & 320{,}324 & & w_b \\ & & & 0 & 0 & & \varphi_b \end{array} \cdot 10^3$$

(8.3c)

$$[k_{a,b}] = \begin{array}{c|cccccc} & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \hline u_a & \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ w_a & 0 & \frac{3EI}{l^3} & 0 & 0 & -\frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \varphi_a & 0 & 0 & 0 & 0 & 0 & 0 \\ u_b & -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ w_b & 0 & -\frac{3EI}{l^3} & 0 & 0 & \frac{3EI}{l^3} & \frac{3EI}{l^2} \\ \varphi_b & 0 & -\frac{3EI}{l^2} & 0 & 0 & \frac{3EI}{l^2} & \frac{3EI}{l} \end{array}$$

bc

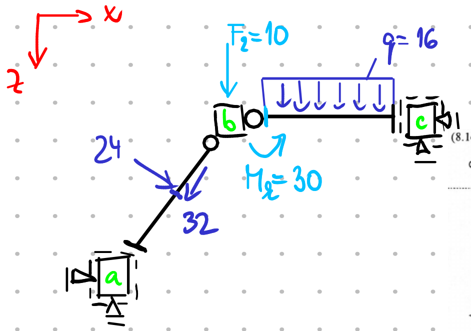


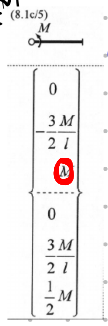
$\gamma_{bc} = 0° \Rightarrow glob = lok$ $\quad l_{bc} = 5\ m$

$$[k^*_{bc}] = [k_{bc}] = \begin{array}{cccccc|c} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c & \\ \hline 500 & 0 & & & & & u_b \\ 0 & 0{,}9 & & & & & w_b \\ 0 & 0 & & & & & \varphi_b \\ -500 & 0 & & & & & u_c \\ 0 & -0{,}9 & & & & & w_c \\ 0 & -4{,}5 & & & & & \varphi_c \end{array} \cdot 10^3$$

globální matice prutové soustavy

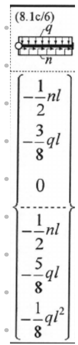
$$[k] = \begin{array}{cc|c} u_b & w_b & \\ \hline 180{,}576+500 & -239{,}568+0 & u_b \\ -239{,}568+0 & 320{,}324+0{,}9 & w_b \end{array} \cdot 10^3 = \begin{bmatrix} 680{,}576 & -239{,}568 \\ -239{,}568 & 321{,}224 \end{bmatrix} \cdot 10^3$$





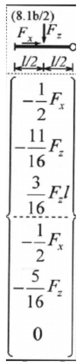
4) $\{F\}=\{S\}-\{\bar R\}$

– primární vektory koncových sil

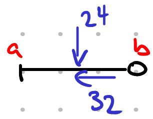


(bc) 

$$\begin{array}{ccccccc} & u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \\ \{\bar R^*_{bc}\}_q=\{\bar R_{bc}\}_q= & \{0 & -30 & 0 & 0 & -50 & -50\}^T \\ \{\bar R^*_{bc}\}_M=\{\bar R_{bc}\}_M= & \{0 & -9 & 0 & 0 & 9 & 15\}^T \\ \{\bar R^*_{bc}\}=\{\bar R_{bc}\}= & \{0 & -39 & 0 & 0 & -41 & -35\}^T \end{array}$$



$$\begin{Bmatrix} X^*_{a,b}\,c - Z^*_{a,b}\,s \\ X^*_{a,b}\,s + Z^*_{a,b}\,c \\ M^*_{a,b} \\ \hdashline X^*_{b,a}\,c - Z^*_{b,a}\,s \\ X^*_{b,a}\,s + Z^*_{b,a}\,c \\ M^*_{b,a} \end{Bmatrix}$$



$c_{ab}=\cos\gamma_{ab}=+\frac{3}{5}=0{,}6$

$s_{ab}=\sin\gamma_{ab}=-\frac{4}{5}=-0{,}8$

(ab) 

$$\begin{array}{ccccccc} & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \{\bar R^*_{ab}\}= & \{16 & -16{,}5 & 22{,}5 & 16 & -7{,}5 & 0\}^T \\ & \bar X^*_{ab} & \bar Z^*_{ab} & \bar M^*_{ab} & \bar X^*_{ba} & \bar Z^*_{ba} & \bar M^*_{ba} \end{array}$$

$$\{\bar R_{ab}\}=\{-3{,}6 \;\; -22{,}7 \;\; 22{,}5 \;\; 3{,}6 \;\; -17{,}3 \;\; 0\}^T$$

$$\{F\}=\{S\}-\{\bar R\}=\begin{Bmatrix}0\\10\end{Bmatrix}-\begin{Bmatrix}3{,}6+0\\-17{,}3-39\end{Bmatrix}\begin{matrix}u_b\\w_b\end{matrix}=\begin{Bmatrix}-3{,}6\\66{,}3\end{Bmatrix}$$

$$\{\bar R\}=\{\bar R_{ab}\}+\{\bar R_{bc}\}$$

5) řešení soustavy $[K]\{x\}=\{F\}$

$$\begin{bmatrix}680{,}576 & -239{,}568\\ -239{,}568 & 321{,}224\end{bmatrix}\cdot 10^3 \begin{Bmatrix}u_b\\w_b\end{Bmatrix}=\begin{Bmatrix}-3{,}6\\66{,}3\end{Bmatrix}$$

$$\boxed{\begin{aligned}u_b&=91{,}34437\cdot 10^{-6}\ \mathrm{m}\\ w_b&=274{,}5224\cdot 10^{-6}\ \mathrm{m}\end{aligned}}$$

$\oplus \to u_b$, $\downarrow w_b$

6) výpočet koncových sil

$$\{R_{ab}\}=\{\bar R_{ab}\}+\{\hat R_{ab}\}=\{\bar R_{ab}\}+[k_{ab}]\cdot\{r_{ab}\}$$

$$\{R_{ab}\}=\begin{Bmatrix}-3{,}6\\-22{,}7\\22{,}5\\3{,}6\\-17{,}3\\0\end{Bmatrix}+\begin{array}{c} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array} \\ \begin{bmatrix} & & & -180{,}576 & 239{,}568 & \\ & & & 239{,}568 & -320{,}324 & \\ & & & 3{,}6 & 2{,}7 & \\ & & & 180{,}576 & -239{,}568 & \\ & & & -239{,}568 & 320{,}324 & \\ & & & 0 & 0 & \end{bmatrix} \end{array}\begin{matrix}u_a\\w_a\\\varphi_a\\u_b\\w_b\\\varphi_b\end{matrix}\cdot 10^3\begin{Bmatrix}0\\0\\0\\91{,}34437\\274{,}5224\\0\end{Bmatrix}\cdot 10^{-6}$$

10^{-3}

$$=\begin{Bmatrix}45{,}672\\-88{,}753\\23{,}570\\-45{,}672\\48{,}453\\0\end{Bmatrix}\begin{matrix}X_{ab}\\Z_{ab}\\M_{ab}\\X_{ba}\\Z_{ba}\\M_{ba}\end{matrix}$$

$$\{R^*_{ab}\}=\begin{Bmatrix} X_{a,b}\,c + Z_{a,b}\,s \\ -X_{a,b}\,s + Z_{a,b}\,c \\ M_{a,b} \\ \hdashline X_{b,a}\,c + Z_{b,a}\,s \\ -X_{b,a}\,s + Z_{b,a}\,c \\ M_{b,a} \end{Bmatrix}=\begin{Bmatrix}98{,}406\\-16{,}714\\23{,}570\\-66{,}405\\-71{,}286\\0\end{Bmatrix}\begin{matrix}X^*_{ab}\\Z^*_{ab}\\M^*_{ab}\\X^*_{ba}\\Z^*_{ba}\\M^*_{ba}\end{matrix}$$

$c_{ab}=\cos\gamma_{ab}=+\frac{3}{5}=0{,}6$

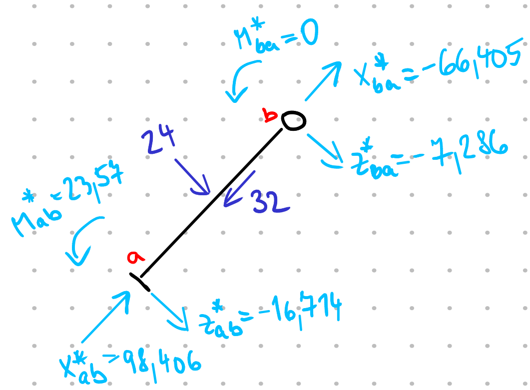
$s_{ab}=\sin\gamma_{ab}=-\frac{4}{5}=-0{,}8$

$$\{R_{bc}\} = \{\bar{R}_{bc}\} + \{\hat{R}_{bc}\} = \{\bar{R}_{bc}\} + [k_{bc}]\{r_{bc}\}$$

$$\{R^*_{bc}\} = \{R_{bc}\} = \begin{Bmatrix} 0 \\ -39 \\ 0 \\ 0 \\ -41 \\ -35 \end{Bmatrix} + \begin{array}{c} \begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \end{array} \\ \begin{bmatrix} 500 & 0 & & & & \\ 0 & 0{,}9 & & & & \\ 0 & 0 & & & & \\ -500 & 0 & & & & \\ 0 & -0{,}9 & & & & \\ 0 & -4{,}5 & & & & \end{bmatrix} \end{array} \begin{array}{c} u_b \\ w_b \\ \varphi_b \\ u_c \\ w_c \\ \varphi_c \end{array} \cdot 10^3 \begin{Bmatrix} 91{,}34437 \\ 274{,}5224 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \cdot 10^{-6} = \begin{Bmatrix} 45{,}672 \\ -38{,}753 \\ 0 \\ -45{,}672 \\ -41{,}247 \\ -36{,}235 \end{Bmatrix} \begin{array}{c} X_{bc} \\ Z_{bc} \\ M_{bc} \\ X_{cb} \\ Z_{cb} \\ M_{cb} \end{array}$$

7) kontrola rovnováhy na prutech a styčnících

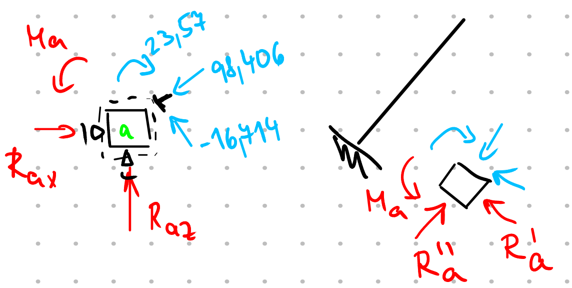
- zatížení
- lokální
- globální

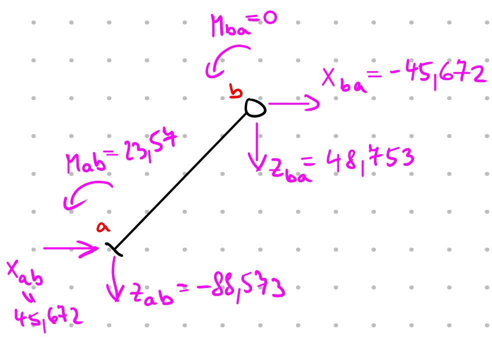


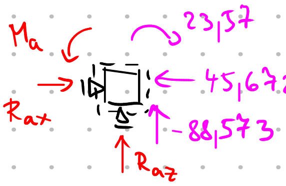
$\Sigma F_i^{\perp}=0$ $\quad 98{,}406-32+(-66{,}405)=0$ ✓

$\Sigma F_i^{\parallel}=0$ $\quad (-16{,}714)+24+(-7{,}286)=0$ ✓

$\Sigma M_{ai}=0$ $\quad 23{,}57-24\cdot 2{,}5-(-7{,}286)\cdot 5=0$ ✓





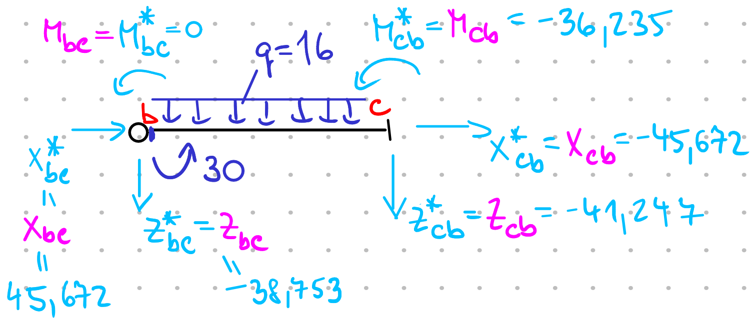


styčník a:

$\Sigma F_x=0 \Rightarrow R_{ax}=45{,}672$ kN

$\Sigma F_z=0 \Rightarrow R_{az}=88{,}573$ kN

$\Sigma M_a=0 \Rightarrow M_a=23{,}57$ kNm

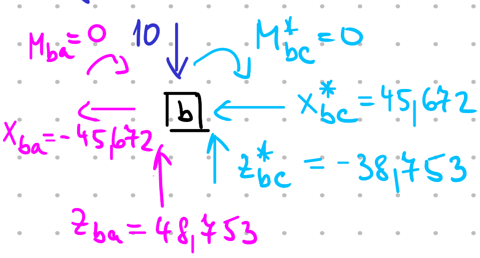


$\Sigma \vec{F}_x=0$ $\quad 45{,}672+(-45{,}672)=0$ ✓

$\Sigma F_z=0\downarrow$ $\quad (-38{,}753)+16\cdot 5+(-41{,}247)=0$ ✓

$\Sigma M_b=0$ $\quad 30-16\cdot 5\cdot 2{,}5-(-41{,}247)\cdot 5+(-36{,}235)=0$ ✓

styčník b:

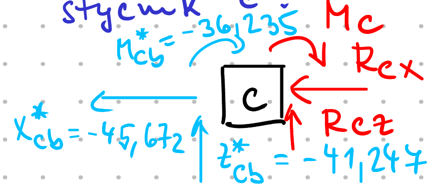


$\Sigma F_x=0\leftarrow$ $\quad (-45{,}672)+45{,}672=0$ ✓

$\Sigma F_z=0\uparrow$ $\quad 48{,}753+(-38{,}753)-10=0$ ✓

$\Sigma M=0$ ✓

styčník c:

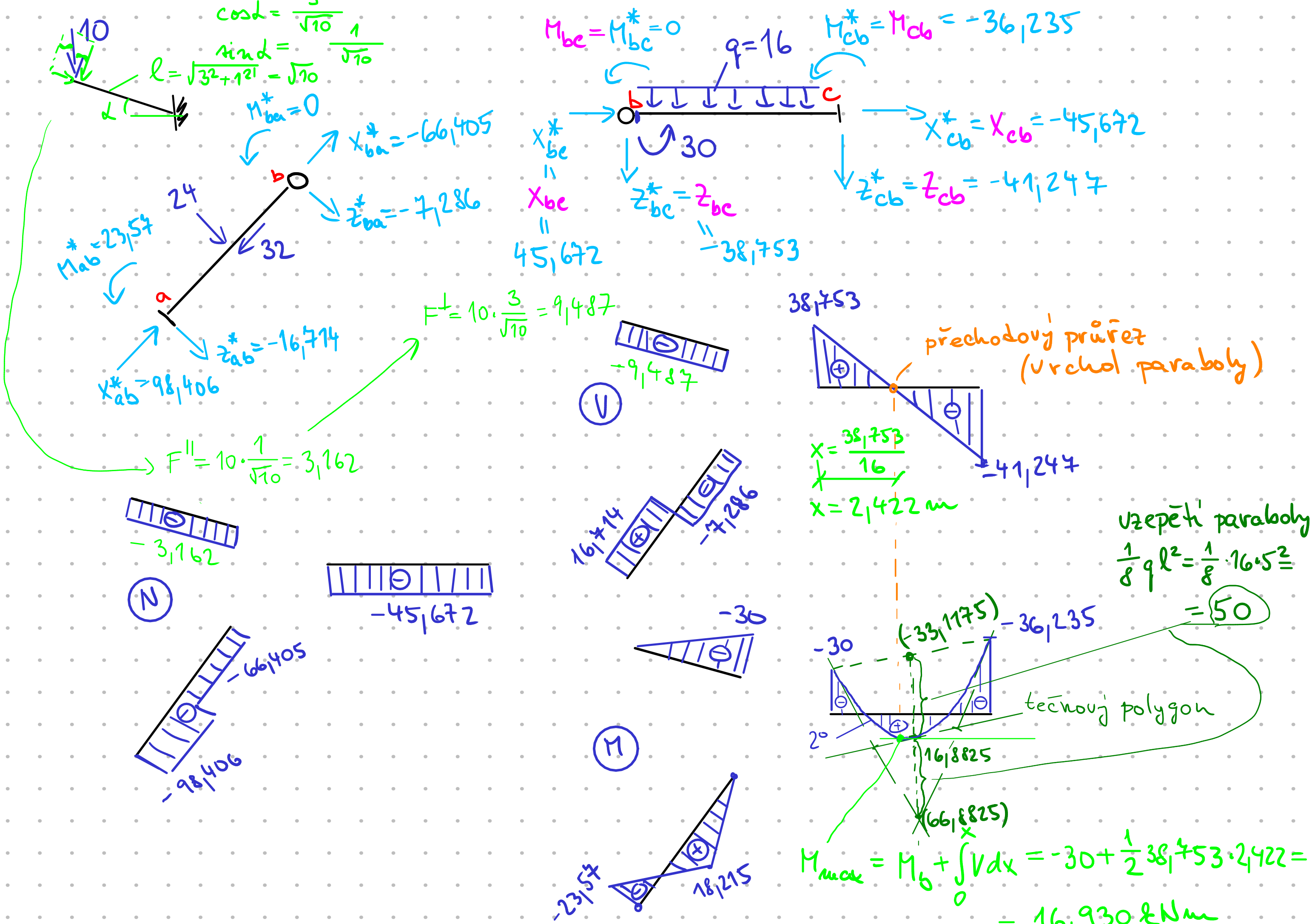


$\Sigma F_x=0 \Rightarrow R_{cx}=45{,}672$ kN

$\Sigma F_z=0 \Rightarrow R_{cz}=41{,}247$ kN

$\Sigma M=0 \Rightarrow M_c=36{,}235$ kNm

f) vykreslení vnitřních sil



$$M_{max} = M_b + \int_0^x V\,dx = -30 + \frac{1}{2}\,38{,}753 \cdot 2{,}422 = \underline{16{,}930\ kNm}$$