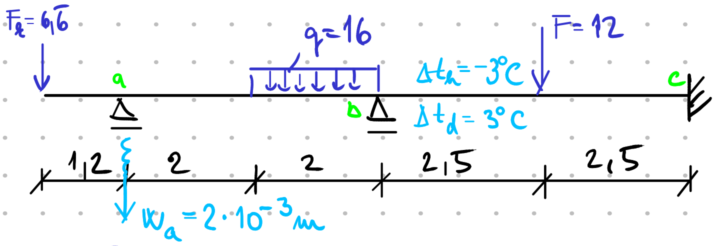
ODM – spojitý nosník se silovým a deformačním zatížením

$[m, kN, kNm, kN/m]$



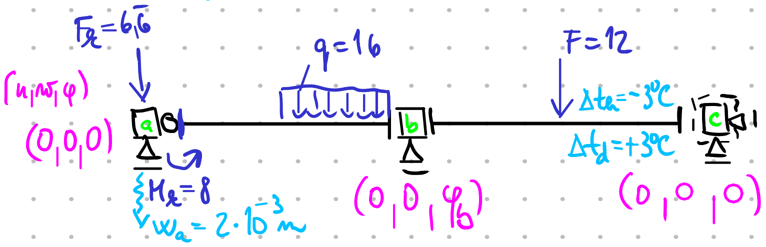
0,4
0,3

$A = 0{,}3 \cdot 0{,}4 = 0{,}12\ m^2$

$I = \frac{1}{12} b h^3 = 1{,}6 \cdot 10^{-3}\ m^4$

$E = 25\ GPa = 25 \cdot 10^9\ Pa$

$\alpha_T = 10^{-5}\ °C^{-1}$



1) výpočtový model, $n_p = 1$

2)

$$[k] \cdot \{r\} = \{F\}$$

vektor neznámých deformací

silový vektor

matice tuhosti prutové soustavy

– sestavení $\{r\} = \{\varphi_b\}$

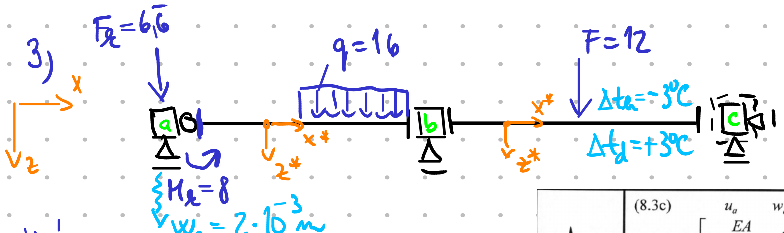
$\{S\} = \{0\}$

$$\{F\} = \{S\} - \{\bar{R}\}$$

vektor uzlových zatížení

primární vektor koncových sil

3)



globální matice tuhosti

lokální

$[k_{ab}] = [k^*_{ab}] =$ $l_{ab} = 4\,m$, $E_{ab} = E$, $A_{ab} = A$, $I_{ab} = I$ $=$

(8.3c)

$$[k_{a,b}] = \begin{array}{c|cccccc} & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \hline u_a & \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ w_a & 0 & \frac{3EI}{l^3} & 0 & 0 & -\frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \varphi_a & 0 & 0 & 0 & 0 & 0 & 0 \\ u_b & -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ w_b & 0 & -\frac{3EI}{l^3} & 0 & 0 & \frac{3EI}{l^3} & \frac{3EI}{l^2} \\ \varphi_b & 0 & -\frac{3EI}{l^2} & 0 & 0 & \frac{3EI}{l^2} & \frac{3EI}{l} \end{array}$$

$$= \begin{array}{c} u_a \\ w_a \\ \varphi_a \\ u_b \\ w_b \\ \varphi_b \end{array} \begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array}$$

$$= \left[\begin{array}{cccccc} & 0 & & & & 0 \\ & 1{,}875 & & & & -7{,}5 \\ & 0 & & & & 0 \\ & 0 & & & & 0 \\ & -1{,}875 & & & & 7{,}5 \\ & -7{,}5 & & & & 30 \end{array} \right] \cdot 10^6$$

$[k_{bc}] = [k^*_{bc}] =$ $l_{bc} = 5\,m$, $E_{bc} = E$, $A_{bc} = A$, $I_{bc} = I$ $=$

(8.3a)

$$[k_{a,b}] = \begin{array}{c|cccccc} & u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \\ \hline u_a & \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ w_a & 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} & 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ \varphi_a & 0 & -\frac{6EI}{l^2} & \frac{4EI}{l} & 0 & \frac{6EI}{l^2} & \frac{2EI}{l} \\ u_b & -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ w_b & 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ \varphi_b & 0 & -\frac{6EI}{l^2} & \frac{2EI}{l} & 0 & \frac{6EI}{l^2} & \frac{4EI}{l} \end{array}$$

$$= \begin{array}{c|cccccc} & u_b & w_b & \varphi_b & u_c & w_c & \varphi_c \\ \hline u_b & & & 0 & & & \\ w_b & & & -9{,}6 & & & \\ \varphi_b & & & 32 & & & \\ u_c & & & 0 & & & \\ w_c & & & 9{,}6 & & & \\ \varphi_c & & & 16 & & & \end{array} \cdot 10^6$$

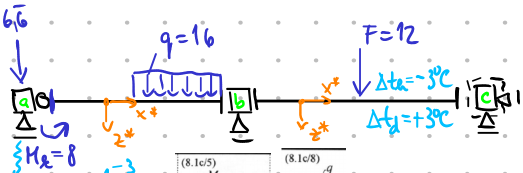
$$[k] = \overset{\varphi_b}{\left[30 + 32\right]} \varphi_b \cdot 10^6 = [62] \cdot 10^6$$

4) sestavení $\{F\} = \{S\} - \{\bar{R}\}$; sestavení primárních vektorů koncových sil

$$\{\bar{R}_{ab}\}_{M_R} = \{\bar{R}^*_{ab}\}_{M_R} = \overset{\begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b \end{array}}{\{0\ ;\ -3\ ;\ 8\ ;\ 0\ ;\ 3\ ;\ \boxed{4}\}^T} \cdot 10^3$$

$$\{\bar{R}_{ab}\}_q = \{\bar{R}^*_{ab}\}_q = \{0\ ;\ -35\ ;\ 0\ ;\ 0\ ;\ -285\ ;\ \boxed{-18}\}^T \cdot 10^3$$

$$\{\tilde{r}_{ab}\} = \{\tilde{r}^*_{ab}\} = \{0\ ;\ 2\ ;\ 0\ ;\ 0\ ;\ 0\ ;\ 0\}^T \cdot 10^{-3}$$



(8.1c/5): 0; $\frac{3}{2}\frac{M}{l}$; M; 0; $-\frac{3}{2}\frac{M}{l}$; $\frac{1}{2}M$

(8.1c/8): $-\frac{1}{8}nl$; $-\frac{7}{128}ql$; 0; $-\frac{3}{8}nl$; $-\frac{57}{128}ql$; $-\frac{9}{128}ql^2$

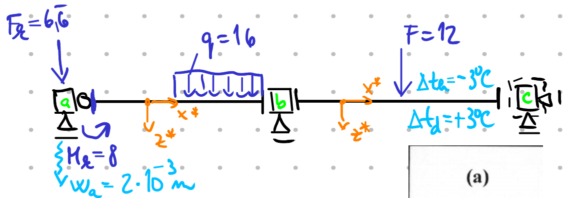
$n = 0$

$$\{\tilde{R}_{ab}\}=\{\tilde{R}^*_{ab}\}=[k_{ab}]\cdot\{\tilde{r}_{ab}\}=\begin{array}{c}\begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b\end{array}\\ \left[\begin{array}{cccccc} & 0 & & & & \\ & 1{,}875 & & & & \\ & 0 & & & & \\ & 0 & & & & \\ & -1{,}875 & & & & \\ & -7{,}5 & & & & \end{array}\right]\end{array}\cdot 10^6\begin{Bmatrix}0\\2\\0\\0\\0\\0\end{Bmatrix}\cdot 10^{-3}=\begin{Bmatrix}0\\3{,}75\\0\\0\\-3{,}75\\\boxed{-15}\end{Bmatrix}\cdot 10^3\ \begin{array}{c}u_a\\w_a\\\varphi_a\\u_b\\w_b\\\varphi_b\end{array}$$

$$\{\bar{R}_{ab}\}=\{\bar{R}_{ab}\}_{M_k}+\{\bar{R}_{ab}\}_q+\{\tilde{R}_{ab}\}=$$

$$\{0\,;-2{,}75\,;\,8\,;\,0\,;-29{,}25\,;\,\overset{\varphi_b}{\boxed{-29}}\}^T\cdot 10^3$$

(8.1a/2) F_x, F_z; $l/2$, $l/2$	(8.1a/10) h, ΔT_1, ΔT_0
$-\frac{1}{2}F_x$	$EA\alpha_T\Delta T_0$
$-\frac{1}{2}F_z$	0
$\frac{1}{8}F_z l$	$\frac{EI}{h}\alpha_T\Delta T_1$
$-\frac{1}{2}F_x$	$-EA\alpha_T\Delta T_0$
$-\frac{1}{2}F_z$	0
$-\frac{1}{8}F_z l$	$-\frac{EI}{h}\alpha_T\Delta T_1$



$$\begin{array}{c}\begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c\end{array}\end{array}$$

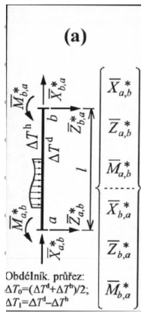
$$\{\bar{R}_{bc}\}_F=\{\bar{R}^*_{bc}\}_F=\{0\,;-6\,;\boxed{7{,}5}\,;\,0\,;-6\,;-7{,}5\}^T\cdot 10^3$$

$$\{\bar{R}_{bc}\}_T=\{\bar{R}^*_{bc}\}_T=\{0\,;\,0\,;\boxed{6}\,;\,0\,;\,0\,;-6\}^T\cdot 10^3$$

$$\{\bar{R}_{bc}\}=\{\bar{R}_{bc}\}_F+\{\bar{R}_{bc}\}_T=\{0\,;-6\,;\boxed{13{,}5}\,;\,0\,;-6\,;-13{,}5\}^T\cdot 10^3$$

$$\Delta T_1=T_d-T_h=3-(-3)=6°C$$

$$\Delta T_0=(T_d+T_h)/2=0°C$$



$$\{F\}=\{S\}-\{\bar{R}\}=\{0\}^{\varphi_b}-\overbrace{\{-29+13{,}5\}}^{\{\bar{R}_{ab}\}+\{\bar{R}_{bc}\}}{}^{\varphi_b}\cdot 10^3=\underline{\{15{,}5\}\cdot 10^3}$$

5) vyřešení soustavy rovnic

$$[k]\cdot\{r\}=\{F\}$$

$$[62]\cdot 10^6\cdot\{\varphi_b\}=\{15{,}5\}\cdot 10^3$$

$$\varphi_b=\frac{15{,}5\cdot 10^3}{62\cdot 10^6}=\underline{0{,}25\cdot 10^{-3}\,rad}$$

6) vektor koncových sil

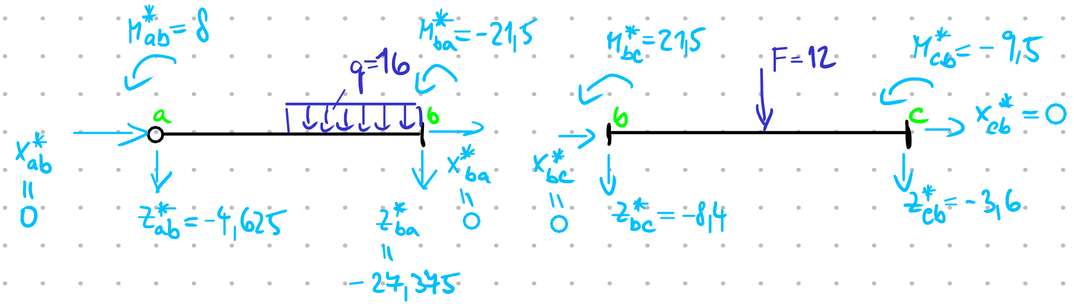
$$\{R_{ab}\}=\{R^*_{ab}\}=\{\bar{R}_{ab}\}+\{\overset{\text{sekundární}}{\hat{R}_{ab}}\}=\{\bar{R}_{ab}\}+[k_{ab}]\cdot\{r_{ab}\}$$

$$\{R_{ab}\}=\begin{Bmatrix}X_{ab}\\Z_{ab}\\M_{ab}\\X_{ba}\\Z_{ba}\\M_{ba}\end{Bmatrix}=\begin{Bmatrix}0\\-2{,}75\\8\\0\\-29{,}25\\-29\end{Bmatrix}\cdot 10^3+\begin{array}{c}\begin{array}{cccccc} u_a & w_a & \varphi_a & u_b & w_b & \varphi_b\end{array}\\ \left[\begin{array}{cccccc} & & & & & 0\\ & & & & & -7{,}5\\ & & & & & 0\\ & & & & & 0\\ & & & & & 7{,}5\\ & & & & & 30\end{array}\right]\end{array}\cdot 10^6\begin{Bmatrix}0\\0\\0\\0\\0\\0{,}25\end{Bmatrix}\cdot 10^{-3}=\begin{Bmatrix}0\\-4{,}625\\8\\0\\-24{,}375\\-21{,}5\end{Bmatrix}\begin{array}{c}u_a\\w_a\\\varphi_a\\u_b\\w_b\\\varphi_b\end{array}\cdot 10^3$$

$$\{R_{bc}\}=\{R^*_{bc}\}=\{\bar{R}_{bc}\}+\{\hat{R}_{bc}\}=\{\bar{R}_{bc}\}+[k_{bc}]\cdot\{r_{bc}\}$$

$$\{R_{bc}\}=\begin{Bmatrix}X_{bc}\\Z_{bc}\\M_{bc}\\X_{cb}\\Z_{cb}\\M_{cb}\end{Bmatrix}=\begin{Bmatrix}0\\-6\\13{,}5\\0\\-6\\-13{,}5\end{Bmatrix}\cdot 10^3+\begin{array}{c}\begin{array}{cccccc} u_b & w_b & \varphi_b & u_c & w_c & \varphi_c\end{array}\\ \left[\begin{array}{cccccc} & & 0 & & & \\ & & -9{,}6 & & & \\ & & 32 & & & \\ & & 0 & & & \\ & & 9{,}6 & & & \\ & & 16 & & & \end{array}\right]\end{array}\cdot 10^6\begin{Bmatrix}0\\0\\0{,}25\\0\\0\\0\end{Bmatrix}\begin{array}{c}u_b\\w_b\\\varphi_b\\u_c\\w_c\\\varphi_c\end{array}\cdot 10^{-3}=\begin{Bmatrix}0\\-8{,}4\\21{,}5\\0\\-3{,}6\\-9{,}5\end{Bmatrix}\cdot 10^3$$

Reakce a vykreslení vnitřních sil



kontrola rovnováhy na prutech:

(ab) $\Sigma F_{xi} = 0 \quad \oplus\rightarrow$

$X^*_{ab} + X^*_{ba} = 0$ ✓

$\Sigma F_{zi} = 0 \quad \oplus\downarrow$

$Z^*_{ab} + Z^*_{ba} + Q = 0$

$-4,625 - 27,375 + 16 \cdot 2 = 0$

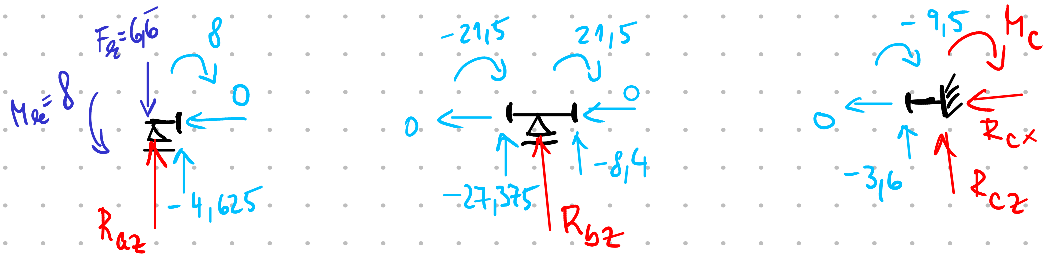
$0 = 0$ ✓

$\Sigma M_{ai} = 0 \quad \oplus\circlearrowleft$

$M^*_{ab} - Q \cdot 3 - Z^*_{ba} \cdot 4 + M^*_{ba} = 0$

$8 - 16 \cdot 2 \cdot 3 + 27,375 \cdot 4 - 21,5 = 0$

$0 = 0$ ✓



$\Sigma F_{xi} = 0$ ✓

$\Sigma F_{zi} = 0 \quad \oplus\uparrow$

$R_{az} + (-4,625) - 6,\bar{6} = 0$

$\underline{R_{az} = +11,29\bar{1}\bar{6}\ kN}$

$\Sigma F_{zi} = 0$

$\Downarrow$

$\underline{R_{bz} = 35,775\ kN}$

$\Sigma M_{bi} = 0 \quad \oplus\circlearrowright$

$(-21,5) + 21,5 = 0$ ✓

$\Sigma F_{xi} = 0 \quad \oplus\leftarrow$

$R_{cx} + 0 = 0$

$\underline{R_{cx} = 0}$

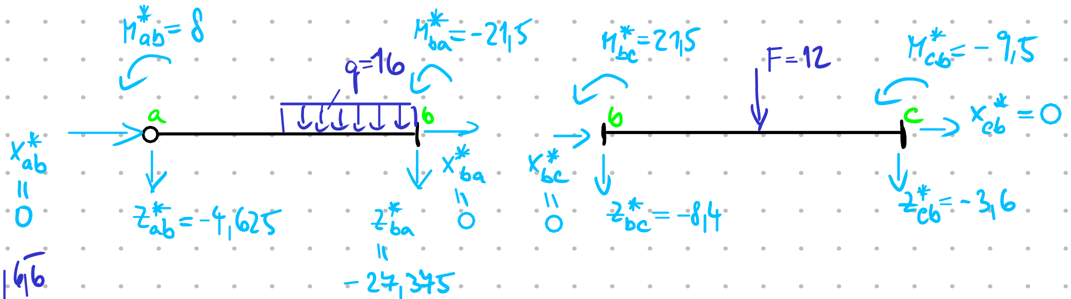
$\Sigma F_{zi} = 0 \Rightarrow \underline{R_{cz} = 3,6\ kN}$

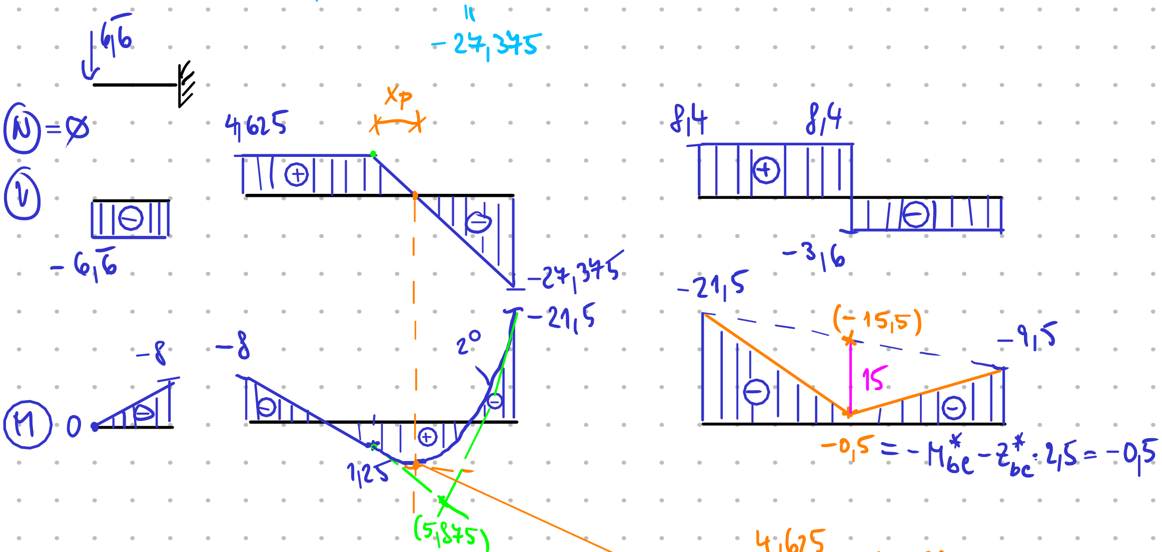
$\Sigma M_{ci} = 0 \quad \oplus\circlearrowright$

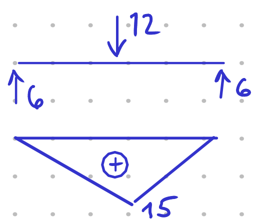
$M_c + (-9,5) = 0$

$\underline{M_c = 9,5\ kNm}$

(bc) — ✓

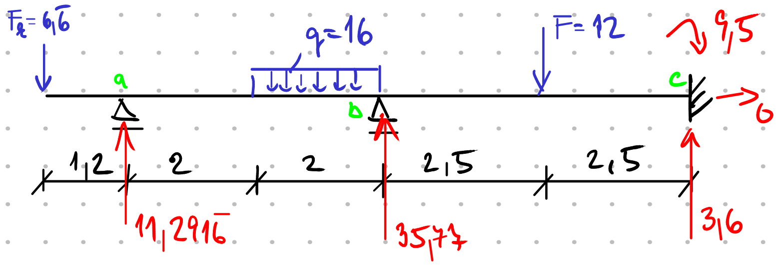






$x_p = \frac{4,625}{16} = 0,289\ m$

$M_{max} = 1,25 + \int_{x_p} V dx = 1,25 + \frac{1}{2} x_p \cdot 4,625 = \underline{\underline{1,918\ kNm}}$



F₁ = 6,6
q=16
F= 12
9,5
a
b
c
6
1,2
2
2
2,5
2,5
11,2916
35,77
3,6