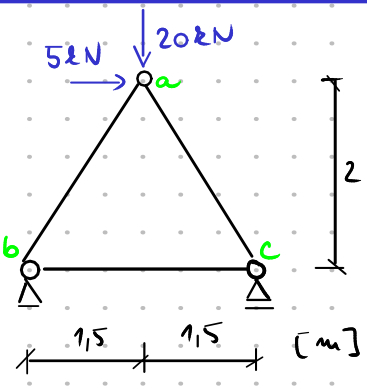
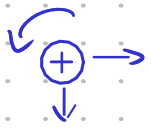
Přihradová konstrukce – ODM



$A = 1{,}5 \cdot 10^{-3}\ m^2$

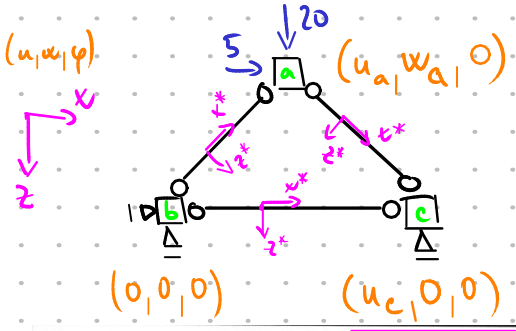
$E = 210\ GPa$



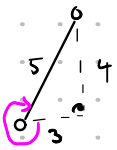
1) model $\Rightarrow n_p = 3$

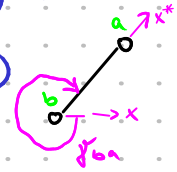
2) vektor neznámých deformací a styčníkových zatížení

$$\{r\} = \begin{Bmatrix} u_a \\ w_a \\ u_c \end{Bmatrix} \qquad \{S\} = \begin{Bmatrix} 5 \\ 20 \\ 0 \end{Bmatrix}$$



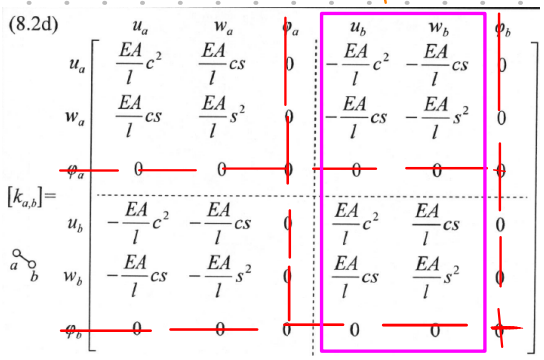
3) matice tuhosti



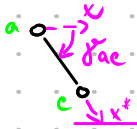
(ba) 

$l_{ba} = 2{,}5\ m \qquad s = -\frac{4}{5} \qquad c = +\frac{3}{5}$

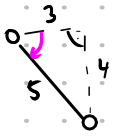
$EA = 315000\ kPa\ m^2$

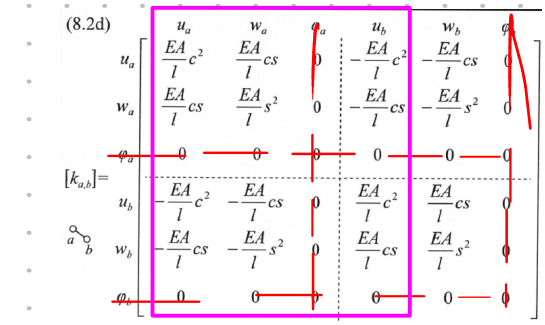


$$[k_{ba}] = \begin{array}{c} \\ u_b \\ w_b \\ u_a \\ w_a \end{array} \begin{array}{c} \begin{array}{cccc} u_b & w_b & u_a & w_a \end{array} \\ \begin{bmatrix} -45{,}36 & 60{,}48 & 45{,}36 & -60{,}48 \\ 60{,}48 & -80{,}64 & -60{,}48 & 80{,}64 \\ 45{,}36 & -60{,}48 & -45{,}36 & 60{,}48 \\ -60{,}48 & 80{,}64 & 60{,}48 & -80{,}64 \end{bmatrix} \end{array} \cdot 10^3$$

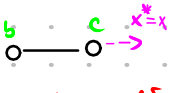
(ac) 

$l_{ac} = 2{,}5\ m \qquad s = \frac{4}{5} \qquad c = \frac{3}{5}$

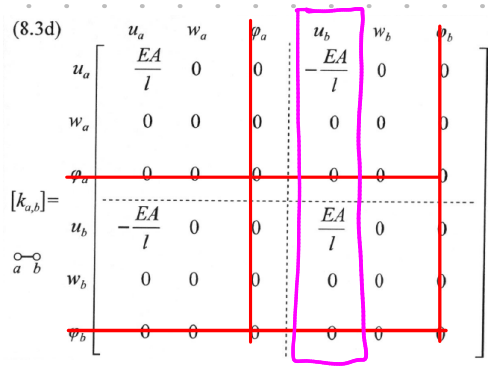




$$[k_{ac}] = \begin{array}{c} \\ u_a \\ w_a \\ u_c \\ w_c \end{array} \begin{array}{c} \begin{array}{cccc} u_a & w_a & u_c & w_c \end{array} \\ \begin{bmatrix} 45{,}36 & 60{,}48 & -45{,}36 & -60{,}48 \\ 60{,}48 & 80{,}64 & -60{,}48 & -80{,}64 \\ -45{,}36 & -60{,}48 & 45{,}36 & 60{,}48 \\ -60{,}48 & -80{,}64 & 60{,}48 & 80{,}64 \end{bmatrix} \end{array} \cdot 10^3$$

(bc) 

$l_{bc} = 3\ m \qquad \gamma_{bc} = 0 \qquad g/ob = lok$



$$[k_{bc}] = [k^*_{bc}] = \begin{array}{c} \\ u_b \\ w_b \\ u_c \\ w_c \end{array} \begin{array}{c} \begin{array}{cccc} u_b & w_b & u_c & w_c \end{array} \\ \begin{bmatrix} -105 & 0 & 105 & 0 \\ 0 & 0 & 0 & 0 \\ 105 & 0 & -105 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \cdot 10^3$$

$$[k] = \begin{array}{c} \\ u_a \\ w_a \\ u_c \end{array} \begin{array}{c} \begin{array}{ccc} u_a & w_a & u_c \end{array} \\ \begin{bmatrix} 45{,}36 & -60{,}48 & -45{,}36 \\ +45{,}36 & +60{,}48 & \\ -60{,}48 & 80{,}64 & -60{,}48 \\ +60{,}48 & +80{,}64 & \\ -45{,}36 & -60{,}48 & 45{,}36 \\ & & +105 \end{bmatrix} \end{array} \cdot 10^3 = \begin{array}{c} \\ u_a \\ w_a \\ u_c \end{array} \begin{array}{c} \begin{array}{ccc} u_a & w_a & u_c \end{array} \\ \begin{bmatrix} 90{,}72 & 0 & -45{,}36 \\ 0 & 161{,}28 & -60{,}48 \\ -45{,}36 & -60{,}48 & 150{,}36 \end{bmatrix} \end{array} \cdot 10^3$$

4) řešení soustavy

$$[r]\cdot\{n\}=\{F\} \qquad \{F\}=\{S\}-\{\bar{R}\} \qquad \{\bar{R}\}=\emptyset$$

$$[r]\cdot\{n\}=\{S\}$$

$$\begin{bmatrix} 90{,}72 & 0 & -45{,}36 \\ 0 & 161{,}28 & -60{,}48 \\ -45{,}36 & -60{,}48 & 150{,}36 \end{bmatrix}\cdot 10^3 \begin{Bmatrix} u_a \\ w_a \\ u_c \end{Bmatrix} = \begin{Bmatrix} 5 \\ 20 \\ 0 \end{Bmatrix}$$

$$\left[\begin{array}{ccc|c} 90{,}72 & 0 & -45{,}36 & 5 \\ 0 & 161{,}28 & -60{,}48 & 20 \\ -45{,}36 & -60{,}48 & 150{,}36 & 0 \end{array}\right] \cdot 2$$

$$\left[\begin{array}{ccc|c} 90{,}72 & 0 & -45{,}36 & 5 \\ 0 & 161{,}28 & -60{,}48 & 20 \\ 0 & -120{,}96 & 255{,}36 & 5 \end{array}\right] \begin{matrix} \\ \\ \cdot 120{,}96 \\ \cdot 161{,}28 \end{matrix} \rightarrow \left[\begin{array}{ccc|c} 90{,}72 & 0 & -45{,}36 & 5 \\ 0 & 161{,}28 & -60{,}48 & 20 \\ 0 & 0 & 33868{,}8 & 32256 \end{array}\right]$$

u_a $10^3 w_a$ u_c

$$\Rightarrow u_a = 0{,}1027337\cdot 10^{-3}\ m$$
$$\Rightarrow w_a = 0{,}15972\cdot 10^{-3}\ m$$
$$\Rightarrow u_c = 0{,}0952381\cdot 10^{-3}\ m$$

5) výpočet koncových sil

$$\{R_{ba}\} = \{\bar{R}_{ba}\} + \{\hat{R}_{ba}\} = \emptyset + [r_{ba}]\{r_{ba}\}$$

$$\{R_{ba}\} = \left[\begin{array}{cc|cc} & & u_a & w_a \\ & & -45{,}36 & 60{,}48 \\ & & 60{,}48 & -80{,}64 \\ \hline & & 45{,}36 & -60{,}48 \\ & & -60{,}48 & 80{,}64 \end{array}\right] \begin{matrix} \\ \\ u_b \\ w_b \\ \hline u_a \\ w_a \end{matrix} \cdot 10^3 \begin{Bmatrix} 0 \\ 0 \\ 0{,}1027337 \\ 0{,}15972 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 5 \\ -6{,}\overline{6} \\ -5 \\ 6{,}\overline{6} \end{Bmatrix}$$

u_b w_b

$\Delta = -\frac{4}{5}$

$c = +\frac{3}{5}$

$$\begin{Bmatrix} X_{a,b}\,c + Z_{a,b}\,s \\ -X_{a,b}\,s + Z_{a,b}\,c \\ \hline M_{a,b} \\ \hline X_{b,a}\,c + Z_{b,a}\,s \\ -X_{b,a}\,s + Z_{b,a}\,c \\ \hline M_{b,a} \end{Bmatrix}$$

$$\{R^*_{ba}\} = \begin{Bmatrix} 8{,}\overline{3} \\ 0 \\ -8{,}\overline{3} \\ 0 \end{Bmatrix}$$

$$\{R_{ac}\} = \{\hat{R}_{ac}\} = [r_{ac}]\{r_{ac}\}$$

$$\{R_{ac}\} = \left[\begin{array}{ccc|c} u_a & w_a & u_c & w_c \\ \hline 45{,}36 & 60{,}48 & -45{,}36 & \\ 60{,}48 & 80{,}64 & -60{,}48 & \\ -45{,}36 & -60{,}48 & 45{,}36 & \\ \hline -60{,}48 & -80{,}64 & 60{,}48 & \end{array}\right] \begin{matrix} u_a \\ w_a \\ u_c \\ \hline w_c \end{matrix} 10^3 \cdot \begin{Bmatrix} 0{,}1027337 \\ 0{,}15972 \\ 0{,}0952381 \\ 0 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} 10 \\ 13{,}\overline{3} \\ -10 \\ -13{,}\overline{3} \end{Bmatrix}$$

$\Delta = \frac{4}{5}$

$c = \frac{3}{5}$

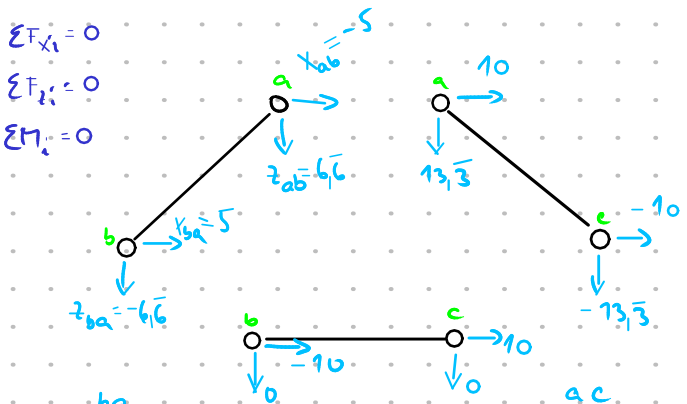
$$\begin{Bmatrix} X_{a,b}\,c + Z_{a,b}\,s \\ -X_{a,b}\,s + Z_{a,b}\,c \\ \hline M_{a,b} \\ \hline X_{b,a}\,c + Z_{b,a}\,s \\ -X_{b,a}\,s + Z_{b,a}\,c \\ \hline M_{b,a} \end{Bmatrix}$$

$$\{R^*_{ac}\} = \begin{Bmatrix} 16{,}\overline{6} \\ 0 \\ -16{,}\overline{6} \\ 0 \end{Bmatrix}$$

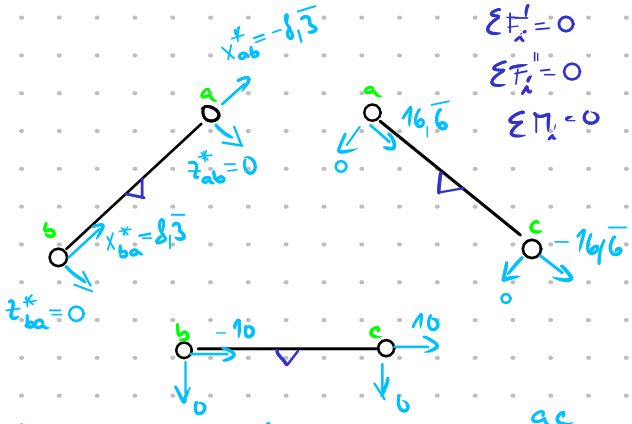
$$\{R_{bc}\} = \{\hat{R}_{bc}\} = [k_{bc}]\{u_{bc}\}$$

$$\{R_{bc}\} = \{R^*_{bc}\} = \begin{bmatrix} u_b & w_b & u_c & w_c \\ & & -105 & \\ & & 0 & \\ & & 105 & \\ & & 0 & \end{bmatrix} \begin{matrix} u_b \\ w_b \\ u_c \\ w_c \end{matrix} \cdot 10^3 \begin{Bmatrix} 0 \\ 0 \\ 0{,}0952381 \\ 0 \end{Bmatrix} \cdot 10^{-3} = \begin{Bmatrix} -10 \\ 0 \\ 10 \\ 0 \end{Bmatrix}$$

$\Sigma F_{xi} = 0$
$\Sigma F_{zi} = 0$
$\Sigma M_i = 0$

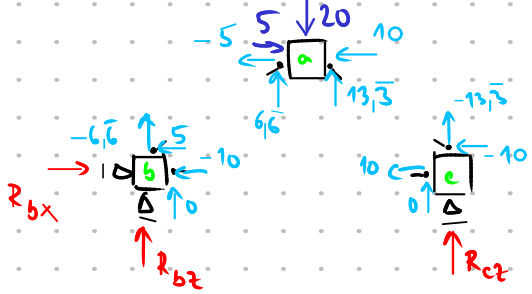


$\Sigma F'_{xi} = 0$
$\Sigma F''_{xi} = 0$
$\Sigma M_i = 0$

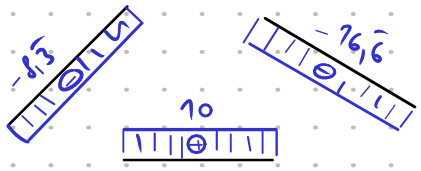


$$ba\begin{Bmatrix} 5 \\ -6{,}\overline{6} \\ -5 \\ 6{,}\overline{6} \end{Bmatrix}\begin{matrix} X_{ba} \\ Z_{ba} \\ X_{ab} \\ Z_{ab} \end{matrix} \quad bc\begin{Bmatrix} -10 \\ 0 \\ 10 \\ 0 \end{Bmatrix} \quad ac\begin{Bmatrix} 10 \\ 13{,}\overline{3} \\ -10 \\ -13{,}\overline{3} \end{Bmatrix}$$

$$ba\begin{Bmatrix} 8{,}\overline{3} \\ 0 \\ -8{,}\overline{3} \\ 0 \end{Bmatrix}\begin{matrix} X^*_{ba} \\ Z^*_{ba} \\ X^*_{ab} \\ Z^*_{ab} \end{matrix} \quad bc\begin{Bmatrix} -10 \\ 0 \\ 10 \\ 0 \end{Bmatrix} \quad ac\begin{Bmatrix} 16{,}\overline{6} \\ 0 \\ -16{,}\overline{6} \\ 0 \end{Bmatrix}$$



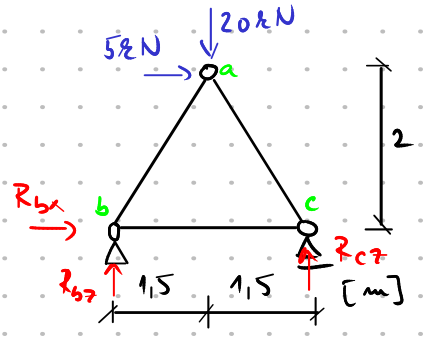
$\Sigma F_x = 0$
$\Sigma F_z = 0$



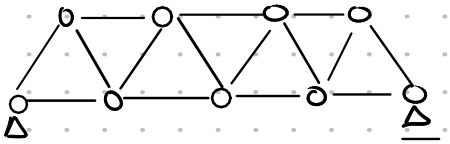
$\Sigma F_{xi} = 0$
$R_{bx} - 5 - (-10) = 0$
$R_{bx} = -5\,kN$

$\Sigma F_{zi} = 0$
$R_{bz} + (-6{,}\overline{6}) = 0 \Rightarrow R_{bz} = 6{,}\overline{6}\,kN$

$\Sigma F_{zi} = 0$
$R_{cz} + (-13{,}\overline{3}) = 0$
$R_{cz} = 13{,}\overline{3}\,kN$



$\Sigma F_{xi} = 0 \circlearrowright$
$R_{bx} + 5 = 0 \Rightarrow R_{bx} = -5\,kN$



$m_p = 7 \cdot 2 + 1 = 15$