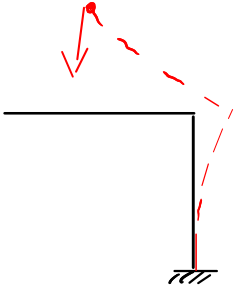
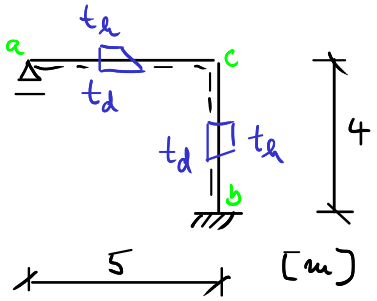
Staticky neurčitý rám - vliv změny teploty





$\Delta t_d = 20°C$

$\Delta t_k = 10°C$

$\alpha_T = 10^{-5}\ °C^{-1}$

$E = 30\ GPa$

$A = 0{,}12\ m^2$

$I = \frac{1}{12} b h^3 = 1{,}6 \cdot 10^{-3}\ m^4$

0,4 = h, b = 0,3

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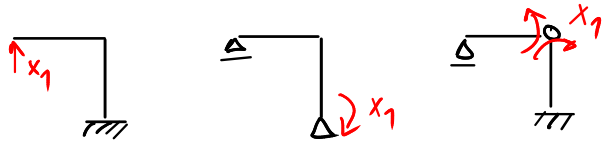
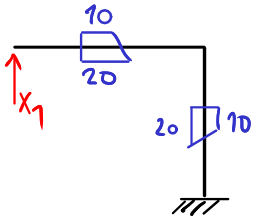
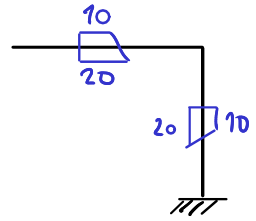
1) $n_s = ?$ a ZSUS

$n_s = 1x$



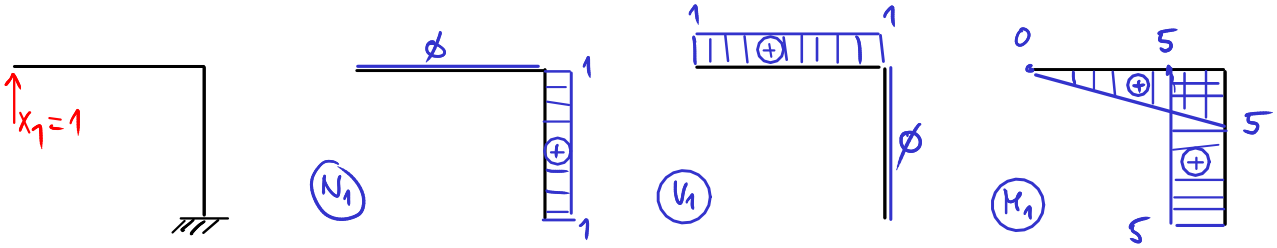




0

2) vykreslení vnitřních sil pro jednotlivé zat. stavy

- není silové zatížení $\Rightarrow N_0, V_0, M_0 = 0$



3) sestavení rovnic a výpočet δ

$$\delta_{11} X_1 + \delta_{10} = 0$$

$$\delta_{11} = \frac{1}{EI} \int M_1 M_1 \, dx = \frac{1}{EI} \left[+\frac{1}{2} 5 \cdot 5 \cdot \left(\frac{2}{3} 5\right) + 4 \cdot 5 \cdot (5) \right] = \frac{141{,}6}{EI}$$

$$\delta_{10} = \int N_1 \alpha_T \Delta t_0 \, dx + \int M_1 \alpha_T \frac{\Delta t_1}{h} dx = \left| \begin{array}{l} \Delta t_0 = \frac{t_k + t_d}{2} = \frac{10+20}{2} = 15°C \\ \Delta t_1 = t_d - t_k = 20 - 10 = 10°C \end{array} \right| =$$

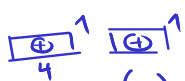
$$= \underbrace{10^{-5} \cdot 15 \cdot \left[0 + 4 \cdot 1 \right]}_{6 \cdot 10^{-4}} + \underbrace{10^{-5} \cdot \frac{10}{0{,}4} \left[\frac{1}{2} 5 \cdot 5 + 4 \cdot 5 \right]}_{8{,}125 \cdot 10^{-3}} = 8{,}725 \cdot 10^{-3}$$

$$\frac{141{,}6}{EI} X_1 + 8{,}725 \cdot 10^{-3} = 0$$

$$X_1 = - \frac{8{,}725 \cdot 10^{-3} \cdot EI}{141{,}6} = -2{,}956\ kN$$

$EI = 48000\ kPa\ m^4$ $\qquad$ $EA = 3{,}6 \cdot 10^6\ kPa\ m^2$

4) přidání normálového členu δ_{11}



$$^N\delta_{11} = \frac{1}{EA}\int N_1 N_1 \, dx = \frac{1}{EA}\left[0 + 4\cdot 1\cdot(1)\right] = \frac{4}{EA}$$

$$^N\delta_{11} = 1{,}1\bar{1}\cdot 10^{-6}$$

$$^M\delta_{11} = \frac{141{,}\bar{6}}{EI} = 2{,}95138\cdot 10^{-3}$$

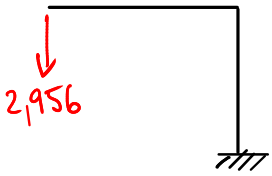
$$X_1 = -\frac{8{,}725\cdot 10^{-3}}{^N\delta_{11} + {}^M\delta_{11}} = -2{,}9558\,kN$$

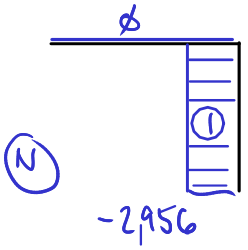
5) průběh vnitřních sil

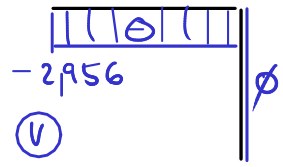
$$N = N_0 + N_1 X_1$$

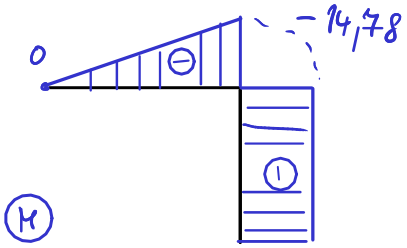
$$V = V_0 + V_1 X_1$$

$$M = M_0 + M_1 X_1$$

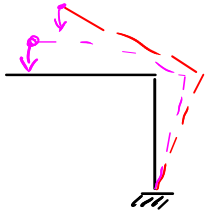


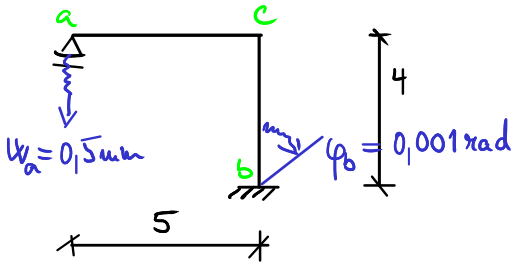






Rám – posun (popuštění) podpor

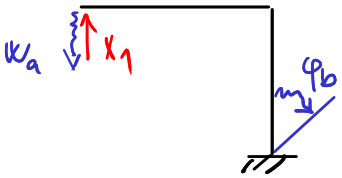


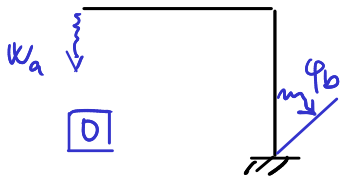


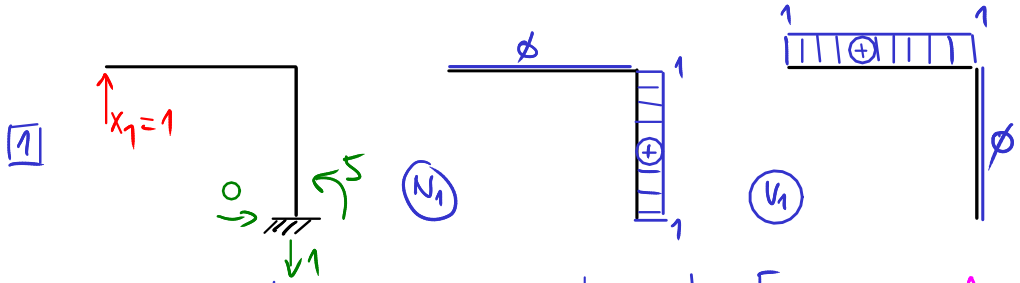
1) ns a ZSUS
viz předchozí

2) výpočet reakcí a vnitřních sil

– není silové zatížení $\Rightarrow N_0, V_0, M_0 = 0$





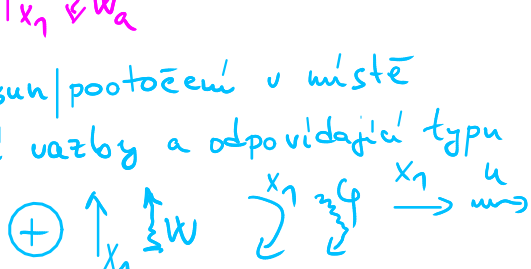


3) sestavení rovnic a výpočet δ

$$\delta_{11} X_1 + (\delta_{10} + \delta_{10}^{PP}) = -w_a$$

δ_{10}^{PP} … ostatní deformace

$-w_a$ … posun/pootočení v místě odebrané vazby a odpovídající typu vazby



– vazbě neodpovídá žádná def $\Rightarrow 0$

$$\delta_{11} = \frac{141{,}6}{EI} = 2{,}95138 \cdot 10^{-3}$$

$$\delta_{10}^{PP} = -\sum_r \left({}^1R_{rx} u_r + {}^1R_{rz} w_r + {}^1M_r \varphi_r\right) = \left| r \in \{b\} \right| =$$

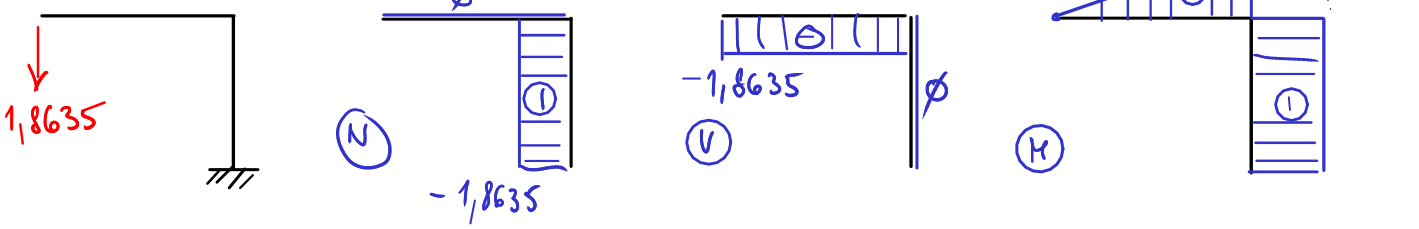
$5 \cdot \varphi_b$

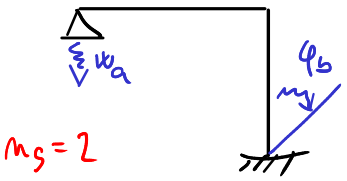
$$= -\left({}^1R_{bx} u_b + {}^1R_{bz} w_b + {}^1M_b \varphi_b\right) = -\left(0 \cdot 0 + 1 \cdot 0 - 5 \cdot 0{,}001\right) = \underline{0{,}005}$$

$$2{,}95138 \cdot 10^{-3} X_1 + 0{,}005 = -0{,}5 \cdot 10^{-3}$$

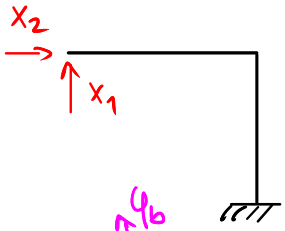
$$\underline{X_1 = -1{,}8635\,kN}$$

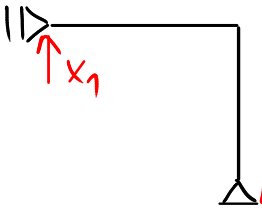
4) průběh N, V, M





$n_s = 2$





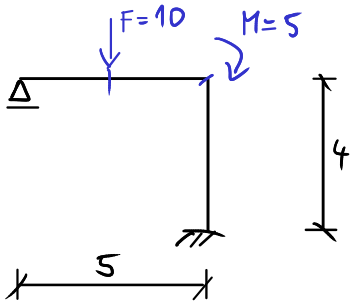
φ_b

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{10}^{PP} = -w_a$$
$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{20}^{PP} = 0$$

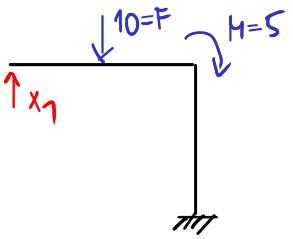
φ_b

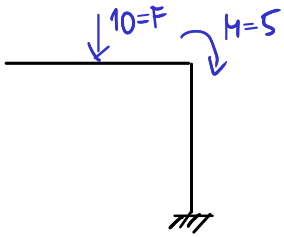
$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{10}^{PP} = -w_a$$
$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{10}^{PP} = +\varphi_b$$

0 0



1) $n_s = 1$, ZSUS





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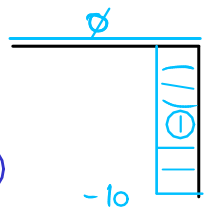

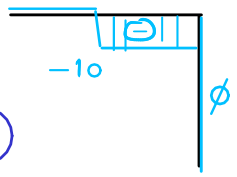

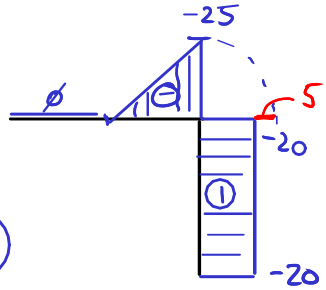

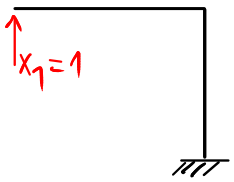

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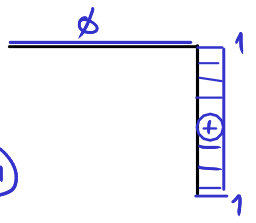
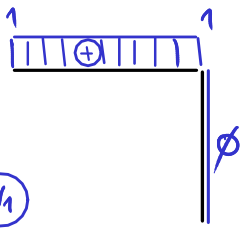
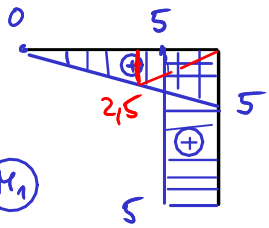


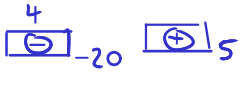


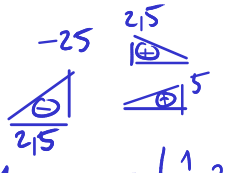


3) sestavení rce a výpočet δ

$$\delta_{11}X_1 + \delta_{10} = 0$$

$$\delta_{11} = \frac{141{,}6}{EI}$$

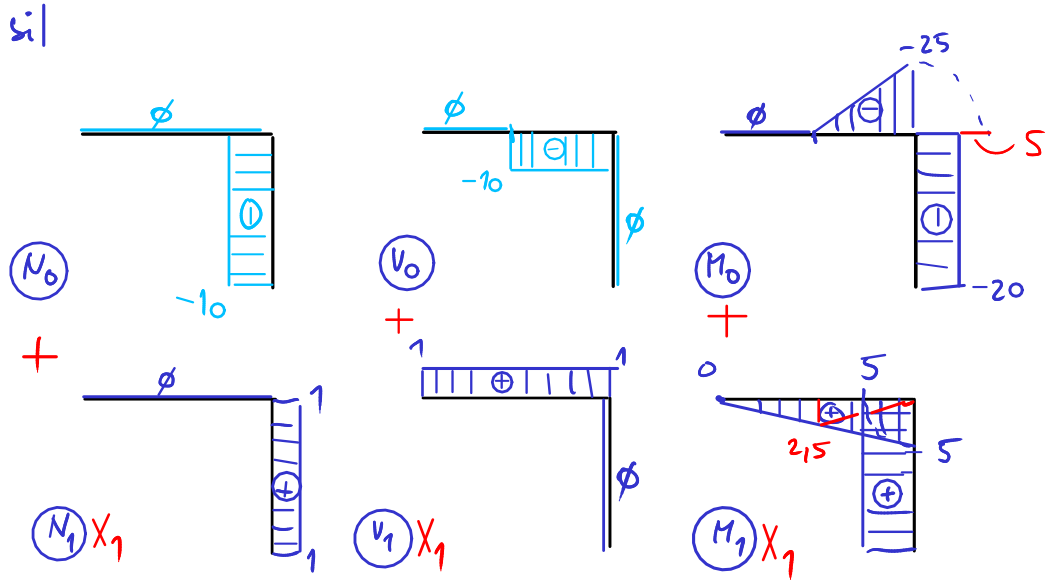




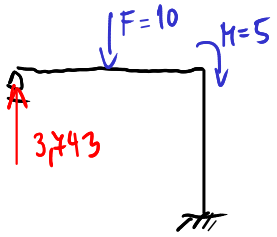
$$\delta_{10} = \frac{1}{EI}\int M_0 M_1 dx = \frac{1}{EI}\left[-20\cdot 4\cdot(5) - \frac{1}{2}2{,}5\cdot 25\cdot\left(\frac{1}{3}2{,}5 + \frac{2}{3}5\right)\right] = -\frac{530{,}208\overline{3}}{EI}$$

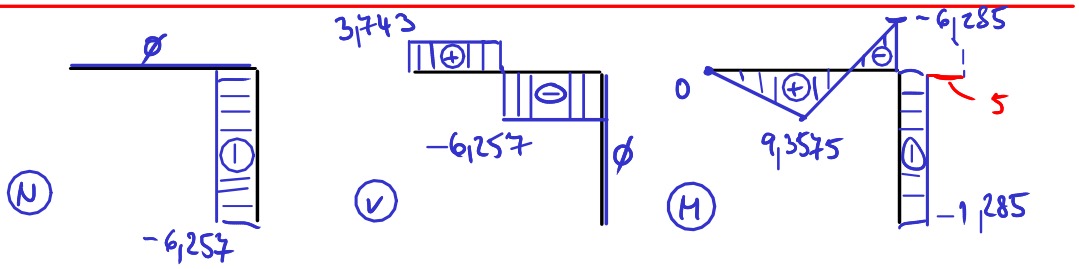
$$\frac{141{,}6}{EI}X_1 - \frac{530{,}208\overline{3}}{EI} = 0 \Rightarrow \underline{X_1 = +3{,}743\,kN}$$

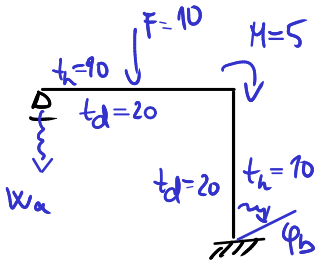
4) průběh vnitřních sil



$X_1 = +3{,}7432\,kN$

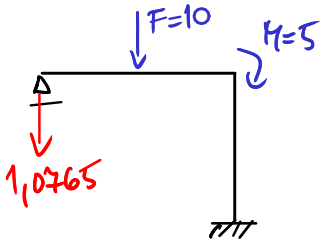


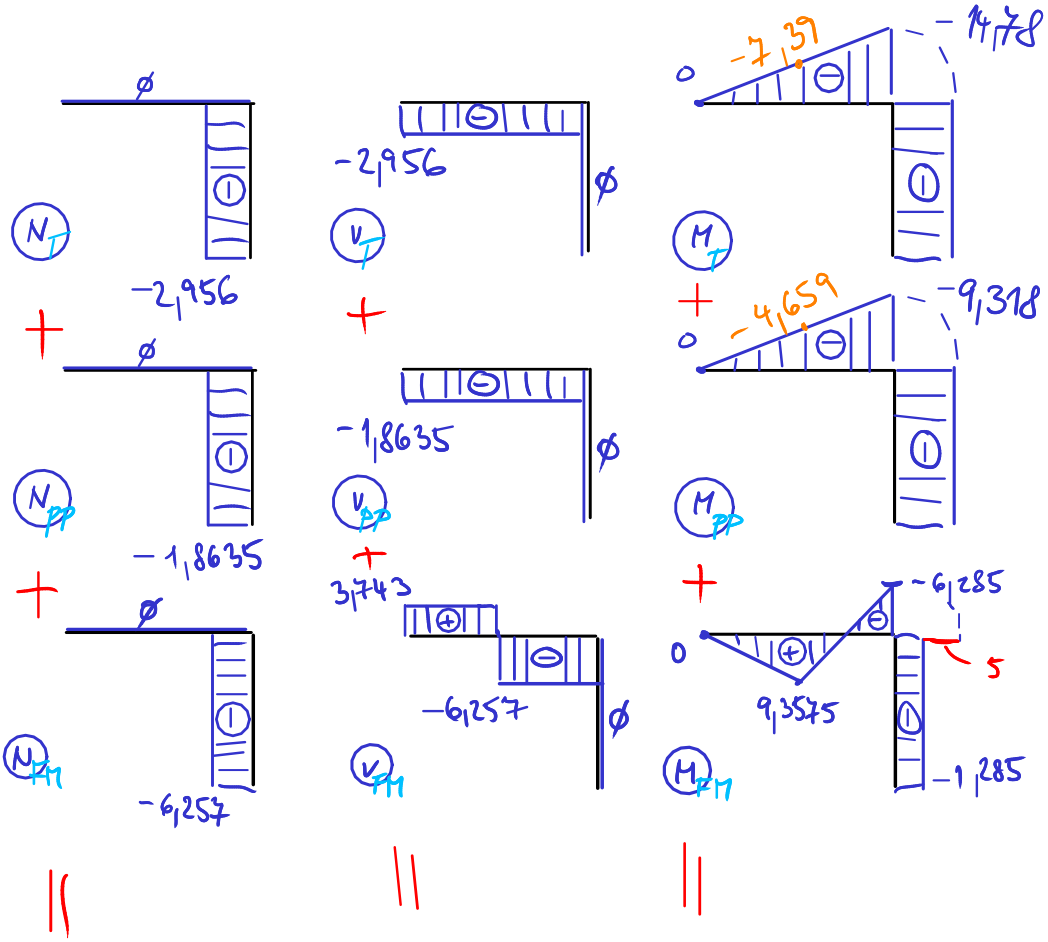




$$\left(f_{11} X_1 + \delta_{10}^{T} + \delta_{10} + \delta_{10}^{PP} = -w_a \right)$$

$$X = -2{,}956 - 1{,}8635 + 3{,}743 = -1{,}0465\,kN$$





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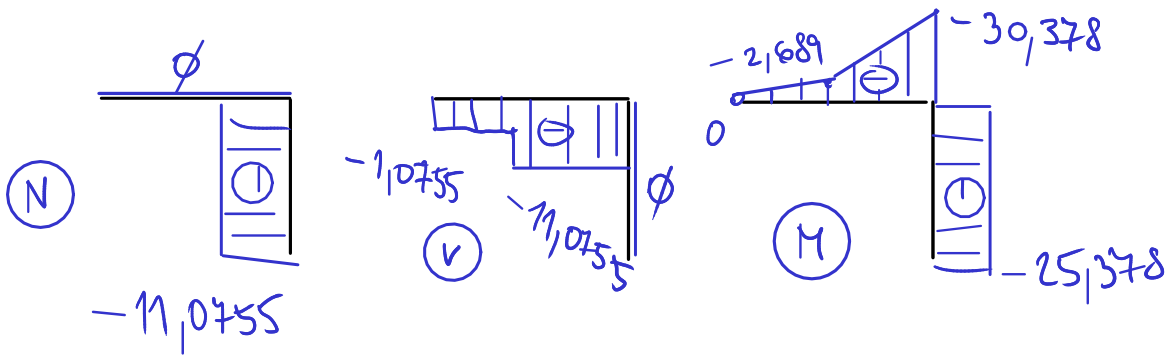
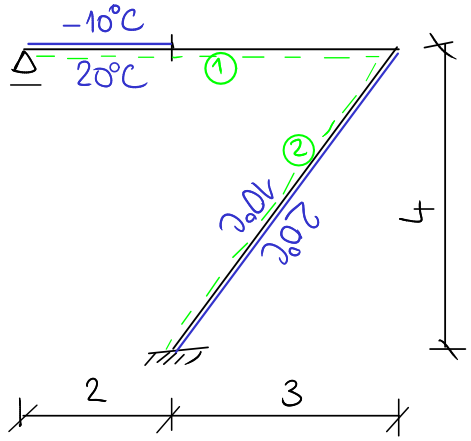

N
Ø
−11,0755
V
−1,0755
−11,0755
Ø
M
−2,689
0
−30,378
−25,378

ZATÍŽENÍ TEPLOTOU



① $0{,}3 \times 0{,}3$ < $I_1 = 0{,}675 \cdot 10^{-3}\ m^4$, $A_1 = 0{,}09\ m^2$

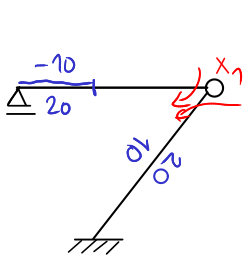
② $\square_{0{,}3}^{0{,}4}$ < $I_2 = 1{,}6 \cdot 10^{-3}\ m^4$, $A_2 = 0{,}12\ m^2$

konvence ⊕ ohýbá prut na stranu vláken

$E = 30\ GPa \qquad \alpha_T = 10^{-5}\ °C^{-1}$

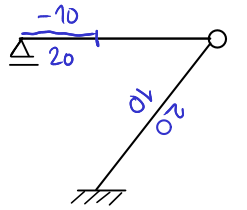
$\Delta^1 t_0 = \frac{^1t_d + {}^1t_h}{2} = \frac{20-10}{2} = 5°C \qquad \Delta^1 t_1 = {}^1t_d - {}^1t_h = 20-(-10) = 30°C$

$\Delta^2 t_0 = \frac{^2t_d + {}^2t_h}{2} = \frac{10+20}{2} = 15°C \qquad \Delta^2 t_1 = {}^2t_d - {}^2t_h = 10-20 = -10°C$



ZSUS $n_s = 1$

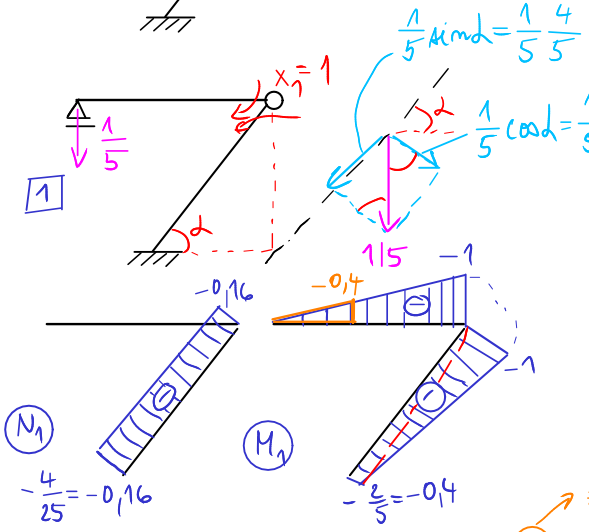
$$\delta_{11} X_1 + \delta_{10} = 0$$



$$^M\delta_{11} = \frac{1}{EI}\int M_1 M_1 dx = \frac{1}{EI_1}\left[+\frac{1}{2}5\cdot 1\cdot\left(\frac{2}{3}1\right)\right] +$$

$$+\frac{1}{EI_2}\left[+\frac{1}{2}5\cdot 0{,}4\cdot\left(\frac{2}{3}0{,}4+\frac{1}{3}1\right)+\frac{1}{2}5\cdot 1\left(\frac{2}{3}1+\frac{1}{3}0{,}4\right)\right] =$$

$$= \underbrace{\frac{1{,}6}{EI_1}}_{30\cdot 10^6,\ 0{,}675\cdot 10^{-3}} + \underbrace{\frac{2{,}6}{EI_2}}_{30\cdot 10^6,\ 1{,}6\cdot 10^{-3}} = 82{,}305\cdot 10^{-6} + 54{,}166\cdot 10^{-6} = \underline{136{,}471\cdot 10^{-6}}$$



$$^N\delta_{11} = \frac{1}{EA}\int N_1 N_1 dx = \frac{1}{EA_2}\left[5\cdot 0{,}16\cdot 0{,}16\right] = \frac{0{,}128}{EA_2} =$$

$(30\cdot 10^6,\ 0{,}12)$

$$= \underline{0{,}035\cdot 10^{-6}} \ll {}^M\delta_{11} = 136{,}471\cdot 10^{-6}$$

můžeme zanedbat (nepočítat)

zatížení je pouze na délce 2m pro prut ①

$$^M\delta_{10} = \alpha_T \frac{\Delta t_1}{h}\int M_1 dx = \alpha_T \frac{\Delta^1 t_1}{h_1}\int_0^{\textcircled{2}} {}^1M_1 dx + \alpha_T \cdot \frac{\Delta^2 t_1}{h_2}\int_0^5 {}^2M_1\, dx =$$

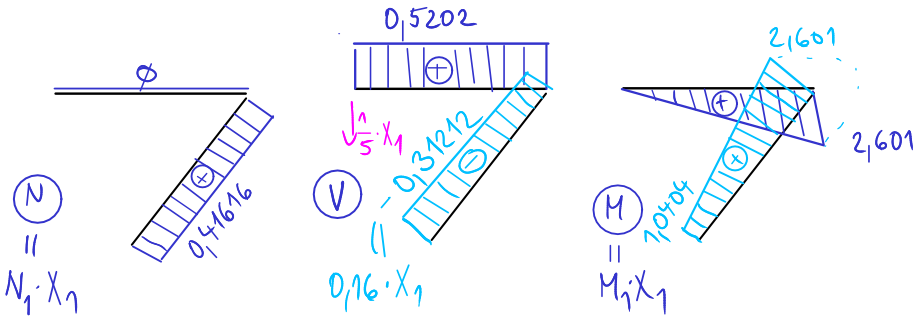
$$= 10^{-5}\cdot\frac{30}{0{,}3}\cdot\left[-\frac{1}{2}2\cdot 0{,}4\right] + 10^{-5}\cdot\frac{(-10)}{0{,}4}\cdot\left[-\frac{0{,}4+1}{2}\cdot 5\right] = \underline{0{,}475\cdot 10^{-3}}$$

$$^N\delta_{10} = \alpha_T \Delta t_0 \int N_1 dx = \alpha_T \cdot \Delta^2 t_0 \cdot \int_0^5 {}^2N_1 dx = 10^{-5}\cdot 15\cdot\left[-5\cdot 0{,}16\right] = \underline{-0{,}12\cdot 10^{-3}}$$

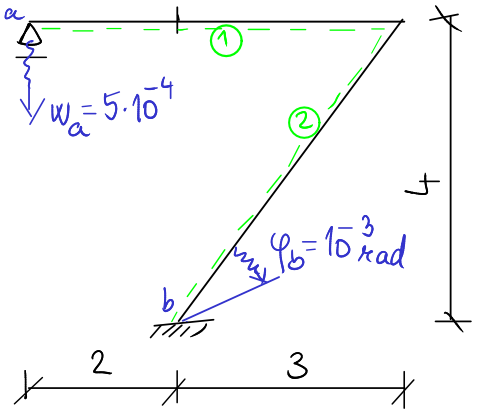
$$\delta_{11} X_1 + \delta_{10} = 0$$

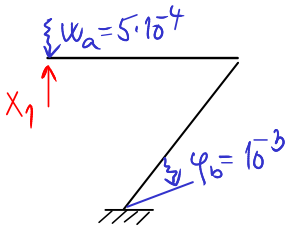
$$136{,}471\cdot 10^{-6} X_1 + \left(0{,}475\cdot 10^{-3} - 0{,}12\cdot 10^{-3}\right) = 0$$

$$X_1 = \frac{-0{,}355\cdot 10^{-3}}{136{,}471\cdot 10^{-6}} = \underline{-2{,}601\ kNm}$$



Popuštění podpor

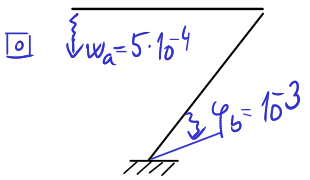




ZSUS $n_s = 1$ $\Rightarrow$ $\delta_{11} X_1 + \delta_{10} = d_1$

$$M\delta_{11} = \frac{1}{EI_1}\left[\frac{1}{2}5 \cdot 5 \cdot \left(\frac{2}{3}5\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}5 \cdot 5 \cdot \left(\frac{1}{3}2 + \frac{2}{3}5\right) + \frac{1}{2}2 \cdot 5 \cdot \left(\frac{1}{3}5 + \frac{2}{3}2\right)\right] =$$

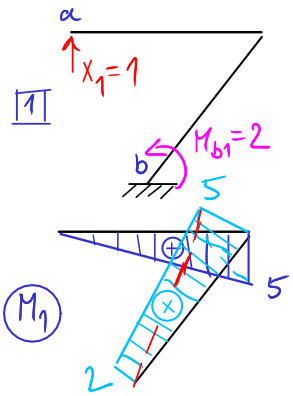
$$= \frac{41{,}\overline{6}}{EI_1} + \frac{65}{EI_2}$$



$$\delta_{10} = -\sum_n \bar{R}_n \cdot \delta_n = -\sum_n \left(\bar{R}_{nx} \cdot u_n + \bar{R}_{nz} w_n + \bar{M}_n \varphi_n\right) =$$

$$= -\left(M_{b1} \varphi_b\right) = -\left(\ominus 2 \cdot 10^{-3}\right) = 2 \cdot 10^{-3}$$

(2 $\curvearrowright$, $\downarrow 10^{-3}$; $\ominus$ opačný směr M a φ)



$\delta_{11} X_1 + \delta_{10} = d_1$ $\rightarrow$ deformace v místě a ve směru odebrané vazby

($\uparrow X_1$, $\downarrow w_a = 5 \cdot 10^{-4}$)

$$\left(\frac{41{,}\overline{6}}{EI_1} + \frac{65}{EI_2}\right) X_1 + 2 \cdot 10^{-3} = \ominus 5 \cdot 10^{-4}$$

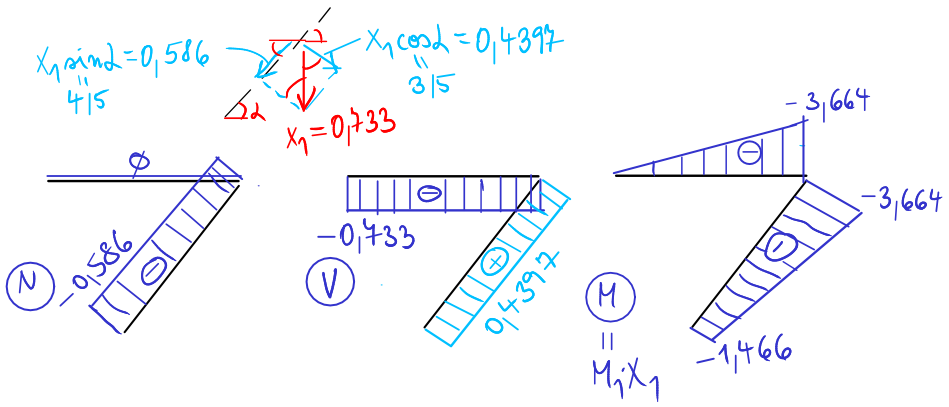
($\ominus$ opačný směr X_1 a w_a)

/ $\cdot E$

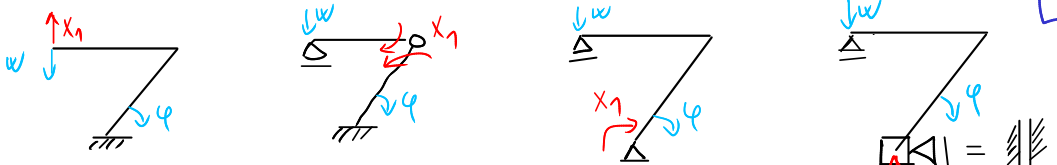
$$\left(\frac{41{,}\overline{6}}{I_1} + \frac{65}{I_2}\right) X_1 = -2{,}5 \cdot 10^{-3} \cdot E$$

$$102\,353{,}3951 X_1 = -75000$$

$$X_1 = -0{,}7332\ \text{kN}$$



Volba ZSUS a vliv na rovnici při popuštění podpor $\delta_{11}X_1 + \delta_{10} = d_1$



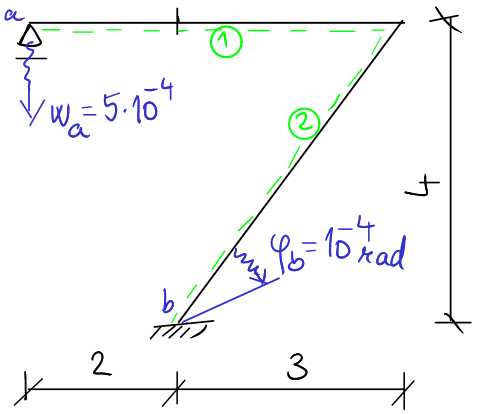
$\delta_{11}X_1 + \delta_{10} = -w$ (φ)

$\delta_{11}X_1 + \delta_{10} = 0$ (w, φ)

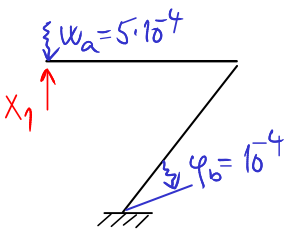
$\delta_{11}X_1 + \delta_{10} = +\varphi$ (w)

$\delta_{11}X_1 + \delta_{10} = 0$ (w, φ)

Popuštění podpor $\varphi_b = 10^{-4}$



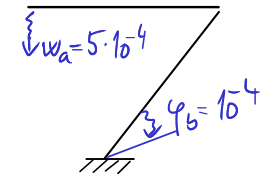
ZSUS $n_s = 1$ $\Rightarrow \delta_{11} X_1 + \delta_{10} = d_1$



$$M_{\delta_{11}} = \frac{1}{EI_1}\left[\frac{1}{2}5 \cdot 5 \cdot \left(\frac{2}{3}5\right)\right] + \frac{1}{EI_2}\left[\frac{1}{2}5 \cdot 5 \cdot \left(\frac{1}{3}2 + \frac{2}{3}5\right) + \frac{1}{2}2 \cdot 5 \cdot \left(\frac{1}{3}5 + \frac{2}{3}2\right)\right] =$$

$$= \frac{41{,}\overline{6}}{EI_1} + \frac{65}{EI_2}$$

[0]

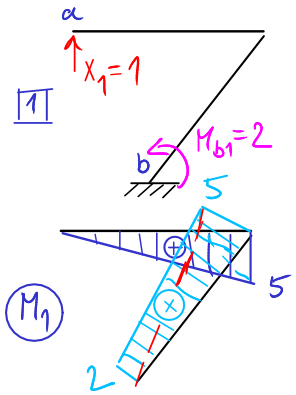


$$\delta_{10} = -\sum_r \bar{R}_r \cdot \delta_r = -\sum_r \left(\bar{R}_{rx} \cdot u_r + \bar{R}_{rz} w_r + \bar{M}_r \varphi_r\right) =$$

$$= -\left(M_{b1} \varphi_b\right) = -\left(\ominus 2 \cdot 10^{-4}\right) = 2 \cdot 10^{-4}$$

(2 ↷, 10^{-4} ↝; $\ominus$ = opačný směr M a φ)

[1]



$\delta_{11} X_1 + \delta_{10} = d_1$ → deformace v místě a ve směru odebrané vazby

$\uparrow X_1$ $\downarrow w_a = 5 \cdot 10^{-4}$

$$\left(\frac{41{,}\overline{6}}{EI_1} + \frac{65}{EI_2}\right) X_1 + 2 \cdot 10^{-4} = \ominus 5 \cdot 10^{-4}$$

($\ominus$ = opačný směr X_1 a w_a)

/ · E

$$\left(\frac{41{,}\overline{6}}{I_1} + \frac{65}{I_2}\right) X_1 = -7 \cdot 10^{-4} \cdot E$$

$$102\,353{,}3951 X_1 = -21000$$

$$X_1 = -0{,}205 \text{ kN}$$