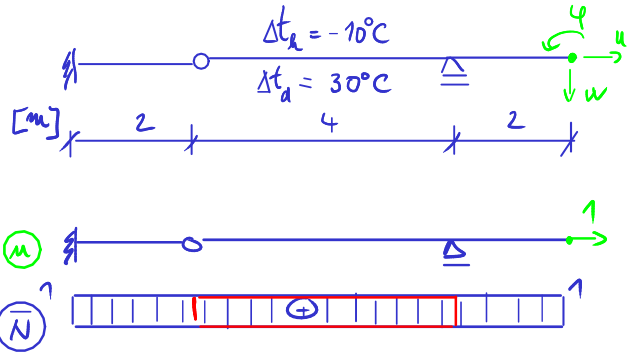
Výpočet posunutí a pootočení

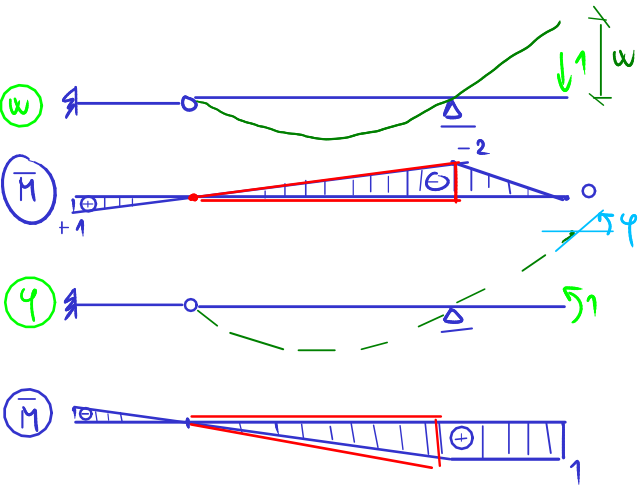
Maxwell-Mohrův vzorec

$$\delta = (u, w, \varphi) = \int_0^s \frac{N\bar{N}}{EA}\,ds + \int_0^s \varkappa \frac{V\bar{V}}{GA}\,ds + \int_0^s \frac{M\bar{M}}{EI}\,ds + \int_0^s \bar{N}\alpha_t \Delta t_0\,ds + \int_0^s \bar{M}\alpha_t \frac{\Delta t_1}{h}\,ds -$$

$$- \sum (\bar{R}_{kr} u_r + \bar{R}_{zr} w_r + \bar{M}_r \varphi_r)$$

Vypočítejte deformace u, w, φ volného konce.



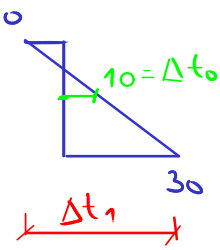


$\alpha_T = 10^{-5} K^{-1} = 10^{-5} °C^{-1}$

$$\delta = \int_0^s \bar{N}\alpha_T \Delta t_0\,ds + \int_0^s \bar{M}\alpha_T \frac{\Delta t_1}{h}\,ds$$

$$\Delta t_0 = \frac{\Delta t_d + \Delta t_h}{2} = -10°C$$

$$\Delta t_1 = \Delta t_d - \Delta t_h = 40°C$$

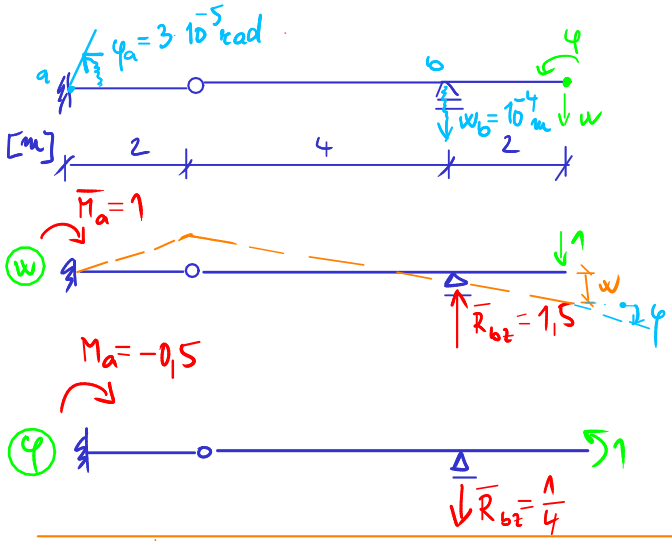


$$u = \alpha_T \Delta t_0 \int_0^s \bar{N}\,ds = \quad \left(\bar{N} = 1,\ s = 4\right)$$

$$= 10^{-5} \cdot 10 \cdot (1 \cdot 4) = \underline{4 \cdot 10^{-4}\ m}$$

$$w = \alpha_T \frac{\Delta t_1}{h} \int_0^s \bar{M}\,ds = 10^{-5} \cdot \frac{40}{0,3} \cdot \left(\frac{1}{2} \cdot 4 \cdot (-2)\right) = \underline{-5,3 \cdot 10^{-3}\ m}$$

$$\varphi = \alpha_T \frac{\Delta t_1}{h} \int_0^s \bar{M}\,ds = 10^{-5} \cdot \frac{40}{0,3} \cdot \left(\frac{1}{2} \cdot 4 \cdot 1\right) = \underline{2,6 \cdot 10^{-3}\ rad}$$



$$\delta = -\sum_{n = a,b} (\bar{R}_{nx} u_r + \bar{R}_{rz} w_r + \bar{M}_r \varphi_r)$$

$$w = -\left(\bar{M}_a \varphi_a + \bar{R}_{bz} w_b\right) =$$

$$\bar{M}_a = 1 \uparrow \varphi_a \qquad \uparrow \bar{R}_{bz} = 1,5 \downarrow w_b$$

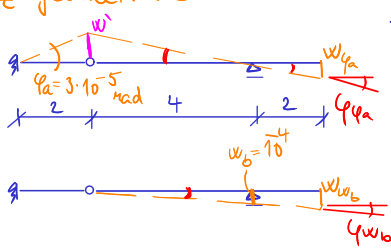
$$= -\left(-1 \cdot 3 \cdot 10^{-5} - 1,5 \cdot 10^{-4}\right) = \underline{1,8 \cdot 10^{-4}\ m}$$

$$\varphi = -\left(\bar{M}_a \varphi_a + \bar{R}_{bz} w_b\right) =$$

$$\bar{M}_a = -0,5 \uparrow \varphi_a \qquad \downarrow \bar{R}_{bz} = \frac{1}{4} \downarrow w_b$$

$$= -\left(-(-0,5) \cdot 3 \cdot 10^{-5} + \frac{1}{4} \cdot 10^{-4}\right) = \underline{-4 \cdot 10^{-5}\ rad}$$

z geometrie



$$\frac{w'}{2} = \operatorname{tg}\varphi_a \doteq \varphi_a \Rightarrow w' = 6 \cdot 10^{-5}\ m$$

$$\frac{w_{\varphi a}}{2} = \frac{w'}{4} \Rightarrow w_{\varphi a} = \frac{w'}{2} = 3 \cdot 10^{-5}\ m$$

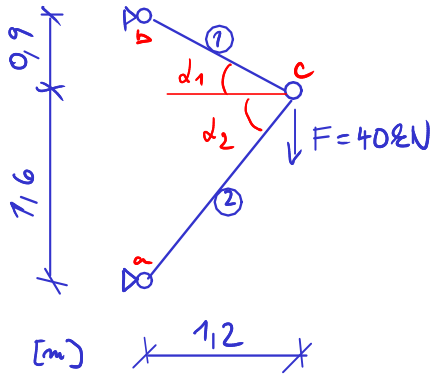
$$\frac{w_b}{4} = \frac{w_{wb}}{6} \Rightarrow w_{wb} = 1,5 \cdot 10^{-4}$$

$$w = w_{wb} + w_{\varphi a} = 1,5 \cdot 10^{-4} + 3 \cdot 10^{-5} = \underline{1,8 \cdot 10^{-4}\ m}\ \checkmark$$

$$\varphi_{\varphi a} = \frac{w_{\varphi a}}{2} = \frac{3}{2} \cdot 10^{-5} = 1,5 \cdot 10^{-5}$$

$$\varphi_{wb} = \frac{w_b}{4} = \frac{w_{wb}}{6} = \frac{1}{4} \cdot 10^{-4}$$

$$\varphi = \varphi_{\varphi a} + \varphi_{wb} = 1,5 \cdot 10^{-5} + \frac{1}{4} \cdot 10^{-4} = \underline{4 \cdot 10^{-5}\ rad}\ \checkmark$$

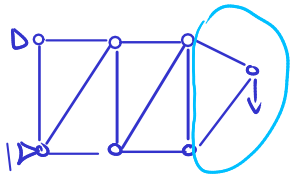


$E = 2 \cdot 10^8\ kPa$

$A_1 = 7{,}5 \cdot 10^{-4}\ m^2$

$A_2 = 20 \cdot 10^{-4}\ m^2$

$w_c, u_c = ?$

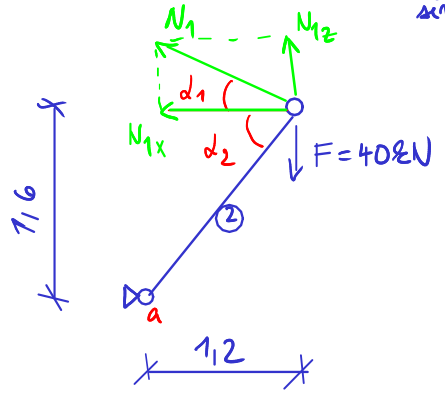


$l_1 = \sqrt{0{,}9^2 + 1{,}2^2} = 1{,}5\ m$

$l_2 = \sqrt{1{,}2^2 + 1{,}6^2} = 2\ m$

$\sin\alpha_1 = \frac{0{,}9}{1{,}5} = 0{,}6 \quad \cos\alpha_1 = \frac{1{,}2}{1{,}5} = 0{,}8$

$\sin\alpha_2 = \frac{1{,}6}{2} = 0{,}8 \quad \cos\alpha_2 = \frac{1{,}2}{2} = 0{,}6$



$\sum M_{ai} = 0 \quad \oplus\circlearrowleft$

$N_{1x} \cdot 1{,}6 + N_{1z} \cdot 1{,}2 - F \cdot 1{,}2 = 0$

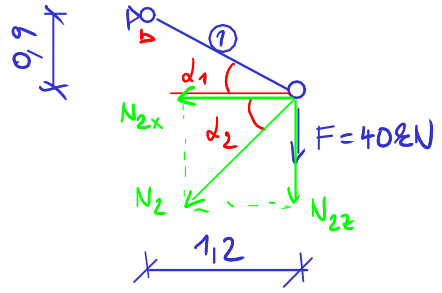
$N_1 = \frac{F \cdot 1{,}2}{1{,}6 \cdot \cos\alpha_1 + 1{,}2 \cdot \sin\alpha_1} = \underline{24\ kN}$

Styčníková metoda

$\sum F_{xi} = 0$

$\sum F_{zi} = 0$

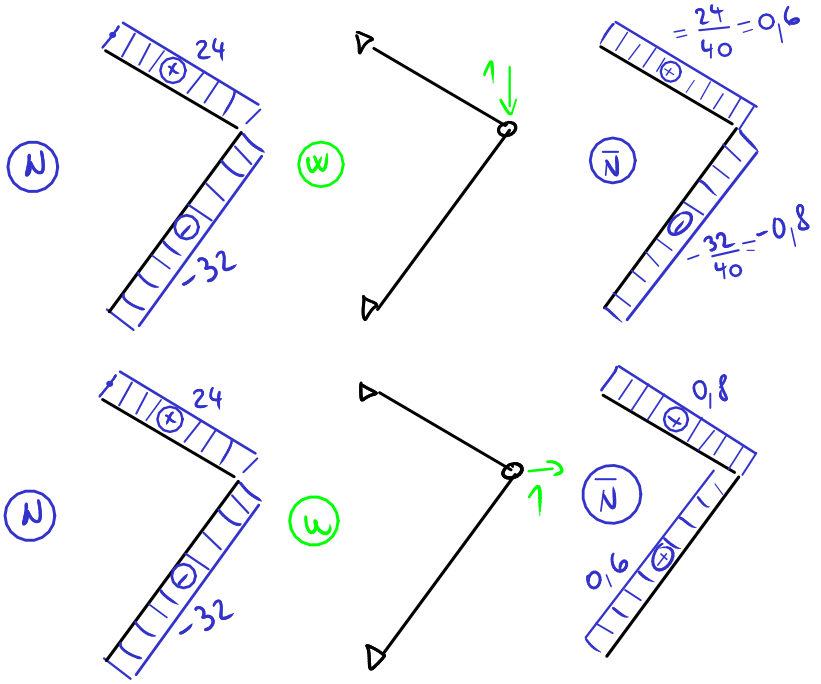
soustava 2 rovnic o 2 neznámých



$\sum M_{bi} = 0 \quad \oplus\circlearrowleft$

$N_{2x} \cdot 0{,}9 + N_{2z} \cdot 1{,}2 + F \cdot 1{,}2 = 0$

$N_2 = \frac{-F \cdot 1{,}2}{0{,}9 \cdot \cos\alpha_2 + 1{,}2 \cdot \sin\alpha_2} = \underline{-32\ kN}$



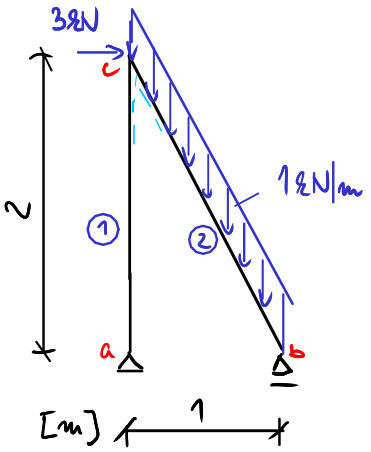
$w_c = \int_0^S \frac{N\bar{N}}{EA}\,ds = \frac{1}{EA_1}\int N\bar{N}\,ds + \frac{1}{EA_2}\int N\bar{N}\,ds$

$w_c = \frac{1}{EA_1}\left(1{,}5 \cdot 24 \cdot 0{,}6\right) + \frac{1}{EA_2}\left(+32 \cdot 2 \cdot 0{,}8\right) =$

$= \frac{21{,}6}{EA_1} + \frac{51{,}2}{EA_2} = \underline{2{,}72 \cdot 10^{-4}\ m}$

$u_c = \frac{1}{EA_1}\left(1{,}5 \cdot 24 \cdot 0{,}8\right) + \frac{1}{EA_2}\left(-2 \cdot 32 \cdot 0{,}6\right)$

$u_c = \frac{28{,}8}{EA_1} - \frac{38{,}4}{EA_2} = \underline{9{,}6 \cdot 10^{-5}\ m}$

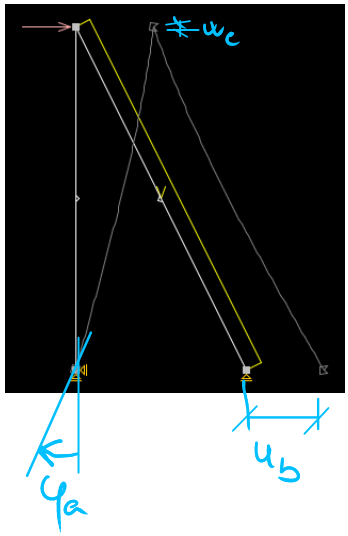


$E = 25\,GPa = 25 \cdot 10^6\ kPa$

$I = 1{,}5 \cdot 10^{-4}\ m^4$

$A = 0{,}03\ m^2$

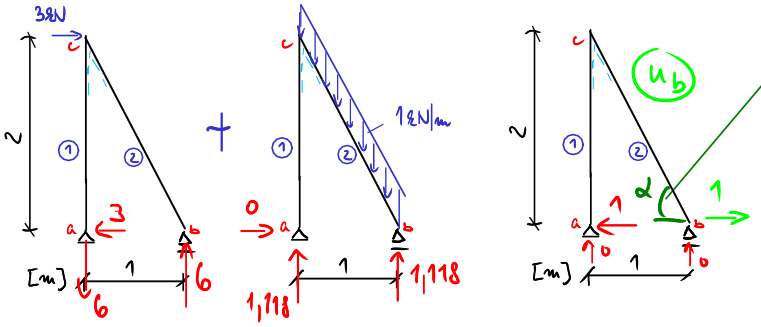
$u_b \mid w_c \mid \varphi_a = ?$

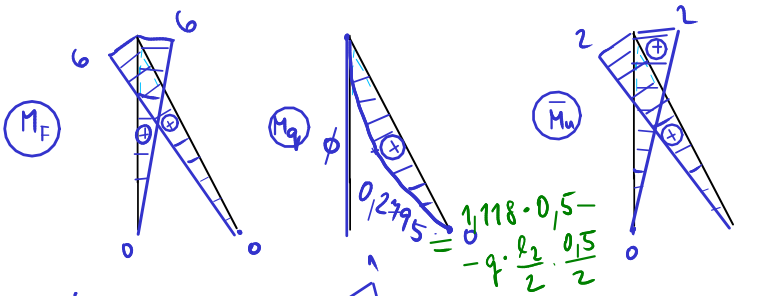


$$\delta = \int \frac{N\bar{N}}{EA}dx + \int \frac{M\bar{M}}{EI}dx$$

$$l_2 = \sqrt{1^2 + 2^2} = \sqrt{5}\ m$$

$$\cos\alpha = \frac{1}{\sqrt{5}} \qquad \sin\alpha = \frac{2}{\sqrt{5}}$$



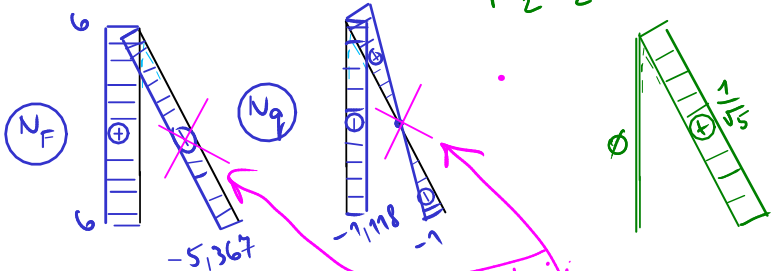


$$u_b = \int_0^S \frac{M\bar{M}}{EI}ds = \frac{1}{EI}\left[+\frac{1}{2}6 \cdot 2 \cdot \left(\frac{2}{3}2\right) + \frac{1}{2}6\sqrt{5} \cdot \left(\frac{2}{3}2\right) + \right.$$

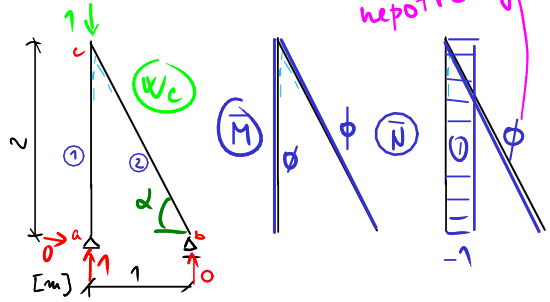
$$\left. + \frac{2}{3}\sqrt{5}\,0{,}2795 \cdot \left(\frac{1}{2}2\right)\right] = \frac{17{,}36}{EI} = 4{,}63 \cdot 10^{-3}\ m$$

$u_b = (\text{příspěvek } N) = \int \frac{N\bar{N}}{EA}dx =$

$$= \frac{1}{EA} \cdot \left[-5{,}367 \cdot \sqrt{5} \cdot \frac{1}{\sqrt{5}}\right] = -\frac{5{,}367}{EI} = -4{,}156 \cdot 10^{-6}\ m$$

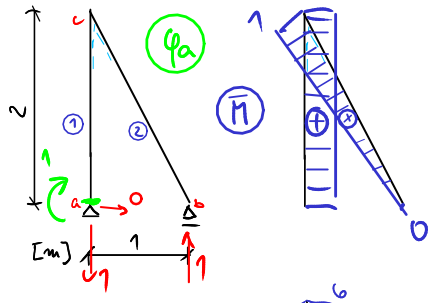


nepotřebuji



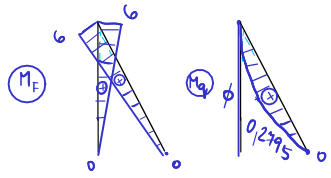
$$w_c = \int_0^S \frac{N\bar{N}}{EA}ds = \frac{1}{EA}\left[-6 \cdot 2 \cdot 1 + 2 \cdot 1{,}118 \cdot 1\right]$$

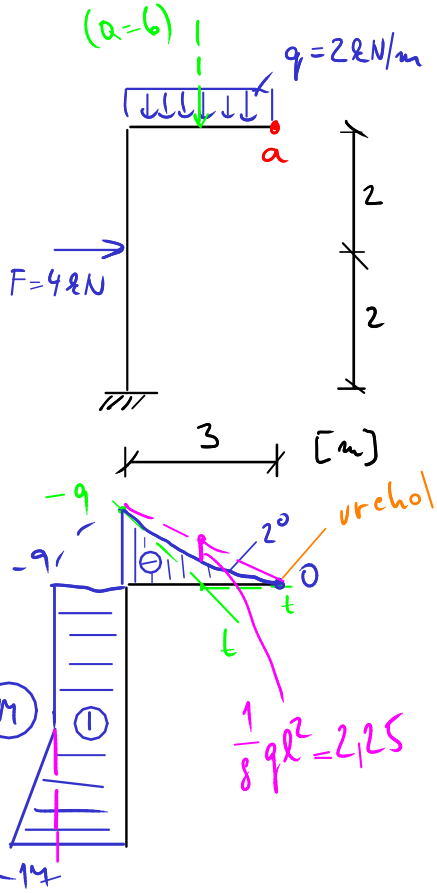
$$= \frac{-9{,}764}{EA} = -1{,}3 \cdot 10^{-5}\ m\ (\downarrow)$$



$$\varphi_a = \int_0^S \frac{M\bar{M}}{EI}ds = \frac{1}{EI}\left[+\frac{1}{2}6 \cdot 2 \cdot 1 + \frac{1}{2}6 \cdot \sqrt{5} \cdot \left(\frac{2}{3}1\right)\right.$$

$$\left. + \frac{2}{3}\sqrt{5} \cdot 0{,}2795 \cdot \left(\frac{1}{2}1\right)\right] = \frac{10{,}68}{EI} = 2{,}848 \cdot 10^{-3}\ rad\ (\circlearrowright)$$

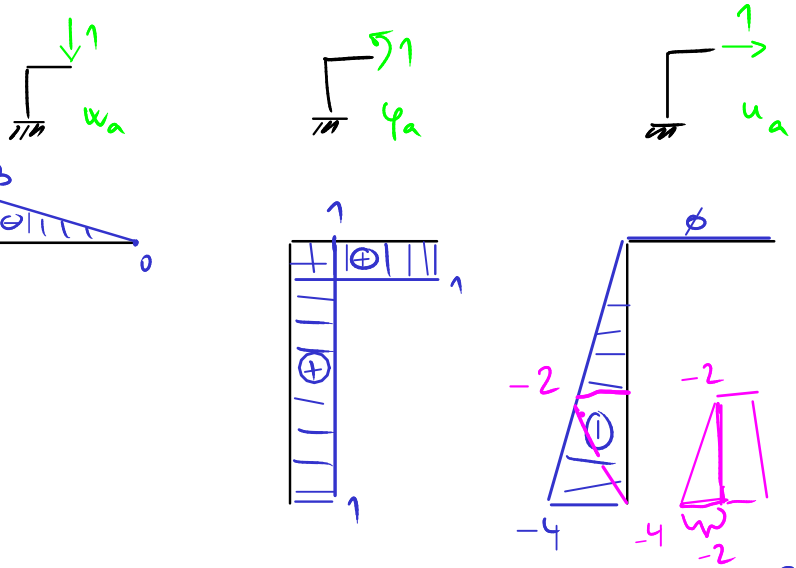




$EI = 10800\ kPa\ m^4$ ⟵ $E = 24\ GPa$, $I = 4{,}5 \cdot 10^{-4}\ m^4$, $A = 0{,}06\ m^2$

$w_a \mid \varphi_a \mid u_a = ?$

$$J = \int_0^{\ell} \frac{M\overline{M}}{EI}\,dx$$



$$w_a = \frac{1}{EI}\left[+\frac{1}{2}\cdot 9\cdot 3\cdot\left(\frac{2}{3}3\right) - \frac{2}{3}\cdot 3\cdot 2{,}25\cdot\left(\frac{1}{2}3\right) + \left(4\cdot 9+\frac{1}{2}\cdot 8\cdot 2\right)\cdot 3 \right]$$

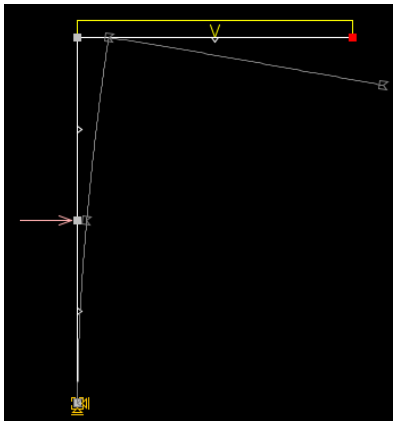
$$\varphi_a = \frac{1}{EI}\left[-\frac{1}{3}\cdot 9\cdot 3\cdot 1 - \left(4\cdot 9+\frac{1}{2}\cdot 8\cdot 2\right)\cdot 1 \right]$$

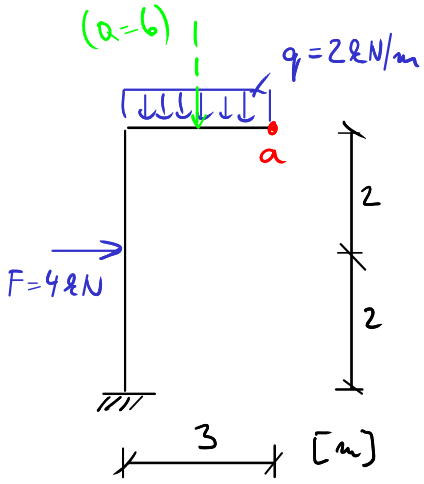
$$u_a = \frac{1}{EI}\left[+9\cdot 4\cdot 2 + \frac{1}{2}\cdot 8\cdot 2\left(\frac{2}{3}4\right) + \frac{1}{2}\cdot 8\cdot 2\cdot\left(\frac{1}{3}2\right) \right]$$

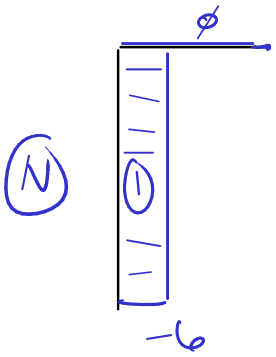
$$\varphi_a = \frac{-53}{EI} \simeq -4{,}9074\cdot 10^{-3}\ rad$$

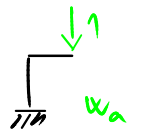
$$w_a = \frac{152{,}25}{EI} = 14{,}0972\cdot 10^{-3}\ m$$

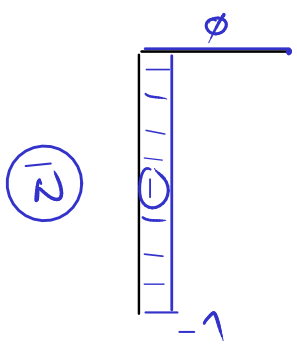
$$u_a = \frac{98{,}6}{EI} = 9{,}1358\cdot 10^{-3}\ m$$

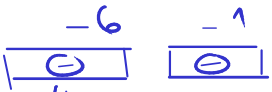












příspěvek (N)

$$w_a = \int \frac{N\bar{N}}{EA} ds = \frac{1}{EA}\left(+ 6 \cdot 4 \cdot 1 \right) = \frac{24}{EA} = \underline{\underline{1{,}6 \cdot 10^{-5}\ m}}$$