Výpočet přetvoření aplikací principu virtuálních prací

Maxwell-Mohrův vzorec

$$\delta = (u,w,\varphi) = \int_0^s \frac{N\bar N}{EA}\,ds + \int_0^s \varkappa \frac{V\bar V}{GA}\,ds + \boxed{\int_0^s \frac{M\bar M}{EI}\,ds} + \int_0^s \bar N \alpha_t \Delta t_0\,ds + \int_0^s \bar M \alpha_t \frac{\Delta t_1}{h}\,ds -$$

$$- \sum\left(\bar R_{xr} u_r + \bar R_{zr} w_r + \bar M_r \varphi_r\right)$$

N, V, M ... vnitřní síly od daného zatížení

$\bar N, \bar V, \bar M$ ... virtuální vnitřní síly od virtuálního zatížení $F=1$ nebo $M=1$

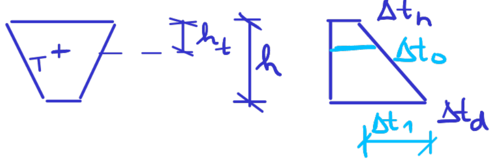
$\bar R$ ... virtuální složky reakcí ——"——

u_r, w_r, φ_r ... popuštění podpor

$\varkappa$ ... součinitel vyjadřující vliv nerovnoměrného rozdělení smykových napětí po výšce

Δt_0 ... rovnoměrné oteplení $\quad \Delta t_0 = \Delta t_h + \frac{h_t}{h}\Delta t_1$

$\Delta t_1 = \Delta t_d - \Delta t_h \qquad \left(h_t = \frac{h}{2} \quad \Delta t_0 = \frac{\Delta t_h + \Delta t_d}{2}\right)$



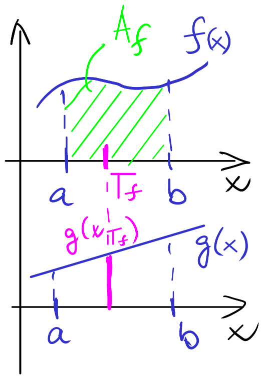
Vereščaginovo pravidlo $\int_0^s M\bar M\,ds$

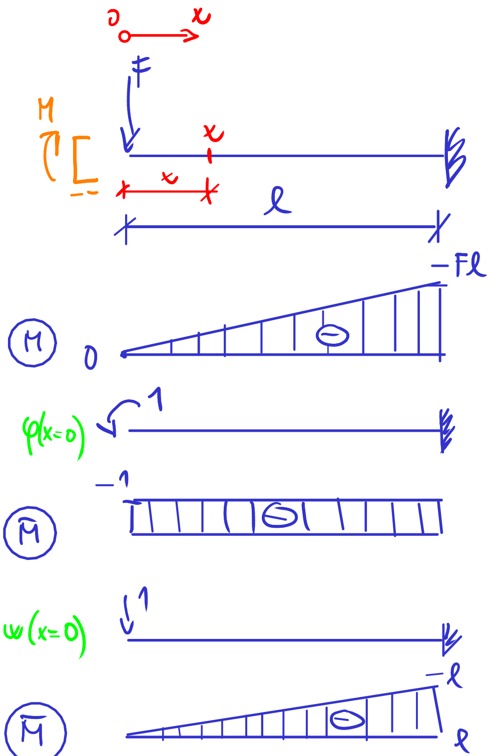
$$\int_a^b f_x g_x\,dx = A_f \cdot g(x_{Tf})$$

plocha pod f

fční hodnota funkce g v místě těžiště funce f

- maximálně lineární
- spojitá
- monotónní v $\langle a,b\rangle$
- integrovatelná
- nenulová plocha na $\langle a,b\rangle$
- nejsou-li předpoklady splněny, lze interval vhodně dělit





$$\delta = \int_0^l \frac{M\bar M}{EI}\,dx \qquad EI = konst.$$

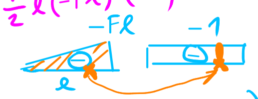
$$M(x) = -F\cdot x$$

$$\bar M(x) = -1$$

$$\varphi(x=0) = \frac{1}{EI}\int_0^l (-Fx)(-1)\,dx = \frac{F}{EI}\int_0^l x\,dx = \frac{F}{EI}\left[\frac{x^2}{2}\right]_0^l$$

$$\varphi(x=0) = \frac{Fl^2}{2EI}$$

$\frac{1}{2}l(-Fl)(-1)$

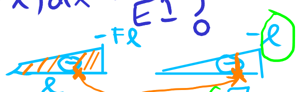


$$\varphi(x=0) = \frac{1}{EI} A_M \cdot g_{\bar M} = \frac{1}{EI}\left(+\frac{1}{2} l Fl \cdot 1\right) = \frac{Fl^2}{2EI}$$

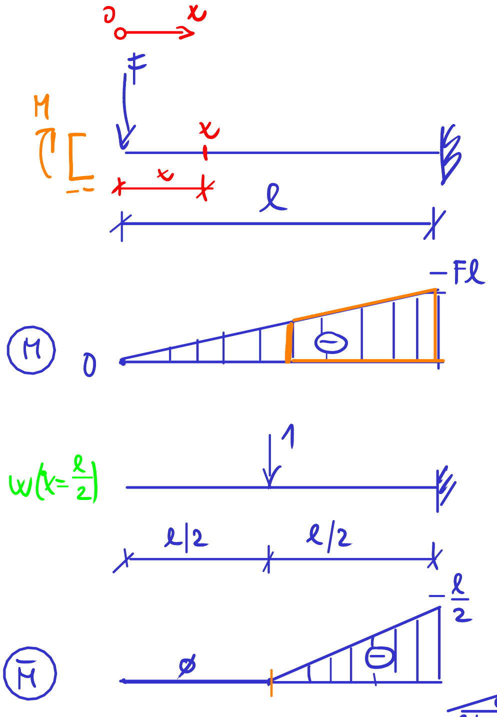
$$M(x) = -Fx$$

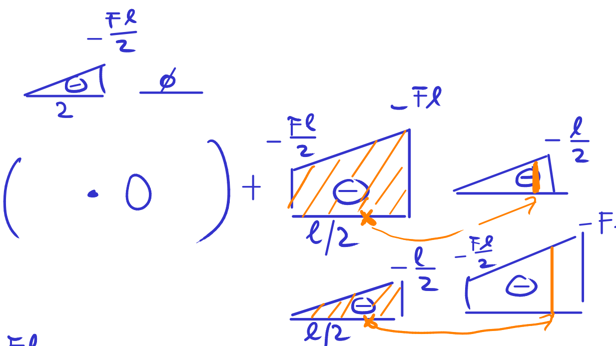
$$\bar M(x) = -x$$

$$w(x=0) = \frac{1}{EI}\int_0^l (-Fx)(-x)\,dx = \frac{F}{EI}\int_0^l x^2\,dx = \frac{F}{EI}\left[\frac{x^3}{3}\right]_0^l = \frac{Fl^3}{3EI}$$

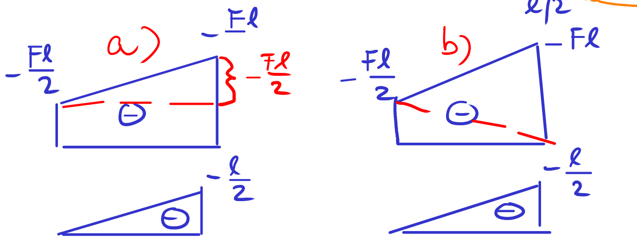


$$w(x=0) = \frac{1}{EI} A_M g_{\bar M} = \frac{1}{EI}\left[+\frac{1}{2} l Fl \cdot \frac{2}{3} l\right] = \frac{Fl^3}{3EI}$$





$$w\left(x=\frac{l}{2}\right)=\frac{1}{EI}\left[\left(\quad\cdot 0\quad\right)+\dots\right]$$



lichoběžník rozdělíme na jednodušší obrazce a), b)

$$w\left(x=\frac{l}{2}\right)\overset{a)}{=}\frac{1}{EI}\left[+\frac{1}{2}\frac{l}{2}\cdot\frac{Fl}{2}\left(\frac{2}{3}\frac{l}{2}\right)+\frac{Fl}{2}\cdot\frac{l}{2}\cdot\left(\frac{1}{2}\frac{l}{2}\right)\right]=\underline{\underline{\frac{5Fl^3}{48EI}}}$$

$$w\left(x=\frac{l}{2}\right)\overset{b)}{=}\frac{1}{EI}\left[+\frac{1}{2}Fl\frac{l}{2}\cdot\left(\frac{2}{3}\cdot\frac{l}{2}\right)+\frac{1}{2}\frac{Fl}{2}\frac{l}{2}\cdot\left(\frac{1}{3}\frac{l}{2}\right)\right]=\underline{\underline{\frac{5Fl^3}{48EI}}}$$

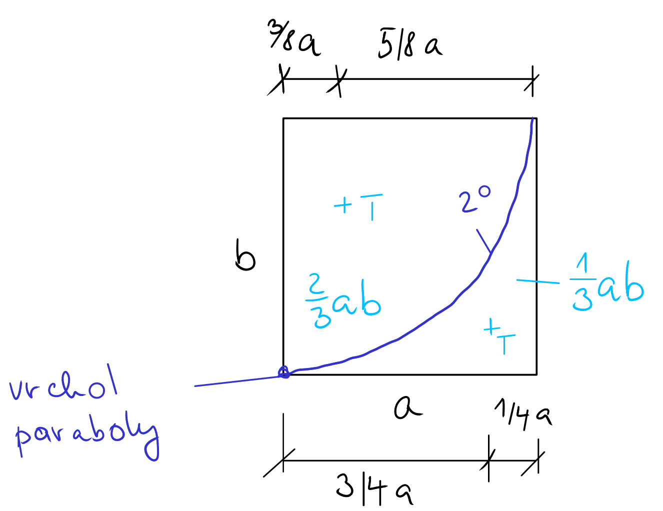
$$M(x)=-Fx$$

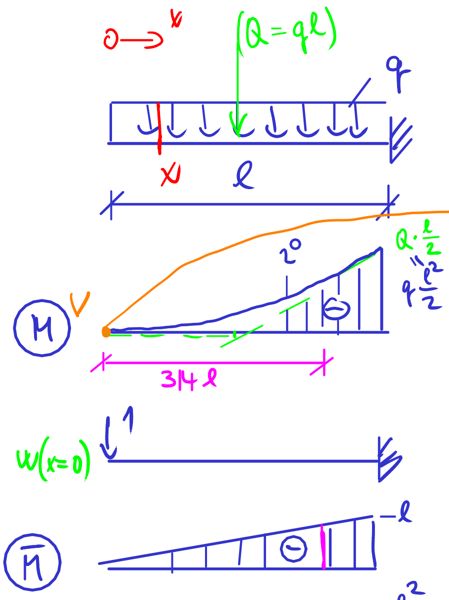
$$\bar{M}(x)=\begin{cases}0 & 0\le x\le\frac{l}{2}\\ -x+\frac{l}{2} & \frac{l}{2}\le x\le l\end{cases}$$

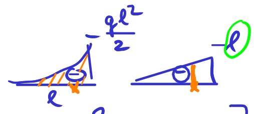
$$w\left(x=\frac{l}{2}\right)=\frac{1}{EI}\int_0^l M\bar{M}\,dx=\frac{1}{EI}\cdot\left[\int_0^{l/2}M\overset{0}{\bar{M}}\,dx+\int_{l/2}^{l}M\bar{M}\,dx\right]=\frac{1}{EI}\cdot\int_{l/2}^{l}(-Fx)\left(-x+\frac{l}{2}\right)dx=$$

$$=\frac{1}{EI}\int_{l/2}^{l}\left(Fx^2-Fx\frac{l}{2}\right)dx=\frac{F}{EI}\left[\frac{x^3}{3}-\frac{l}{2}\frac{x^2}{2}\right]_{l/2}^{l}=$$

$$=\frac{F}{EI}\left[\frac{l^3}{3}-\frac{l^3}{4}-\frac{l^3}{24}+\frac{l^3}{16}\right]=\underline{\underline{\frac{5Fl^3}{48EI}}}$$

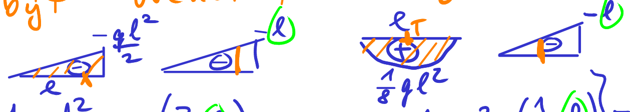




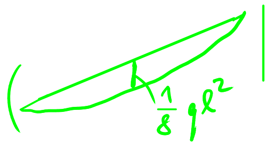


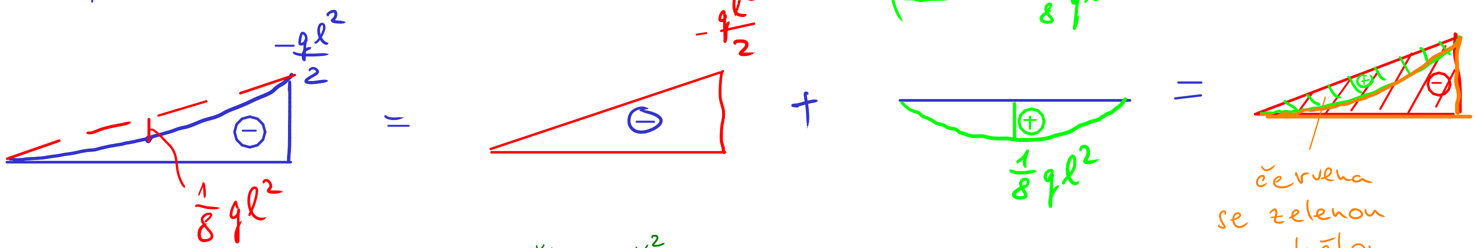
$$w(x=0) = \frac{1}{EI} A_M \eta_{\bar M} = \frac{1}{EI}\left[+\frac{1}{3} l \cdot q\frac{l^2}{2}\left(\frac{3}{4} l\right)\right] = \frac{q l^4}{8 EI}$$

musí být vrchol paraboly (vodorovná tečna)



$$w(x=0) = \frac{1}{EI}\left[+\frac{1}{2}\frac{ql^2}{2} l \cdot \left(\frac{2}{3} l\right) - \frac{2}{3} l \cdot \frac{1}{8} q l^2 \cdot \left(\frac{1}{2} l\right)\right] = \frac{q l^4}{8 EI}$$





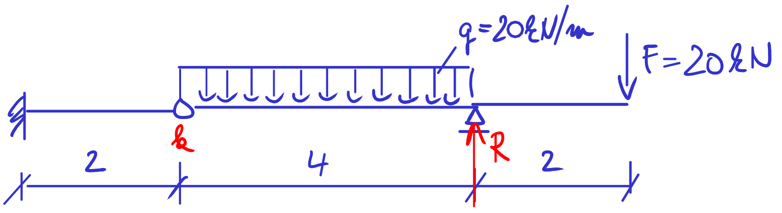
červená se zelenou se odečtou a získáme původní obrazec

$$M(x) = -q x \cdot \frac{x}{2} = -q\frac{x^2}{2}$$

$$\bar M(x) = -x$$

$$w(x=0) = \frac{1}{EI}\int_0^l \left(-q\frac{x^2}{2}\right)(-x)\,dx = \frac{q}{2EI}\int_0^l x^3\,dx = \frac{q}{2EI}\left[\frac{x^4}{4}\right]_0^l$$

$$w(x=0) = \frac{q l^4}{8 EI}$$

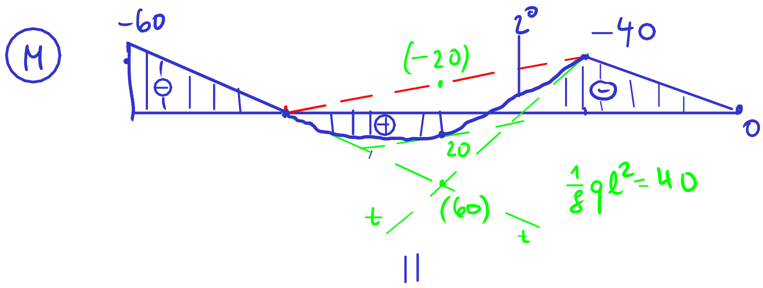


$$E = 210\,GPa = 210 \cdot 10^{6}\,kPa$$

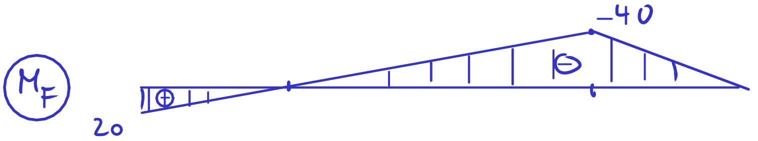
$$I = 9{,}49 \cdot 10^{-5}\,m^{4} \quad (I\,300)$$

$$\Sigma M_{ki}^{P} = 0 \quad \oplus$$

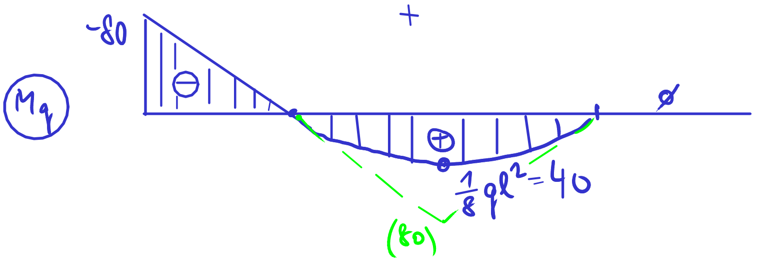
$$q \cdot 4 \cdot 2 + F \cdot 6 - R \cdot 4 = 0 \Rightarrow R = 70\,kN$$



||

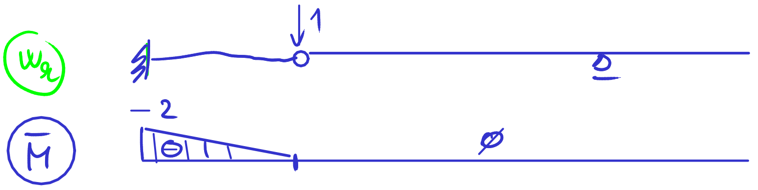


+



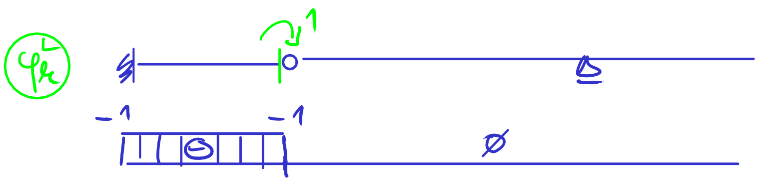
$$w_k = \frac{1}{EI}\left[+\frac{1}{2} 60 \cdot 2 \cdot \left(\frac{2}{3} 2\right)\right] = \frac{80}{EI} = \frac{80}{210 \cdot 10^{6} \cdot 9{,}49 \cdot 10^{-5}}$$

$$w_k = 3{,}891 \cdot 10^{-3}\,m$$



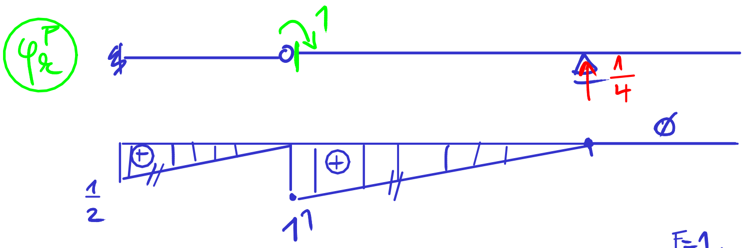
$$\varphi_k^{L} = \frac{1}{EI}\left[+\frac{1}{2} 2 \cdot 60 \cdot (1)\right] = \frac{60}{EI} =$$

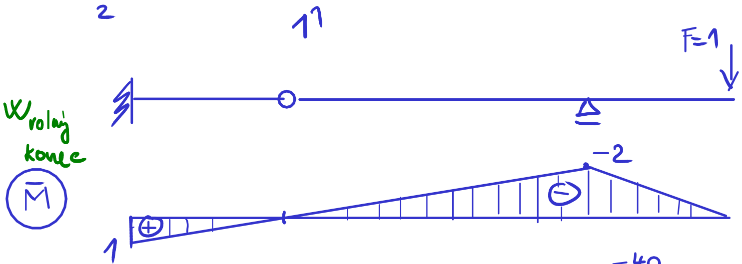
$$= 2{,}918 \cdot 10^{-3}\,rad$$



$$\varphi_k^{P} = \frac{1}{EI}\left[-\frac{1}{2} 2 \cdot 60 \cdot \left(\frac{2}{3} \frac{1}{2}\right) - \frac{1}{2} 4 \cdot 40 \cdot \left(\frac{1}{3} 1\right) + \frac{2}{3} 4 \cdot 40 \cdot \left(\frac{1}{2} 1\right)\right] = \frac{6{,}\overline{6}}{EI}$$

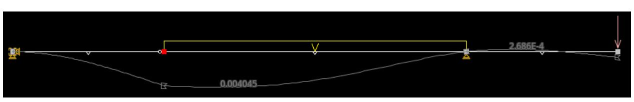
$$\varphi_k^{P} = 3{,}243 \cdot 10^{-4}\,rad$$





$$w_{volný\ konec} = \frac{1}{EI}\left[-\frac{1}{2} 2 \cdot 60 \cdot \left(\frac{2}{3} 1\right) + \frac{1}{2} 4 \cdot 40 \cdot \left(\frac{2}{3} 2\right) - \frac{2}{3} 4 \cdot 40 \cdot \left(\frac{1}{2} 2\right) + \frac{1}{2} 2 \cdot 40 \left(\frac{2}{3} 2\right)\right] =$$

$$= \frac{13{,}\overline{3}}{EI} = 6{,}485 \cdot 10^{-4}\,m$$



tvar ohybové čáry