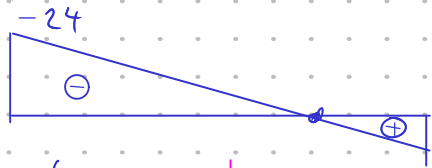


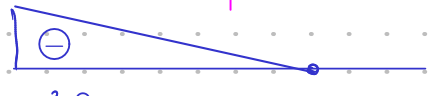
(M)



$$\tilde{M}(x=3) = \frac{1}{2} 30 \cdot 3 \cdot \left(\frac{2}{3} 3\right)$$

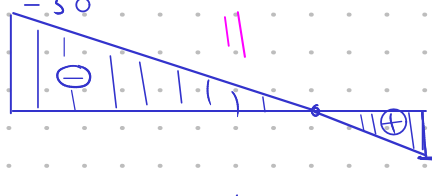
$$8 + \frac{1}{2} 15 \cdot 3 \cdot \left(\frac{1}{3} 3\right) = 112,5 \text{ kNm}$$

(M_F)



$$w(x=3) = \frac{\tilde{M}(x=3)}{EI}$$

(M)



$$= \frac{112,5}{60 \cdot 10^6 \cdot 7,2 \cdot 10^{-3}} = 0,260416 \cdot 10^{-3} \text{ m}$$

fiktivní nosník



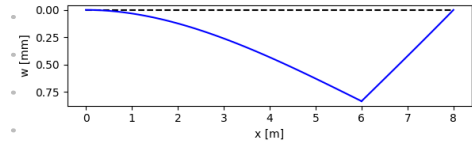
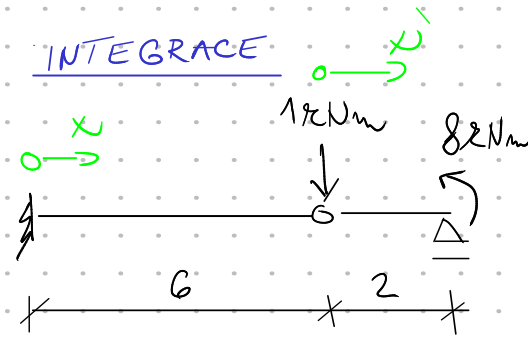
$$w(x=6) = \frac{\tilde{M}(x=6)}{EI} = \frac{360}{60 \cdot 10^6 \cdot 7,2 \cdot 10^{-3}} = 0,83 \cdot 10^{-3} \text{ m}$$

$$\varphi(x=8) = \frac{\tilde{V}(x=8)}{EI} = \frac{-R}{EI} = -\frac{185,3}{-11} = -0,429 \cdot 10^{-3} \text{ rad}$$

$$\oplus \sum M_{ai} = 0: R \cdot 2 - \frac{1}{2} 8 \cdot 2 \cdot \left(\frac{2}{3} 2\right) - \frac{1}{2} 30 \cdot 6 \cdot \left(\frac{2}{3} 6\right) = 0$$

$$R = 185,3 \text{ kNm}$$

INTEGRACE



$$M(x) = -30 + 5x$$

$$M(x') = 4x$$

$$EI w'' = -M(x)$$

$$EI w'' = +30 - 5x$$

$$EI w'' = -4x$$

$$EI w' = C_1 + 30x - \frac{5x^2}{2}$$

$$EI w' = C_3 - 4 \frac{x^2}{2}$$

$$EI w = C_2 + C_1 x + 30 \frac{x^2}{2} - \frac{5x^3}{6}$$

$$EI w = C_4 + C_3 x - 4 \frac{x^3}{6}$$

$$w(x=0) = 0 \Rightarrow C_2 = 0$$

$$w(x'=2) = 0$$

$$w'(x=0) = 0 \Rightarrow C_1 = 0$$

$$360 + C_3 \cdot 2 - 4 \cdot \frac{2^3}{6} = 0$$

$$w(x=6) = w(x'=0)$$

$$C_3 = -177,3$$

$$C_2 + C_1 \cdot 6 + 15 \cdot 6^2 - \frac{5}{6} \cdot 6^3 = C_4$$

$$C_4 = 360$$

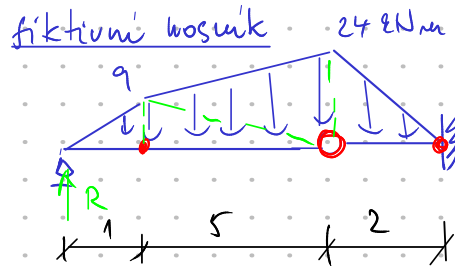
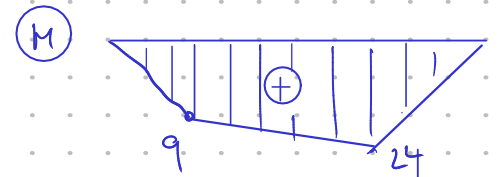
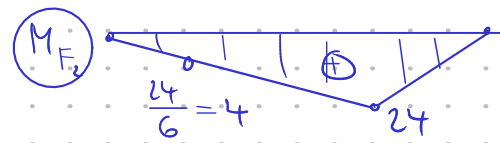
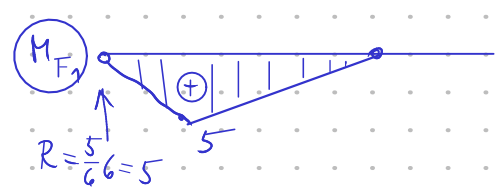
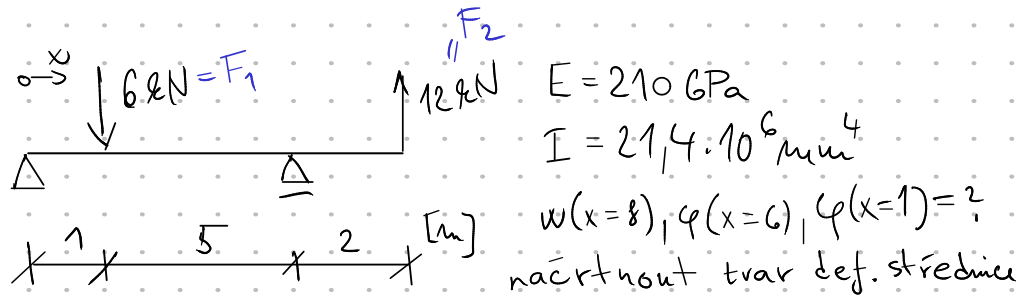
$$\varphi = w'(x) = \left[30x - \frac{5}{2} x^2 \right] \frac{1}{EI} \quad \varphi' = w''(x) = \left[-177,3 - 2x^2 \right] \frac{1}{EI}$$

$$w(x) = \left[15x^2 - \frac{5}{6} x^3 \right] \frac{1}{EI} \quad w(x) = \left[360 - 177,3 - \frac{2}{3} x^3 \right] \frac{1}{EI}$$

$$w(x=6) = \left[15 \cdot 6^2 - \frac{5}{6} \cdot 6^3 \right] \frac{1}{EI} = \frac{360}{EI} \checkmark$$

$$w(x=3) = \left[15 \cdot 3^2 - \frac{5}{6} \cdot 3^3 \right] \frac{1}{EI} = \frac{112,5}{EI} \checkmark$$

$$\varphi(x=8) = \varphi(x'=2) = \left[-177,3 - 2 \cdot 2^2 \right] \frac{1}{EI} = -\frac{185,3}{EI} \checkmark$$



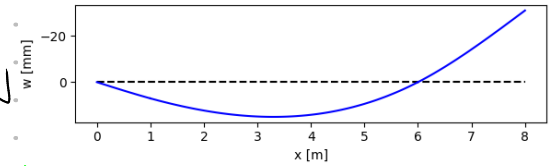
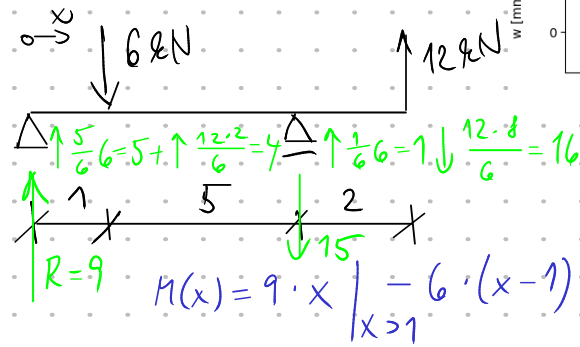
$$\begin{aligned}
 \tilde{M}(x=8) &= R \cdot 8 - \frac{1}{2} \cdot 9 \cdot 1 \cdot \left(4 + \frac{1}{3} \cdot 1\right) - \frac{1}{2} \cdot 9 \cdot 5 \cdot \left(2 + \frac{2}{3} \cdot 5\right) - \frac{1}{2} \cdot 24 \cdot 5 \cdot \left(2 + \frac{1}{3} \cdot 5\right) \\
 &= -139,16 \\
 w(x=8) &= \frac{\tilde{M}(x=8)}{EI} = -\frac{139,16}{210 \cdot 21,4} = -0,0311 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 \sum M_{kloub} &= 0 \quad \oplus \downarrow \\
 R \cdot 6 - \frac{1}{2} \cdot 9 \cdot 1 \cdot \left(5 + \frac{1}{3} \cdot 1\right) - \frac{1}{2} \cdot 9 \cdot 5 \cdot \left(\frac{2}{3} \cdot 5\right) \\
 - \frac{1}{2} \cdot 24 \cdot 5 \cdot \left(\frac{1}{3} \cdot 5\right) &= 0 \\
 R &= 33,16 \text{ kNm}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{V}(x=1) &= 33,16 - \frac{1}{2} \cdot 9 \cdot 1 = 28,16 \text{ kNm} \\
 \varphi(x=1) &= \frac{\tilde{V}(x=1)}{EI} = \frac{28,16}{210 \cdot 21,4} = \\
 &= 6,379 \cdot 10^{-3} \text{ rad}
 \end{aligned}$$

$$\begin{aligned}
 \tilde{V}(x=6) &= 33,16 - \frac{1}{2} \cdot 9 \cdot 1 - \frac{9+24}{2} \cdot 5 = \\
 &= -53,83 \text{ kNm} \\
 \varphi(x=6) &= \frac{\tilde{V}(x=6)}{EI} = \frac{-53,83}{210 \cdot 21,4} = \\
 &= -0,01198 \text{ rad}
 \end{aligned}$$

Clebschova metoda



$$\begin{aligned}
 EI w'' &= -M(x) \\
 EI w'' &= -9x \Big|_{x>1} + 6(x-1) \Big|_{x>6} + 15(x-6) \\
 EI w' &= C_1 - 9 \frac{x^2}{2} \Big|_{x>1} + 3(x-1)^2 \Big|_{x>6} + \frac{15(x-6)^2}{2} \\
 EI w &= C_2 + C_1 x - 3 \frac{x^3}{2} \Big|_{x>1} + (x-1)^3 \Big|_{x>6} + 5 \frac{(x-6)^3}{2}
 \end{aligned}$$

$$w(x=0) = 0 \Rightarrow C_2 + C_1 \cdot 0 - 3 \frac{0^3}{2} = 0 \Rightarrow C_2 = 0$$

$$w(x=6) = 0 \Rightarrow 6C_1 - 3 \frac{6^3}{2} + 5^3 = 0$$

$$C_1 = 33,16$$

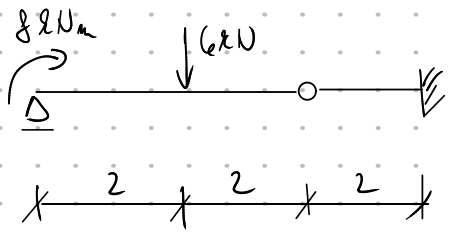
$$w'(x) = \left[33,16 - \frac{9}{2} x^2 \Big|_{x>1} + 3(x-1)^2 \Big|_{x>6} + 15 \frac{(x-6)^2}{2} \right] \frac{1}{EI}$$

$$w(x) = \left[33,16 x - \frac{3}{2} x^3 \Big|_{x>1} + (x-1)^3 \Big|_{x>6} + \frac{5}{2} (x-6)^3 \right] \frac{1}{EI}$$

$$\varphi(x=1) = \left[33,16 - \frac{9}{2} 1^2 \right] \frac{1}{EI} = \frac{28,16}{EI} \checkmark$$

$$\varphi(x=6) = \left[33,16 - \frac{9}{2} 6^2 + 3 \cdot 5^2 \right] \frac{1}{EI} = -\frac{53,83}{EI} \checkmark$$

$$w(x=8) = \left[33,16 \cdot 8 - \frac{3}{2} 8^3 + 4^3 + \frac{5}{2} \cdot 2^3 \right] \frac{1}{EI} = -\frac{139,16}{EI} \checkmark$$



$$E = 210 \text{ GPa}$$

$$I = 21,4 \cdot 10^2 \text{ cm}^4 = 21,4 \cdot 10^{-6} \text{ m}^4$$

$$w(x=4) \mid w(x=5) \mid \varphi(x=2) = ?$$

bez znalosti reakcí:

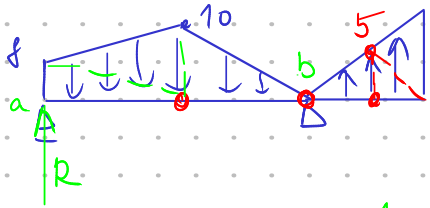
$$\tilde{M}(x=4) = \frac{1}{2} 10 \cdot 2 \cdot \left(\frac{2}{3} 2\right) = 13,3 \text{ kNm}$$

$$w(x=4) = \frac{\tilde{M}(x=4)}{EI} = \frac{13,3}{210 \cdot 21,4} = 2,967 \cdot 10^{-3} \text{ m}$$

$$\tilde{M}(x=5) = \frac{1}{2} 10 \cdot 1 \cdot \left(\frac{2}{3} 1\right) + \frac{1}{2} 5 \cdot 1 \cdot \left(\frac{1}{3} 1\right) = 4,16 \text{ kNm}$$

$$w(x=5) = \frac{\tilde{M}(x=5)}{EI} = \frac{4,16}{210 \cdot 21,4} = 0,9272 \cdot 10^{-3} \text{ m}$$

fiktivní nosník

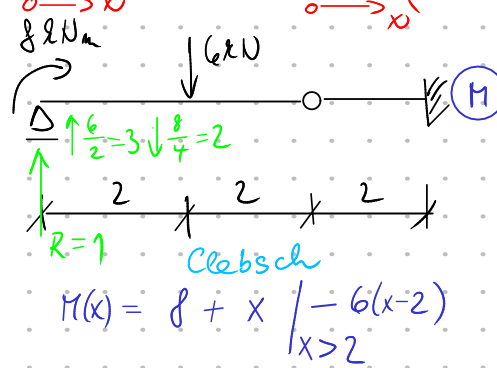


$$\sum M_{bi} = 0: \oplus R \cdot 4 - \frac{1}{2} 8 \cdot 2 \cdot \left(2 + \frac{2}{3} 2\right) - \frac{1}{2} 10 \cdot 2 \cdot \left(2 + \frac{1}{3} 2\right) - \frac{1}{2} 10 \cdot 2 \cdot \left(\frac{2}{3} 2\right) = 20 \text{ kNm}$$

$$\tilde{V}(x=2) = R - \frac{8+10}{2} \cdot 2 = 2 \text{ kN}$$

$$\varphi(x=2) = \frac{\tilde{V}(x=2)}{EI} = \frac{2}{210 \cdot 21,4} = 0,445 \cdot 10^{-3} \text{ rad}$$

Složená nosníková soustava - Integrace



$$M(x) = 8 + x \mid -6(x-2) \quad x > 2$$

$$EI w''(x) = -M(x)$$

$$EI w''(x) = -8 - x \mid +6(x-2) \quad x > 2$$

$$EI w'(x) = C_1 - 8x - \frac{x^2}{2} \mid +6 \frac{(x-2)^2}{2} \quad x > 2$$

$$EI w(x) = C_2 + C_1 x - 4x^2 - \frac{x^3}{6} \mid + (x-2)^3 \quad x > 2$$

$$w(x=0)=0: C_2 + C_1 \cdot 0 - 0 - 0 = 0 \Rightarrow C_2 = 0$$

$$\varphi(x'=2)=0: C_3 + 5 \frac{2^2}{2} = 0 \Rightarrow C_3 = -10$$

$$w(x'=2)=0: C_4 - 10 \cdot 2 + \frac{5}{6} \cdot 2^3 = 0 \Rightarrow C_4 = 13,3$$

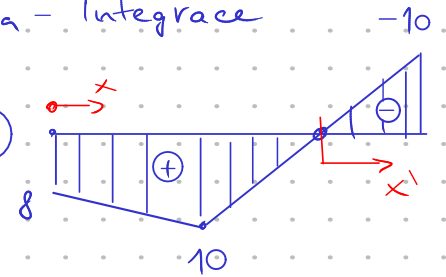
$$w(x=4) = w(x'=0)$$

$$0 + 4C_1 - 4 \cdot 4^2 - \frac{4^3}{6} + 2^3 = 13,3 + 0 + 0 \Rightarrow C_1 = 20$$

$$\varphi(x=2) = w'(x=2) = \left[20 - 8 \cdot 2 - \frac{2^2}{2} \right] \frac{1}{EI} = \frac{2}{EI} \checkmark$$

$$[w(x=4)] = w(x'=0) = C_4 \cdot \frac{1}{EI} = \frac{13,3}{EI} \checkmark$$

$$[w(x=5)] = w(x'=1) = \left[13,3 - 10 \cdot 1 + \frac{5}{6} \cdot 1^3 \right] \frac{1}{EI} = \frac{4,16}{EI} \checkmark$$



$$M(x') = -5x$$

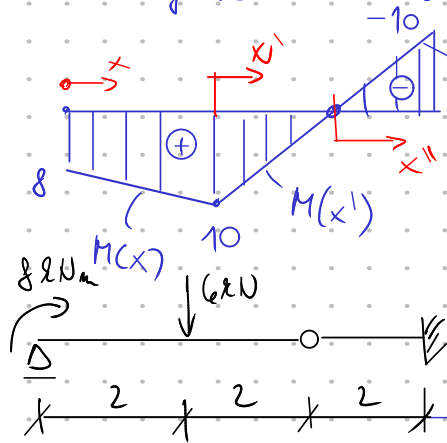
$$EI w''(x') = -M(x')$$

$$EI w''(x') = +5x$$

$$EI w'(x') = C_3 + 5 \frac{x^2}{2}$$

$$EI w(x') = C_4 + C_3 x + \frac{5}{6} x^3$$

- Integrate každá fu zvlášť (nejpráciější)



$$M(x) = 8 - 5x$$

$$EIw''(x) = -M(x)$$

$$EIw''(x) = -8 + 5x$$

$$EIw'(x) = C_1 - 8x + \frac{5x^2}{2}$$

$$EIw(x) = C_2 + C_1x - 4x^2 + \frac{5x^3}{6}$$

$$M(x') = 10 - 5x'$$

$$EIw''(x') = -M(x')$$

$$EIw''(x') = -10 + 5x'$$

$$EIw'(x') = C_3 - 10x' + \frac{5x'^2}{2}$$

$$EIw(x') = C_4 + C_3x' - 5x'^2 + \frac{5x'^3}{6}$$

$$M(x'') = -5x''$$

$$EIw''(x'') = -M(x'')$$

$$EIw''(x'') = +5x''$$

$$EIw'(x'') = C_5 + \frac{5x''^2}{2}$$

$$EIw(x'') = C_6 + C_5x'' + \frac{5x''^3}{6}$$

$$w(x=0) = 0 : C_2 + 0 - 0 - 0 = 0 \Rightarrow C_2 = 0$$

$$\varphi(x''=2) = 0 : C_5 + \frac{5}{2} \cdot 2^2 = 0 \Rightarrow C_5 = -10$$

$$\varphi(x=2) = \varphi(x'=0) : C_1 - 8 \cdot 2 - \frac{2^2}{2} = C_3 - 10 \cdot 0 + 0 \Rightarrow C_1 - C_3 - 18 = 0$$

$$w(x=2) = w(x'=0) : 0 + C_1 \cdot 2 - 4 \cdot 2^2 - \frac{2^3}{6} = C_4 + 0 - 0 + 0 \Rightarrow 2C_1 - C_4 - 17\sqrt{3} = 0$$

$$w(x'=2) = w(x''=0) : C_4 + 2C_3 - 5 \cdot 2^2 + \frac{5}{6} \cdot 2^3 = 13\sqrt{3} - 10 \cdot 0 + 0 \Rightarrow C_4 + 2C_3 - 26\sqrt{6} = 0$$

Soustava rovnic

$$C_1 = 20$$

$$C_4 = 22\sqrt{6}$$

$$C_3 = 2$$

$$\varphi(x=2) = \varphi(x'=0) = \frac{C_3}{EI} = \frac{2}{EI} \checkmark$$

$$[w(x=4)] = w(x'=2) = w(x''=0) = \frac{C_6}{EI} = \frac{13\sqrt{3}}{EI} \checkmark$$

$$[w(x=5)] = w(x''=1) = \left[13\sqrt{3} - 10 \cdot 1 + \frac{5}{6} \cdot 1^3 \right] \frac{1}{EI} = \frac{4,16}{EI} \checkmark$$

