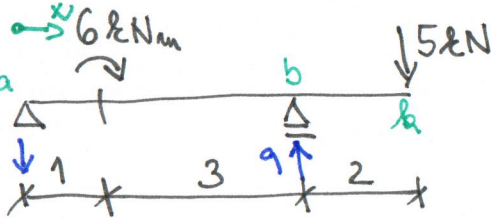
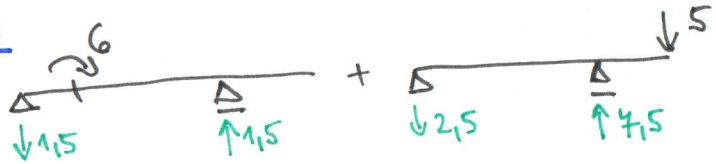


$EI = konst.$

w_c i $\varphi_c = ?$

REAKCE



CLEBSCHOVA METODA

$$M(x) = -4x\Big|_{x>1} + 6\Big|_{x>4} + 9(x-4)$$

$$EIw'' = -M(x)$$

$$EIw'' = 4x\Big|_{x>1} - 6\Big|_{x>4} - 9(x-4)$$

$$EIw' = C_1 + 4\frac{x^2}{2}\Big|_{x>1} - 6(x-1)\Big|_{x>4} - 9\frac{(x-4)^2}{2}$$

$$EIw = C_2 + C_1 x + 2\frac{x^3}{3}\Big|_{x>1} - 6\frac{(x-1)^2}{2}\Big|_{x>4} - 9\frac{(x-4)^3}{2\cdot 3}$$

$$w(x=0)=0 \Rightarrow C_2 + C_1\cdot 0 + 0 = 0 \Rightarrow C_2 = 0$$

$$w(x=4)=0 \Rightarrow 0 + 4C_1 + \frac{2}{3}4^3 - 3(4-1)^2 = 0$$

$$C_1 = -3,916\overline{6}$$

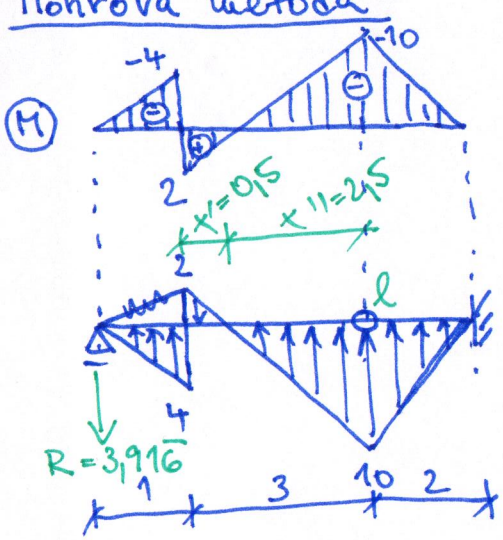
$$w(x) = \frac{1}{EI}\left[-3,916\overline{6}\,x + \frac{2}{3}x^3\Big|_{x>1} - 3(x-1)^2\Big|_{x>4} - \frac{3}{2}(x-4)^3\right]$$

$$\varphi(x) = \frac{1}{EI}\left[-3,916\overline{6} + 2x^2\Big|_{x>1} - 6(x-1)\Big|_{x>4} - \frac{9}{2}(x-4)^2\right]$$

$$w_c = w(x=6) = \left[-3,916\overline{6}\cdot 6 + \frac{2}{3}6^3 - 3\cdot 5^2 - \frac{3}{2}2^3\right]\cdot\frac{1}{EI} = \frac{33,5}{EI}$$

$$\varphi_c = \varphi(x=6) = \left[-3,916\overline{6} + 2\cdot 6^2 - 6\cdot 5 - \frac{9}{2}\cdot 2^2\right]\cdot\frac{1}{EI} = \frac{20,08\overline{3}}{EI}$$

Mohrova metoda



$$\sum M_l = 0 \;\circlearrowleft +$$

$$-R\cdot 4 + \frac{1}{2}\cdot 4\cdot 1\left(3+\frac{1}{3}1\right) - \frac{1}{2}2\cdot x'\left(3-\frac{1}{3}x'\right) + \frac{1}{2}\cdot 10\cdot x''\left(\frac{1}{3}x''\right) = 0$$

$$R = 3,916\overline{6}\ kNm$$

$$\tilde{M}(x=6) = -3,916\overline{6}\cdot 6 + \frac{1}{2}4\cdot 1\left(5+\frac{1}{3}1\right) - \frac{1}{2}2\cdot x'\left(5-\frac{1}{3}x'\right) + \frac{1}{2}10\cdot x''\left(2+\frac{1}{3}x''\right) + \frac{1}{2}2\cdot 10\cdot\left(\frac{2}{3}2\right) = 33,5$$

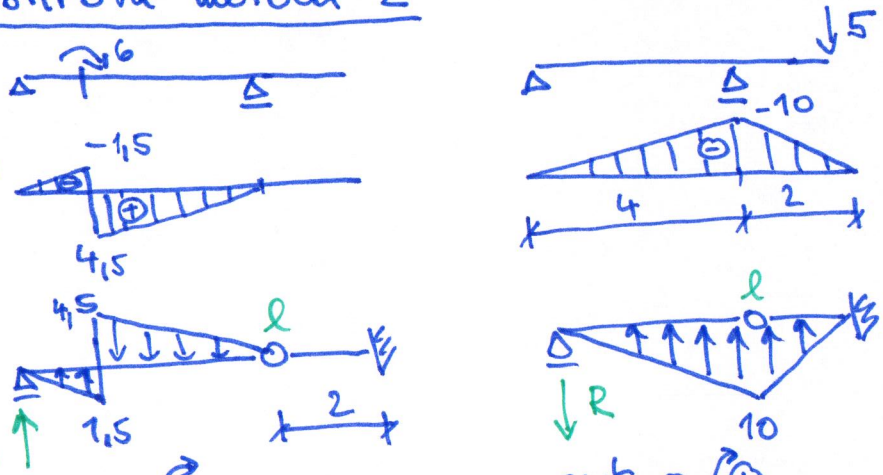
$$\tilde{V}(x=6) = -3,916\overline{6} + \frac{1}{2}4\cdot 1 - \frac{1}{2}\cdot 2\cdot x' + \frac{1}{2}\cdot 10\cdot x'' + \frac{1}{2}\cdot 2\cdot 10 = 20,08\overline{3}$$

$$w = \frac{\tilde{M}}{EI} \quad \text{a} \quad \varphi = \frac{\tilde{V}}{EI}$$

$$w(x=6) = \frac{33,5}{EI}$$

$$\varphi(x=6) = \frac{20,08\overline{3}}{EI}$$

Mohrova metoda 2



$\sum M_l^L = 0$ (+)

$R \cdot 4 + \frac{1}{2} 1{,}5 \cdot 1 \left(3 + \frac{1}{3} 1\right) - \frac{1}{2} 4{,}5 \cdot 3 \cdot \left(\frac{2}{3} 3\right) = 0$

$R = 2{,}75$ kNm

$\tilde{M}(x=6) = 2{,}75 \cdot 6 + \frac{1}{2} \cdot 1{,}5 \cdot 1 \cdot \left(5 + \frac{1}{3} 1\right) - \frac{1}{2} \cdot 4{,}5 \cdot 3 \cdot \left(1 + \frac{2}{3} 3\right) = -6{,}5$

$\tilde{V}(x=6) = 2{,}75 + \frac{1}{2} 1{,}5 \cdot 1 - \frac{1}{2} 4{,}5 \cdot 3 = -3{,}125$

$\sum M_l^L = 0$ (+)

$-R \cdot 4 + \frac{1}{2} 10 \cdot 4 \cdot \left(\frac{1}{3} 4\right) = 0$

$R = 6{,}\overline{6}$ kNm

$\tilde{M}(x=6) = -6{,}\overline{6} \cdot 6 + \frac{1}{2} \cdot 10 \cdot 4 \left(2 + \frac{1}{3} 4\right) + \frac{1}{2} \cdot 10 \cdot 2 \left(\frac{2}{3} 2\right) = 40$

$\tilde{V}(x=6) = -6{,}\overline{6} + \frac{1}{2} 10 \cdot 4 + \frac{1}{2} 10 \cdot 2 = -23{,}\overline{3}$

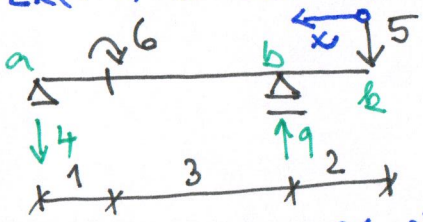
$w(x=6) = \frac{\tilde{M} + \tilde{\tilde{M}}}{EI} = \frac{40 - 6{,}5}{EI} = \frac{33{,}5}{EI}$

$\varphi(x=6) = \frac{\tilde{V} + \tilde{\tilde{V}}}{EI} = \frac{23{,}\overline{3} - 3{,}25}{EI} = \frac{20{,}08\overline{3}}{EI}$

CLEBSCHOVA METODA z volného konce

- není nejvhodnější, protože zde není žádná okrajová podmínka $\Rightarrow C_1$ a $C_2 \neq 0 \Rightarrow$

$\Rightarrow 2\times (w=0) \Rightarrow$ soustava rovnic



$M(x) = -5x \Big|_{x>2} + 9(x-2) \Big|_{x>5}^{-6}$

$EIw'' = -M(x)$

$EIw'' = 5x \Big|_{x>2} - 9(x-2) \Big|_{x>5}^{+6}$

$EIw' = C_1 + 5\frac{x^2}{2} \Big|_{x>2} - 9\frac{(x-2)^2}{2} \Big|_{x>5} + 6(x-5)$

$EIw = C_2 + C_1 x + \frac{5}{2}\frac{x^3}{3} \Big|_{x>2} - \frac{9}{2}\frac{(x-2)^3}{3} \Big|_{x>5} + 6\frac{(x-5)^2}{2}$

$w(x=2) = 0 \quad C_2 + 2C_1 + 6{,}\overline{6} = 0$

$w(x=6) = 0 \quad C_2 + 6C_1 + 87 = 0$

$\Rightarrow C_2 = 33{,}5$

$4C_1 + 80{,}\overline{3} = 0$

$C_1 = -20{,}08\overline{3}$

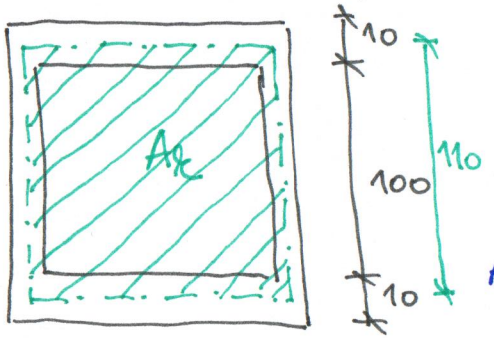
$w(x) = \frac{1}{EI}\left[33{,}5 - 20{,}08\overline{3} \cdot x + \frac{5}{6}x^3 \Big|_{x>2} - \frac{9}{2}(x-2)^3 \Big|_{x>5}^{+3(x-5)^2}\right]$

$\varphi(x) = \frac{1}{EI}\left[-20{,}08\overline{3} + \frac{5}{2}x^2 \Big|_{x>2} - \frac{9}{2}(x-2)^2 \Big|_{x>5}^{+6(x-5)}\right]$

$w_k = w(x=0) = \frac{1}{EI}(33{,}5) = \frac{33{,}5}{EI}$

$\varphi_k = \varphi(x=0) = \frac{-20{,}08\overline{3}}{EI}$

úhel má opačné znaménko, protože jdeme zprava

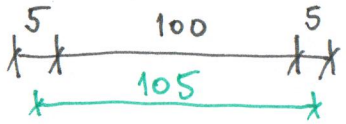


$G = 81\,GPa$

$T = 18\,kNm$

$\ell = 5\,m$

$A_k = 110 \cdot 105 = 11550\,mm^2$



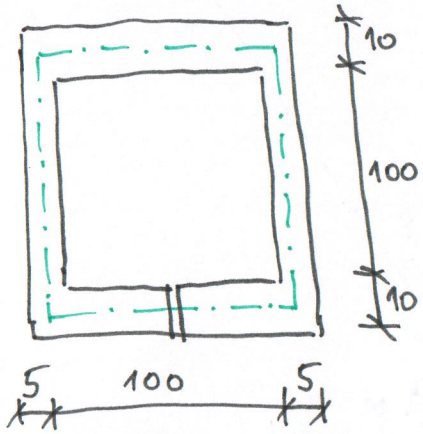
$$w_t = 2 A_k t_{min} = 2 \cdot 11550 \cdot 5 = 115\,500\,mm^3$$

$$I_t = \frac{4 A_k^2}{\sum \frac{h_i}{t_i}} = \frac{4 \cdot 11550^2}{2 \cdot \frac{105}{10} + 2 \cdot \frac{110}{5}} = 8{,}20938 \cdot 10^6\,mm^4$$

$$\tau_{max} = \frac{T}{w_t} = \frac{10^3}{115\,500 \cdot 10^{-9}} = \underline{8{,}658\,MPa}$$

$$\varphi_\ell = \frac{T\ell}{G I_t} = \frac{10^3 \cdot 5}{81 \cdot 10^9 \cdot 8{,}20938 \cdot 10^{-6}} = \underline{7{,}519 \cdot 10^{-3}\,rad}$$

$$(\varphi_\ell = 0{,}43^\circ)$$



$$I_t = \frac{1}{3}\sum t_i^3 h_i = \frac{1}{3}\left(2 \cdot 5^3 \cdot 110 + 2 \cdot 10^3 \cdot 105\right) = 79\,166{,}\overline{6}\,mm^4$$

$$w_t = \frac{I_t}{t_{max}} = \frac{79166{,}\overline{6}}{10} = 7916{,}\overline{6}\,mm^3$$

$$\tau = \frac{T}{w_t} = \frac{10^3}{7916{,}\overline{6} \cdot 10^{-9}} = \underline{126{,}32\,MPa}$$

$$\varphi_\ell = \frac{T\ell}{G I_t} = \frac{10^3 \cdot 5}{81 \cdot 10^9 \cdot 79166{,}\overline{6} \cdot 10^{-12}} = \underline{0{,}7794\,rad}$$

$$(= 44{,}675^\circ)$$