Rovinná napjatost

- hlavní napětí

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\underset{max}{\sigma_1} > \underset{min}{\sigma_2} \qquad \underline{\tau_{xy} = 0}$$

- sklon hlavních napětí - hlavní napětí σ_1 svírá vždy menší úhel s algebraicky větším z daných σ_x, σ_y

$$\operatorname{tg} 2\alpha_{1,2} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

alternativně $\operatorname{tg}\alpha_1 = \frac{\sigma_1 - \sigma_x}{\tau_{xy}} \qquad \operatorname{tg}\alpha_2 = \frac{\sigma_2 - \sigma_x}{\tau_{xy}}$

- extrémní smyková napětí

$$\tau_{\substack{max\\min}} = \pm\frac{1}{2}(\sigma_1 - \sigma_2)$$

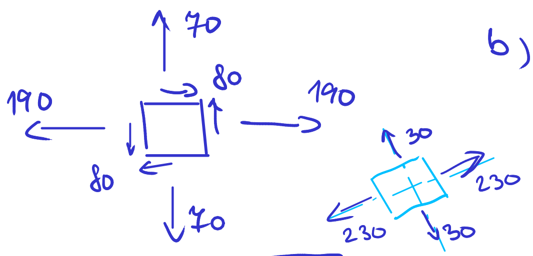
- při graf. řešení odpovídá poloměru Mohrovy kružnice
- napětí působí v rovinách půlících úhel mezi rovinami hlavních napětí (pootočení 45°)

---

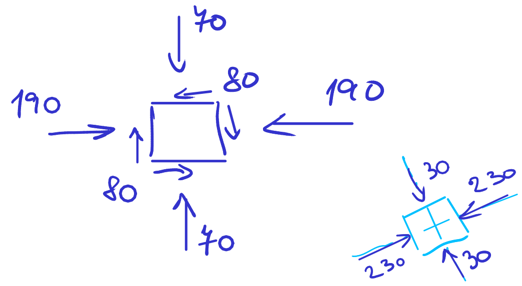
a) $\sigma_x = 190\,MPa$, $\sigma_y = 70\,MPa$, $\tau_{xy} = 80\,MPa$

b) $\sigma_x = -190\,MPa$, $\sigma_y = -70\,MPa$, $\tau_{xy} = -80\,MPa$

a)



b)



a)

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} =$$

$$= \frac{1}{2}(190 + 70) \pm \frac{1}{2}\sqrt{(190-70)^2 + 4\cdot 80^2}$$

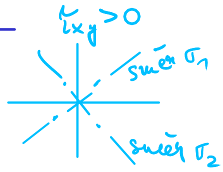
$\underline{\sigma_1 = 230\,MPa}$ $\qquad \sigma_1 > \sigma_2$

$\underline{\sigma_2 = 30\,MPa}$ $\quad (\tau_{xy} = 0)$

$$\tau_{\substack{max\\min}} = \pm\frac{1}{2}(\sigma_1 - \sigma_2) = \underline{\pm 100\,MPa}$$

$(\sigma_x = \sigma_y)$

$$\operatorname{tg} 2\alpha_{1,2} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\cdot 80}{190 - 70}$$



$2\alpha_{1,2} = 53{,}13°$

$\alpha_1 = 26{,}57° \rightarrow$ směr σ_1

$\alpha_2 = -63{,}43° \rightarrow$ směr σ_2

b)

$$\sigma_{1,2} = \frac{1}{2}(-190 - 70) \pm \frac{1}{2}\sqrt{(-190 + 70)^2 + 4(-80)^2}$$

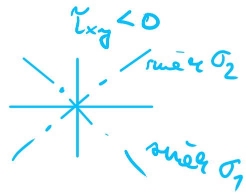
$\underline{\sigma_1 = -30\,MPa}$ $\qquad \sigma_1 > \sigma_2$

$\underline{\sigma_2 = -230\,MPa}$ $\quad (\tau_{xy} = 0)$

$$\tau_{\substack{max\\min}} = \underline{\pm 100\,MPa}$$

$(\sigma_x = \sigma_y)$

$$\operatorname{tg} 2\alpha_{1,2} = \frac{2\cdot(-80)}{-190-(-70)}$$



$2\alpha_{1,2} = 53{,}13°$

$\alpha_2 = 26{,}57° \rightarrow$ směr σ_2

$\alpha_1 = -63{,}43° \rightarrow$ směr σ_1

Invarianty:

1. invariant

$$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$$

$$190 + 70 = 230 + 30 \checkmark \qquad -190 - 70 = -230 - 30 \checkmark$$

2. invariant

$$\sigma_x \cdot \sigma_y - \tau_{xy}^2 = \sigma_1 \sigma_2 (+0^2)$$

$$190 \cdot 70 - 80^2 = 230 \cdot 30$$

$$6900 = 6900 \checkmark$$

- vlastní čísla:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = \begin{bmatrix} 190 & 80 \\ 80 & 70 \end{bmatrix}$$

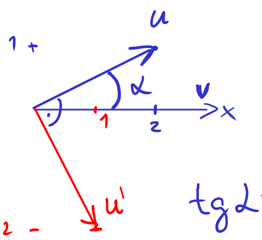
$$\begin{vmatrix} 190-\lambda & 80 \\ 80 & 70-\lambda \end{vmatrix} = (190-\lambda)(70-\lambda) - 80 \cdot 80 = \lambda^2 - 260\lambda + 6900$$

$$D = b^2 - 4ac = 40000$$

$$\lambda_{1,2} = \sigma_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-260) \pm \sqrt{40000}}{2 \cdot 1} = \begin{cases} \underline{230\,\text{MPa}} \\ \underline{30\,\text{MPa}} \end{cases}$$

$$\sigma_1 = 230 \qquad \begin{bmatrix} -40 & 80 \\ 80 & -160 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \emptyset \qquad u = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix}$$

$$\sigma_2 = 30 \qquad u' = \begin{Bmatrix} 1 \\ -2 \end{Bmatrix}$$

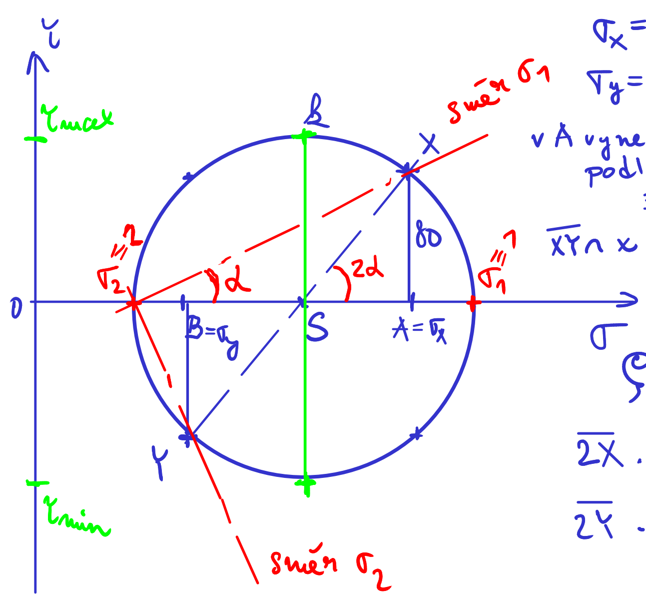


$$v = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$\cos\alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \frac{2 \cdot 1 + 1 \cdot 0}{\sqrt{5} \cdot 1} \Rightarrow \alpha = 26{,}565°$$

$$\operatorname{tg}\alpha = \frac{1}{2} \Rightarrow \alpha = 26{,}565°$$

Mohrova kružnice



$\sigma_x = \overline{0A}$

$\sigma_y = \overline{0B}$

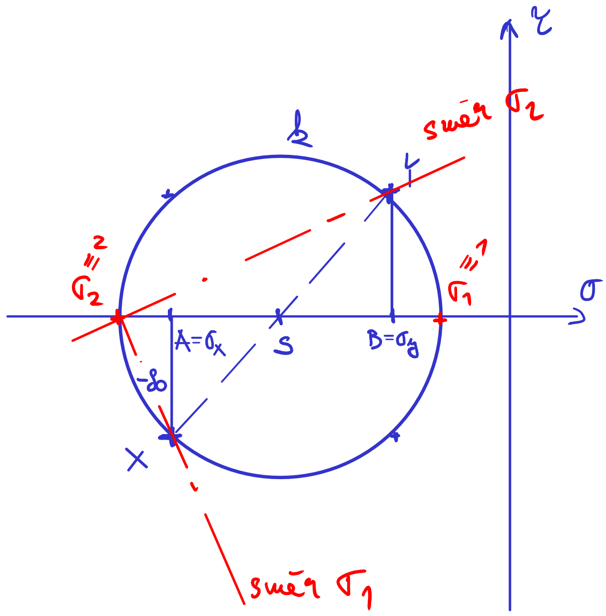
v A vyneseme τ_{xy} podle znaménka $\equiv X$

$\overline{XY} \cap x \equiv S$ střed Mohrovy kružnice k

$\rho = \overline{SX}$ poloměr k

$\overline{2X}$ ... směr σ_1

$\overline{2Y}$ ... směr σ_2



$$\overline{OS} = \frac{\overline{OA} + \overline{OB}}{2} = \frac{\sigma_x + \sigma_y}{2}$$

$$\overline{SX} = \sqrt{\overline{SA}^2 + \overline{AX}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\overline{SA} = \frac{\overline{AB}}{2} = \frac{\overline{OA} - \overline{OB}}{2}$$

$$\operatorname{tg} 2\alpha = \frac{\overline{AX}}{\overline{SA}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$