Mohrova metoda

- analogie

$$\frac{d^2M}{dx^2} = M'' = -q \qquad \frac{d^2w}{dx^2} = w'' = -\frac{M}{EI}$$

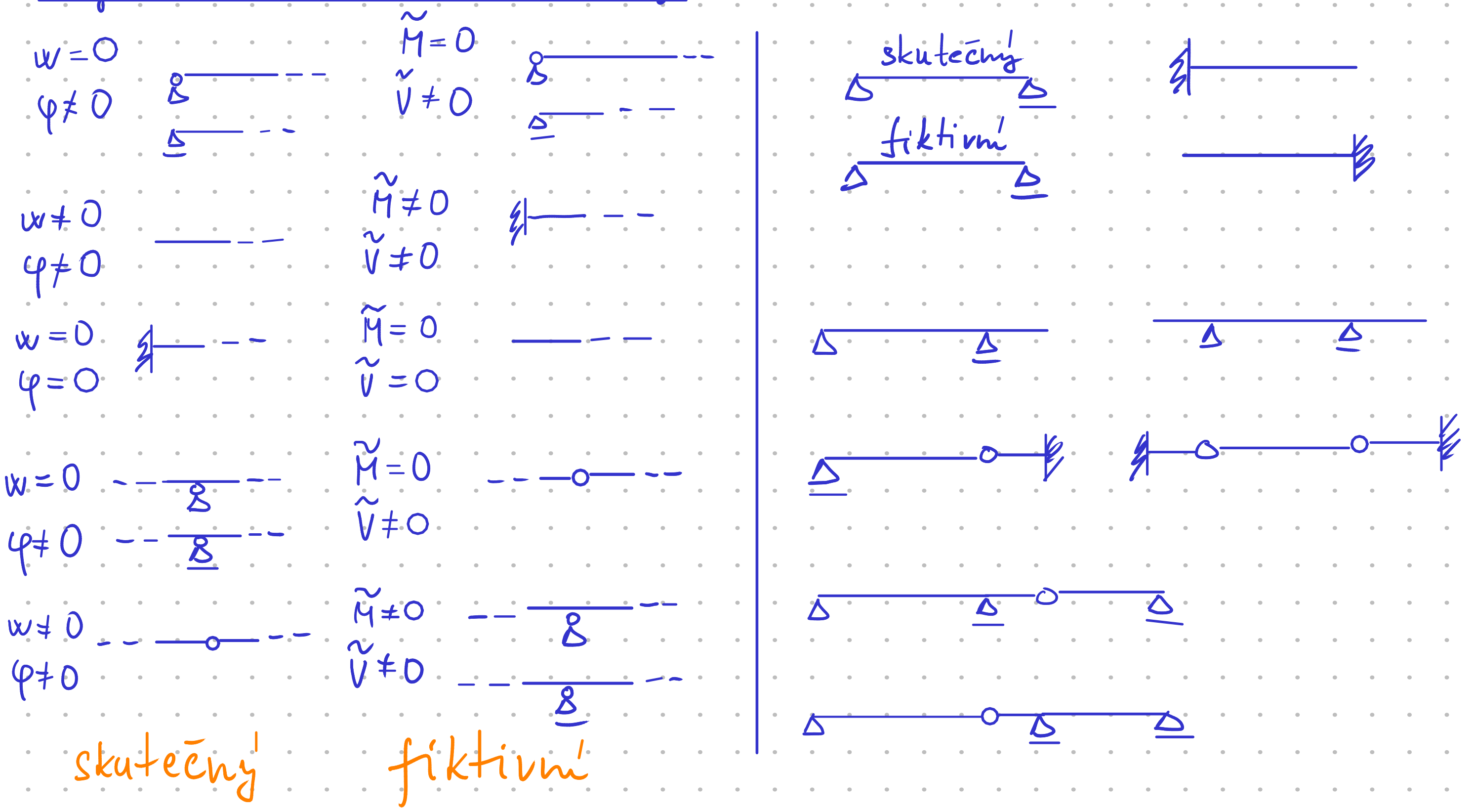
- fiktivní zatížení $\tilde{q} = \frac{M}{EI} \rightarrow$ určíme $\tilde{M}$

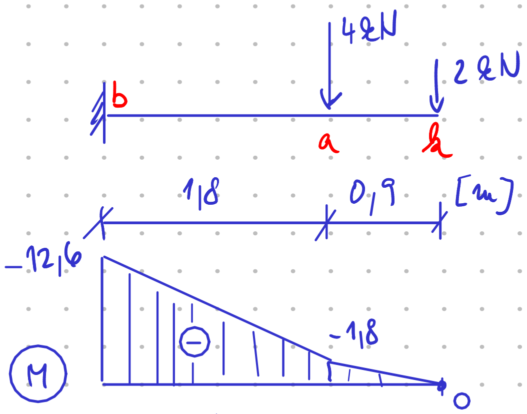
$$\frac{d^2\tilde{M}}{dx^2} = -\tilde{q} = -\frac{M}{EI} = \frac{d^2w}{dx^2}$$

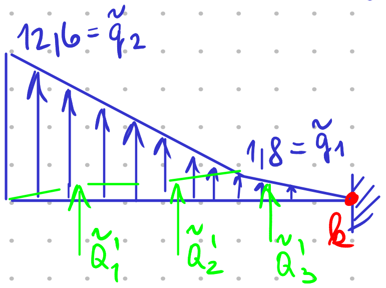
- vyšetřujeme <u>fiktivní nosník</u>, podepřený takovým způsobem, že splňuje vůči $\tilde{M}$ a $\tilde{V}$ tytéž okrajové podmínky jako <u>skutečný nosník</u> vůči w a φ

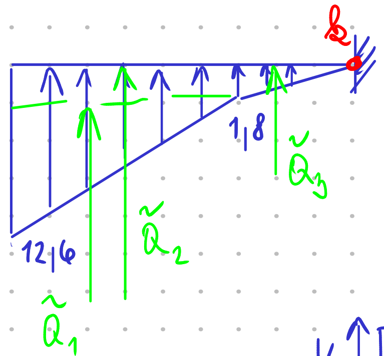
$\Rightarrow w = \tilde{M} \qquad \varphi = w' = \tilde{V}$

<u>Vzájemně si odpovídající vazby</u>









I 180 $\quad I = 14{,}4 \cdot 10^6\ mm^4$

$E = 210\ GPa \qquad w_k = ? = \varphi_k$

1) určíme (M) $\quad \oplus \left.\right] \curvearrowleft M$

$M_k = 0$

$M_a = -2 \cdot 0{,}9 = -1{,}8\ kNm$

$M_b = -2 \cdot 2{,}7 - 4 \cdot 1{,}8 = -12{,}6\ kNm$

2) sestrojíme fiktivní nosník a zatížíme jej momentovou plochou → kladné momenty klademe jako kladné zatížení $\tilde{q}$ (tj. působí směrem dolů)

$\tilde{q}_1 = 1{,}8\ kNm$

$\tilde{q}_2 = 12{,}6\ kNm$

3) vypočteme hodnoty $\tilde{M}$ a $\tilde{V}$ v místech hledaného w a φ

$V \uparrow [\oplus \quad \tilde{V}_k = \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 =$

$= \frac{1}{2} 1{,}8 (12{,}6 - 1{,}8) + 1{,}8 \cdot 1{,}8 + \frac{1}{2} 0{,}9 \cdot 1{,}8 =$

$= \underline{13{,}77\ kNm^2}$

$M \curvearrowright [\oplus \quad \tilde{M}_k = \tilde{Q}_1 \cdot (0{,}9 + \frac{2}{3} 1{,}8) + \tilde{Q}_2 \cdot (0{,}9 + \frac{1}{2} 1{,}8) + \tilde{Q}_3 (\frac{2}{3} 0{,}9) =$

$= 9{,}72 \cdot 2{,}1 + 3{,}24 \cdot 1{,}8 + 0{,}81 \cdot 0{,}6 = \underline{26{,}73\ kNm^3}$

4) hodnoty skutečného w a φ získáme dělením EI

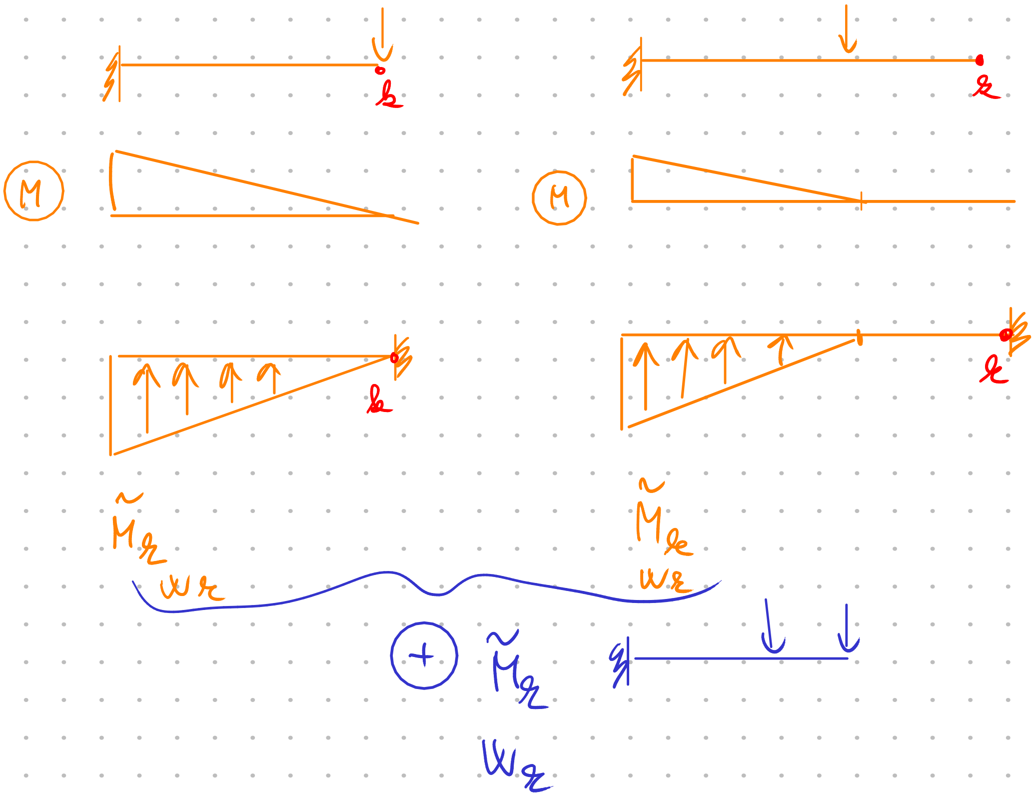
$w = \frac{\tilde{M}}{EI} \qquad [m] = \frac{[kNm^3]}{[\frac{kN}{m^2}\ m^4]} \qquad [kPa] = [k\frac{N}{m^2}]$

$EI = 210 \cdot 10^6 \cdot 14{,}4 \cdot 10^{-6} = 3024\ kNm^2$

$w_k = \frac{\tilde{M}_k}{EI} = \frac{26{,}73}{3024} = \underline{\underline{8{,}84 \cdot 10^{-3}\ m}}$

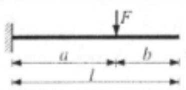
$\varphi_k = \frac{\tilde{V}_k}{EI} = \frac{13{,}77}{3024} = \underline{\underline{4{,}55 \cdot 10^{-3}\ rad}}$

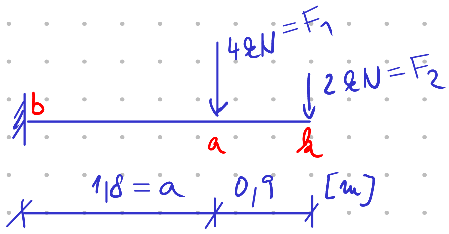
SUPERPOZICE



Kontrola pomocí tabulek

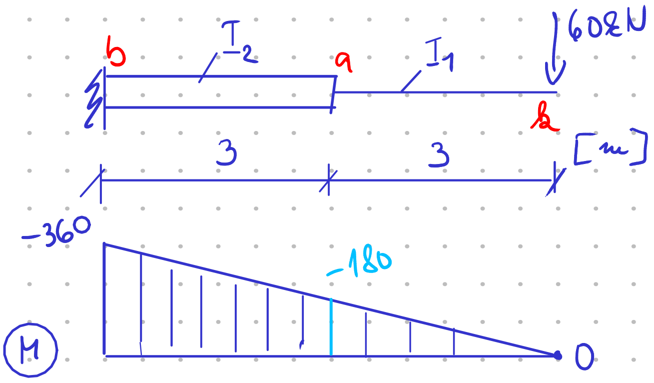
Tabulka 14.1. Deformace konzoly konstantního průřezu

Zatěž. případ	Schéma zatížení	Průhyb volného konce w	Pootočení volného konce φ
1		$\frac{Fa^2}{6EI}(3l-a)$	$\frac{Fa^2}{2EI}$
2		$\frac{Fl^3}{3EI}$	$\frac{Fl^2}{2EI}$



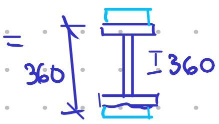
$$w_k = \frac{F_1 a^2}{6EI}(3l-a) + \frac{F_2 l^3}{3EI} = \frac{4\cdot 1{,}8^2}{6EI}(3\cdot 2{,}7 - 1{,}8) + \frac{2\cdot 2{,}7^3}{3EI} = \frac{26{,}73}{EI} \checkmark$$

$$\varphi_k = \frac{F_1 a^2}{2EI} + \frac{F_2 l^2}{2EI} = \frac{4\cdot 1{,}8^2}{2EI} + \frac{2\cdot 2{,}7^2}{2EI} = \frac{13{,}77}{EI} \checkmark$$



$E = 210\,GPa$

$I_1 = I_{I360} = 196{,}1 \cdot 10^6\ mm^4$

$I_2 =$  $+\ 160 \times 10$

2 ks

$w_k = ?$

1) určíme (M)

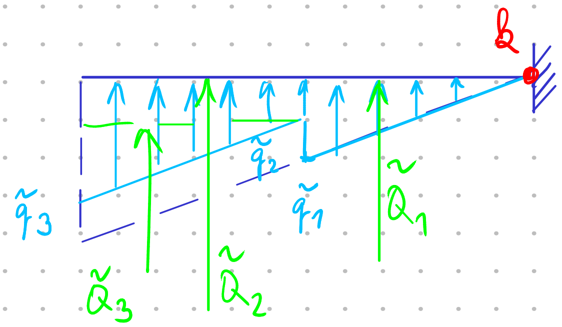
$M_k = 0$

$M_b = -60 \cdot 6 = -360\ kNm$

$M_a = -60 \cdot 3 = -180\ kNm$

2) fiktivní nosník a zatížíme jej <u>redukovaným</u> zatížením $\tilde{q}$

$$I_2 = I_{I360} + \left[\frac{1}{12} 160 \cdot 10^3 + 160 \cdot 10 \cdot 185^2\right] \cdot 2 =$$
$$= 305{,}65 \cdot 10^6\ mm^4$$



$$\tilde{q}_1 = \frac{M_a}{EI_1} = \frac{180 \cdot 10^3}{210 \cdot 10^9 \cdot 196{,}1 \cdot 10^{-6}} = 4{,}37095 \cdot 10^{-3}\ m^{-1}$$

$$\tilde{q}_2 = \frac{M_a}{EI_2} = \frac{180 \cdot 10^3}{210 \cdot 10^9 \cdot 305{,}65 \cdot 10^{-6}} = 2{,}80433 \cdot 10^{-3}\ m^{-1}$$

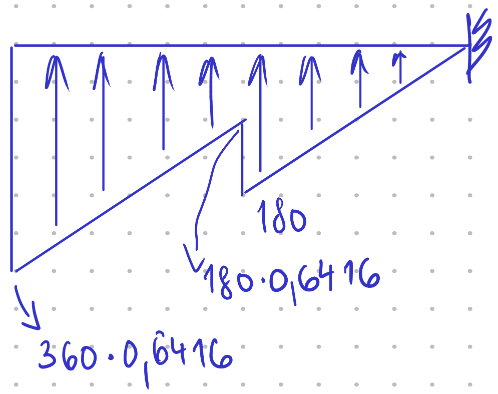
$$\tilde{q}_3 = \frac{M_b}{EI_2} = \frac{360 \cdot 10^3}{210 \cdot 10^9 \cdot 305{,}65 \cdot 10^{-6}} = 5{,}60866 \cdot 10^{-3}\ m^{-1}$$

3) vypočteme $\tilde{M}$ v bodě "k"

$$\tilde{M}_k = w_k = \tilde{Q}_1 \left(\frac{2}{3} \cdot 3\right) + \tilde{Q}_2 \cdot \left(3 + \frac{1}{2} 3\right) + \tilde{Q}_3 \left(3 + \frac{2}{3} 3\right) =$$

$$= \left[\frac{1}{2} 4{,}37095 \cdot 3 \cdot (2) + 2{,}80433 \cdot 3 \cdot (4{,}5) + \frac{1}{2}(5{,}60866 - 2{,}80433) \cdot 3 \cdot (5)\right] \cdot 10^{-3} =$$

$$= \underline{\underline{72 \cdot 10^{-3}\ m}}$$



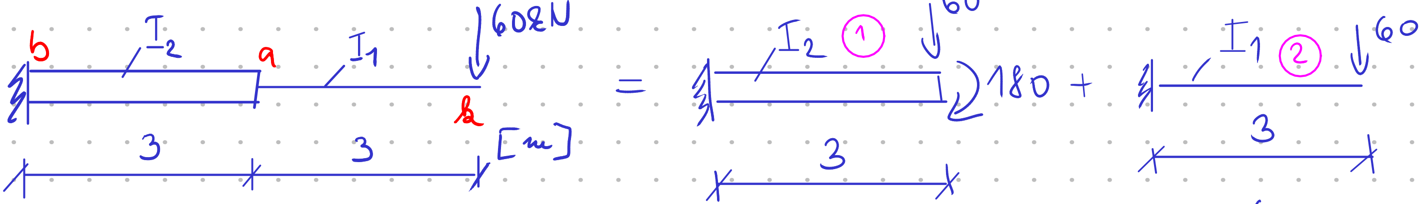
Pozn.: $I_0 = I_1$

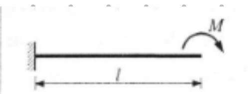
$$\frac{I_0}{I(x)} = \frac{I_1}{I_2} = \frac{196{,}1}{305{,}64} = 0{,}6416$$

3) $\tilde{M}_k = [kN \cdot m^3]$

4) $w_k = \dfrac{\tilde{M}_k}{E I_0} = [m]$

Kontrola s využitím tabulek



2		$w = \frac{Fl^3}{3EI}$	$\frac{Fl^2}{2EI}$
13		$w = \frac{Ml^2}{2EI}$	$\frac{Ml}{EI}$

$E = 210 \cdot 10^6$ kPa

$I_1 = 196{,}1 \cdot 10^{-6}\ m^4$

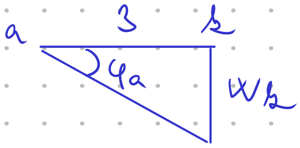
$I_2 = 305{,}65 \cdot 10^{-6}\ m^4$

$$① \quad w_a = \frac{F\ell^3}{3EI_2} + \frac{M\ell^2}{2EI_2} = \frac{60 \cdot 3^3}{3EI_2} + \frac{180 \cdot 3^2}{2EI_2} = \frac{1350}{EI_2} = 21{,}032 \cdot 10^{-3}\ m$$

$$② \quad w_k = \frac{F\ell^3}{3EI_1} = \frac{60 \cdot 3^3}{3EI_1} = \frac{540}{EI_1} = 13{,}113 \cdot 10^{-3}\ m$$

$$① \quad \varphi_a = \frac{F\ell^2}{2EI_2} + \frac{M\ell}{EI_2} = \frac{60 \cdot 3^2}{2EI_2} + \frac{180 \cdot 3}{EI_2} = \frac{810}{EI_2} = 12{,}619 \cdot 10^{-3}\ rad$$

w_k od pootočení

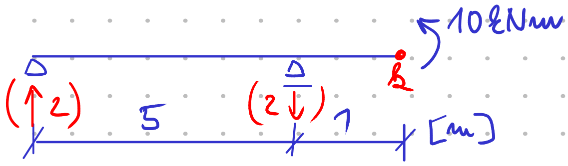


$$\frac{w_k}{3} = tg\, \varphi_a \Rightarrow w_k = 3 \cdot tg\, \varphi_a \approx 3 \cdot \varphi_a = 37{,}857 \cdot 10^{-3}\ m$$

celkové $w_a = w_a^{①} + w_k^{②} + w_k^{\varphi_a} =$

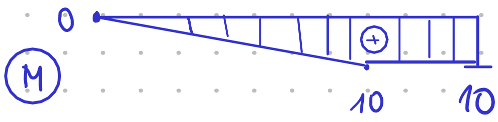
$$= (21{,}032 + 13{,}113 + 37{,}857) \cdot 10^{-3} = \underline{\underline{72 \cdot 10^{-3}\ m}} \ \checkmark$$

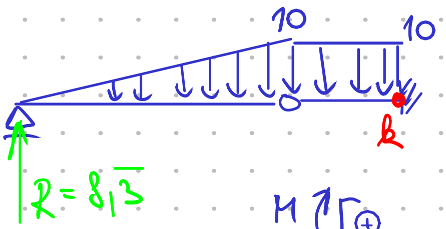
Nosník s převislým koncem



$EI = konst.$

$w_k = ?$





$\sum M^L_{kloub} = 0$ $\quad M \curvearrowleft \oplus$

$$R \cdot 5 - \frac{1}{2} \cdot 5 \cdot 10 \cdot \left(\frac{1}{3} 5\right) = 0$$

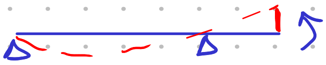
$$R = 8{,}\bar{3}\ kNm^2$$

$M \curvearrowleft \oplus$

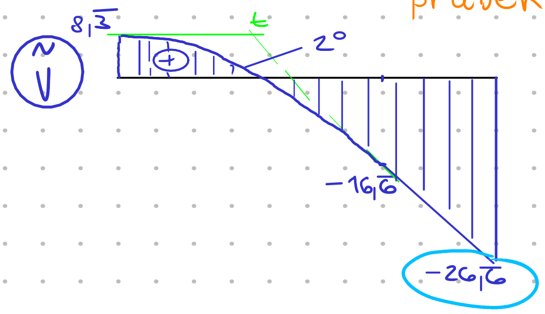
$$\tilde{M}_k = R \cdot 6 - \frac{1}{2} 5 \cdot 10 \left(1 + \frac{1}{3} 5\right) - 10 \cdot 1 \cdot \left(\frac{1}{2} 1\right) =$$

$$= -21{,}\bar{6}\ kNm^3$$

$$w_k = \frac{\tilde{M}_k}{EI} = -\frac{21{,}\bar{6}\ [kNm^3]}{EI}$$



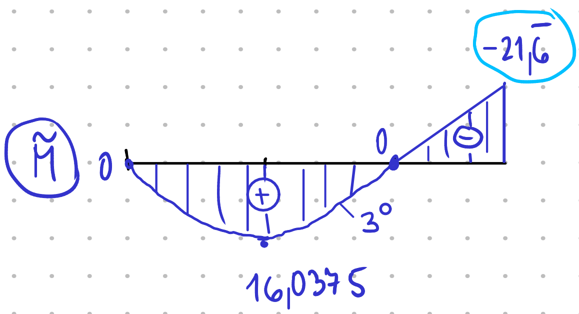
průběh $\tilde{V}$ a $\tilde{M}$ po celé délce nosníku



$$V(x) = 8{,}\bar{3} - \frac{1}{2} q_x \cdot x \qquad q_x = x \frac{10}{5} = 2x$$

$$V(x) = 0$$

$$x^2 = 8{,}\bar{3} \Rightarrow x = \pm 2{,}887\ m$$



$$M_{max} = 8{,}\bar{3} \cdot x - \frac{1}{2} 2x \cdot x \cdot \frac{x}{3} = 16{,}0375\ kNm$$

Clebschova metoda

$$M(x) = 2x \Big|_{x>5} - 2(x-5)$$

$$EIw'' = -2x \Big|_{x>5} + 2(x-5)$$

$$EIw' = C_1 - 2\frac{x^2}{2} \Big|_{x>5} + 2\frac{(x-5)^2}{2}$$

$$EIw = C_2 + C_1 x - \frac{x^3}{3} \Big|_{x>5} + \frac{(x-5)^3}{3}$$

$$w(x=0) = 0 \qquad 0 = C_2 + 0 - 0 \Rightarrow C_2 = 0$$

$$w(x=5) = 0 \qquad 0 = C_1 \cdot 5 - \frac{5^3}{3} \Rightarrow C_1 = \frac{25}{3} \quad (\equiv R = 8{,}\bar{3})$$

$$EIw_k(x=6) = \frac{25}{3} \cdot 6 - \frac{6^3}{3} + \frac{1^3}{3} = -\frac{65}{3} = -21{,}\bar{6}$$

$$EIw'_k(x=6) = \frac{25}{3} - 6^2 + 1^2 = -\frac{80}{3} = -26{,}\bar{6}$$

viz průběhy $\tilde{V}$ a $\tilde{M}$