KROUCENÍ

Kruhový a mezikruhový průřez ○ ◎

- prosté kroucení $M_x = T \neq 0$ $\quad M, V, N = 0$
- smykové napětí $\tau = \dfrac{T}{I_t} \cdot \varrho$

T … kroutící (torzní) moment

I_t … moment tuhosti v kroucení

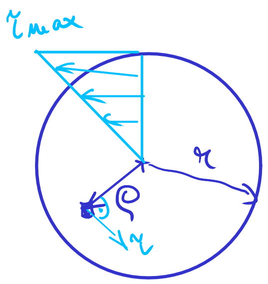
(○ ◎ $I_t = I_p$ polární moment)

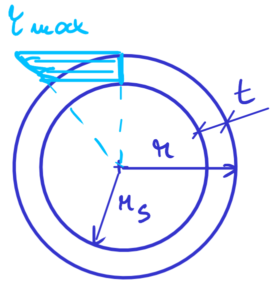
ϱ … průvodič obecného bodu

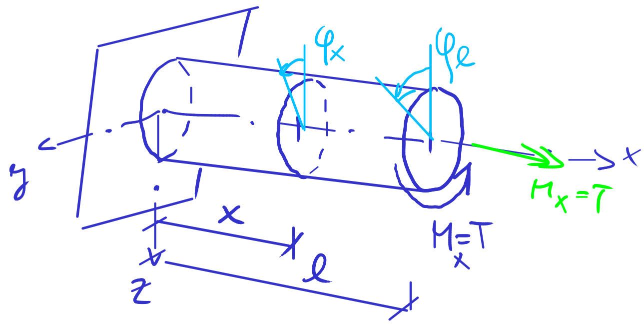
$$\tau_{max} = \frac{T}{I_t} r = \frac{T}{W_t} \qquad W_t = \frac{I_t}{r}$$

kruhový průřez: $I_t = I_p = \dfrac{\pi}{2} r^4 = \dfrac{\pi}{32} d^4$

mezikruhový: $I_t = I_p = \dfrac{\pi}{32}\left(d^4 - d_s^4\right) = \dfrac{\pi}{32}\left[d^4 - (d-2t)^4\right]$



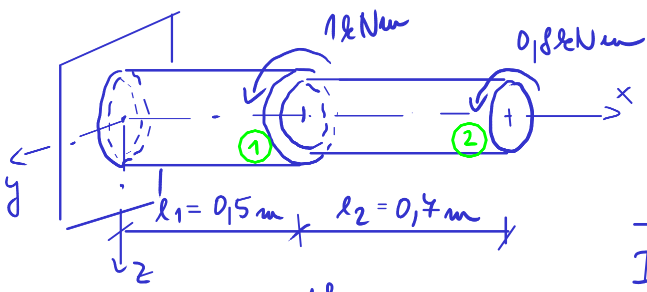




pootočení $\varphi_x(x) = \displaystyle\int_0^x \frac{T}{G I_t}\, dt = \frac{T}{G I_t} x$

natočení konců prutů: $\varphi_\ell = \dfrac{T \ell}{G I_t}$ $\qquad \varphi_\ell = \displaystyle\sum_{i=1}^{n} \frac{T_i \ell_i}{G_i I_{ti}}$

I_t, T, G … po úsecích konstantní

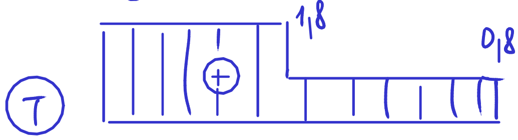


$G = 81$ GPa … modul průřezu ve smyku

$d_1 = 57$ mm $\quad t_1 = 8$ mm $\quad$ TR57x8

$d_2 = 40$ mm

$I_{t1} = I_{p1} = \dfrac{\pi}{32}\left(d_1^4 - d_{s1}^4\right) \qquad d_{s1} = d_1 - 2t_1 = 57 - 2\cdot 8 = 41$ mm



$I_{t1} = \dfrac{\pi}{32}\left(57^4 - 41^4\right) = 0{,}4589 \cdot 10^{+6}\,\text{mm}^4 = 0{,}4589 \cdot 10^{-6}\,\text{m}^4$

$I_{t2} = \dfrac{\pi}{32} d_2^4 = \dfrac{\pi}{32} 40^4 \cdot 10^{-12} = 0{,}2513 \cdot 10^{-6}\,\text{m}^4$

$\tau_{max1} = \dfrac{T_1}{I_t} \cdot r_1 = \dfrac{1{,}8 \cdot 10^3}{0{,}4589 \cdot 10^{-6}} \left(\dfrac{57}{2} \cdot 10^{-3}\right) = \underline{\underline{67{,}6\ \text{MPa}}}$

$\tau_{max2} = \dfrac{T_2}{I_t} r_2 = \dfrac{0{,}8 \cdot 10^3}{0{,}2513 \cdot 10^{-6}} \left(\dfrac{40}{2} \cdot 10^{-3}\right) = \underline{\underline{63{,}7\ \text{MPa}}}$

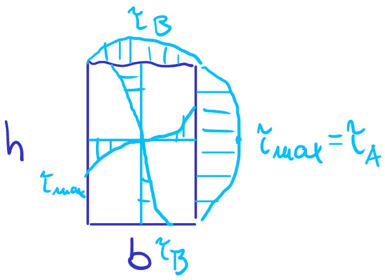
$$\varphi_\ell = \sum_{i=1}^{2} \frac{T_i\, \ell_i}{G\, I_{t i}} = \frac{1{,}8\cdot 10^3 \cdot 0{,}5}{81\cdot 10^9 \cdot 0{,}7589\cdot 10^{-6}} + \frac{0{,}8\cdot 10^3 \cdot 0{,}7}{81\cdot 10^9\cdot 0{,}2513\cdot 10^{-6}} =$$

$$= 0{,}01464 + 0{,}02751 = \underline{\underline{0{,}042\ \text{rad}}} \quad \left(\cdot\frac{180}{\pi} = 2{,}406^\circ\right)$$

Kroucení prutů obecného průřezu

- kroucení
  - volné – umožněno deplanaci ; $\sigma_x = 0$
  - vázané – bráníme deplanaci ; $\sigma_x \neq 0$

- obdélník $b < h$



$$I_t = \frac{2}{G\Theta}\int_A F(y,z)\, dA$$

$$I_{t,obd} = \alpha\, b^3 h$$

$$W_{t,obd} = \beta\, b^2 h$$

α, β podle $\frac{h}{b}$

tab. 5.1 str. 125

$\frac{h}{b} > 10 \quad \alpha, \beta = \frac{1}{3}$

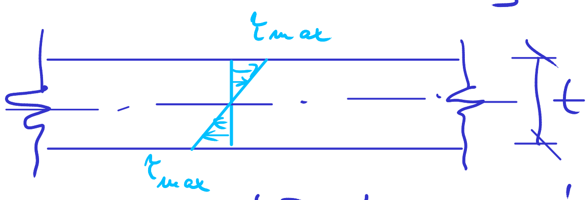
$$\tau_{max} = \frac{T}{W_t}$$

kroucení tenkostěnných prutů – OTEVŘENÉ

– sestává se z protáhlých obdélníků $\quad \alpha = \beta = \frac{1}{3}$

$$I_t = \frac{1}{3} t^3 h$$

$$W_t = \frac{1}{3} t^2 h = \frac{I_t}{t}$$



– moment tuhosti v kroucení průřezu složeného z více obdélníků

$$I_t = \eta \sum_{i=1}^{n} I_{ti} = \frac{\eta}{3}\sum_{i=1}^{n} t_i^3 h_i$$

η... součinitel korekce (zaoblení ; poměr $\frac{h}{t}$, ...)

tab. 5.2 str. 126

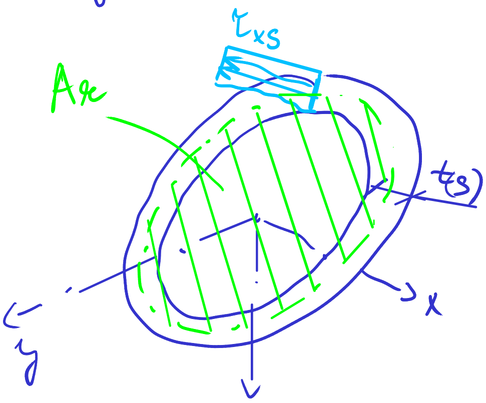
– τ_{max} vznikne v části s t_{max}

$$W_t = W_{t,min} = \frac{I_t}{t_{max}}$$

$$\tau_{max} = \frac{T}{W_t}$$

kroucení tenkostěnného průřezu – UZAVŘENÝ

- je-li t relativně malá, lze přibližně předpokládat τ konstantní podél celé tloušťky t



$$\tau_{xs} = \tau_{xs}(s) = \frac{T}{2 A_g\, t(s)}$$

A_g … plocha ohraničená střednicí průřezu

- τ_{max} vznikne v místě t_{min}

$$W_t = 2 A_g \cdot t_{min} \qquad \text{Bredtův vzorec}$$

$$\tau_{max} = \frac{T}{W_t}$$

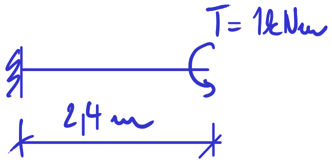
- moment tuhosti v kroucení

$$I_t = \frac{4 A_g^2}{\oint_s \frac{ds}{t(s)}} \qquad \text{2. Bredtův vzorec}$$

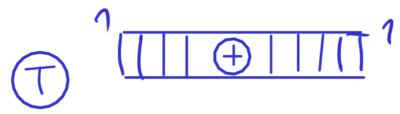
pro $t(s)$ po úsecích konstantní

$$I_t = \frac{4 A_g^2}{\sum_{i=1}^{n} \frac{h_i}{t_i}}$$

h_i … délka úseku
t_i … tloušťka úseku

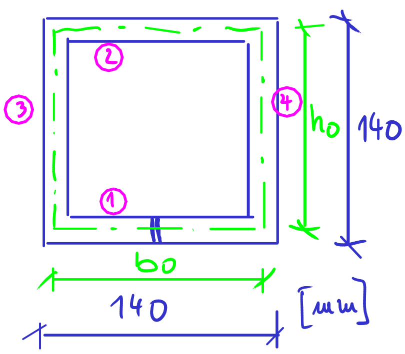


$G = 81\ GPa$



$t = 15\ mm$

$b_0 = h_0 = 140 - 15 = 125\ mm$



$$I_t = \frac{\eta}{3} \sum_{i=1}^{4} t_i^3 h_i \qquad \eta = 1 \text{ úhelník}$$

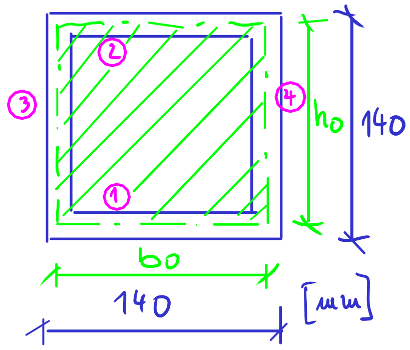
$$I_t = \frac{1}{3}\, 4\left(t_0^3 h_0\right) = \frac{1}{3}\, 4\left(15^3 \cdot 125\right) \cdot 10^{-12} = \underline{\underline{5{,}625 \cdot 10^{-7}\ m^4}}$$

(např. $t_2 = 20\ mm$ $\quad \frac{1}{3}\left[3 t_0^3 h_0 + t_2^3 h_2\right]$ $\quad t_{max} = t_2$)

$$W_t = \frac{I_t}{t_{max}} = \frac{5{,}625 \cdot 10^{-7}}{15 \cdot 10^{-3}} = 3{,}75 \cdot 10^{-5}\ m^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{1 \cdot 10^3}{3{,}75 \cdot 10^{-5}} = \underline{\underline{26{,}67\ MPa}}$$

$$\varphi_l = \frac{T l}{G I_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 5{,}625 \cdot 10^{-7}} = \underline{\underline{0{,}053\ rad}} \ (3{,}037°)$$



$t = 15\,mm$

$b_0 = h_0 = 125\,mm$

$A_r = b_0 h_0$

$$I_t = \frac{4 A_r^2}{\sum_{i=1}^{4} \frac{h_i}{t_i}} = \frac{4\,(0{,}125^2)^2}{4 \cdot \frac{125 \cdot 10^{-3}}{15 \cdot 10^{-3}}} = \underline{2{,}9297 \cdot 10^{-5}\,m^4}$$

$$W_t = 2\,A_r \cdot t_{min} = 2 \cdot 0{,}125^2 \cdot 15 \cdot 10^{-3} = \underline{4{,}6875 \cdot 10^{-4}\,m^3}$$

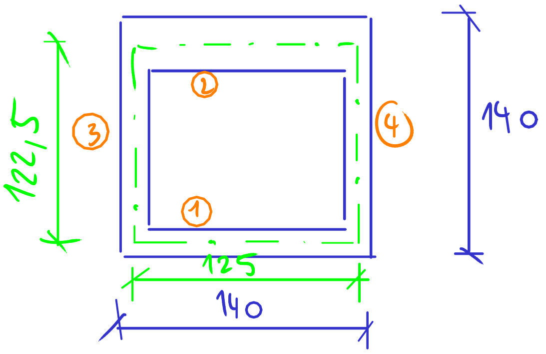
např.: $t_2 = 20\,mm$

$$3 \cdot \frac{h_0}{t_0} + \frac{h_2}{t_2}$$

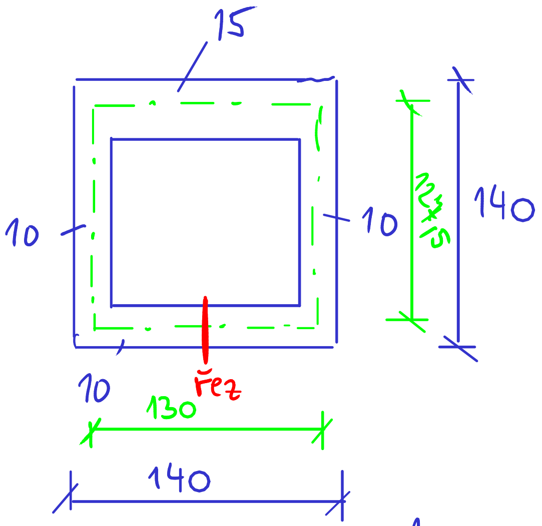
$$t_{min} = 15\,mm$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{4{,}6875 \cdot 10^{-4}} = \underline{\underline{2{,}13\ MPa}}$$

$$\varphi_\ell = \frac{T\,\ell}{G I_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 2{,}9297 \cdot 10^{-5}} = 0{,}001\,rad \quad (0{,}057°)$$



$$2 \cdot \frac{122{,}5}{15} + \frac{125}{15} + \frac{125}{20}$$

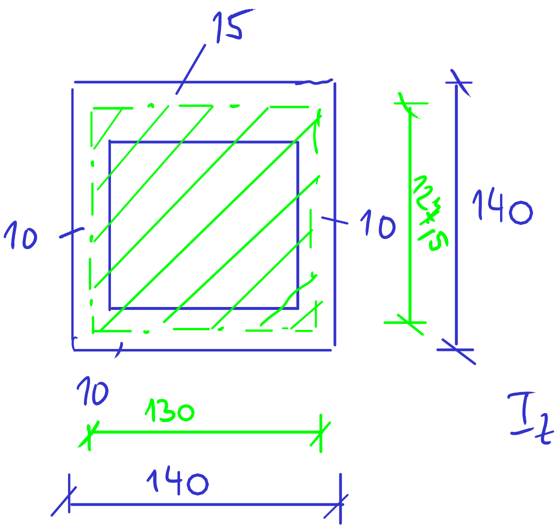


$$I_t = \frac{\eta}{3}\sum_{i=1}^{4} t_i^3 \cdot h_i = \frac{1}{3}\left[15^3 \cdot 130 + 2 \cdot 10^3 \cdot 127{,}5 + 10^3 \cdot 130\right] = 274{,}58\overline{3} \cdot 10^3\ mm^4$$

$$w_t = \frac{I_t}{t_{max}} = \frac{\qquad}{15} = 18{,}30\overline{5} \cdot 10^3\ mm^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{18{,}30\overline{5} \cdot 10^3 \cdot 10^{-9}} = \underline{\underline{54{,}628\ MPa}}$$

$$\varphi_l = \frac{Tl}{GI_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 274{,}58\overline{3} \cdot 10^3 \cdot 10^{-12}} = \underline{\underline{0{,}108\ rad}}$$



$$A_r = 130 \cdot 127{,}5 = 16\,575\ mm^2$$

$$I_t = \frac{4A_r^2}{\sum_{i=1}^{4} \frac{h_i}{t_i}}$$

$$I_t = \frac{4 \cdot 16575^2}{\frac{130}{15} + \frac{130}{10} + 2 \cdot \frac{127{,}5}{10}} = 23{,}299 \cdot 10^6\ mm^4$$

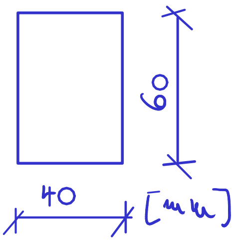
$$w_t = 2 A_r t_{min} = 2 \cdot 16575 \cdot 10 = 331{,}5 \cdot 10^3\ mm^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{331{,}5 \cdot 10^3 \cdot 10^{-9}} = \underline{\underline{3{,}017\ MPa}}$$

$$\varphi_l = \frac{T\,l}{G\,I_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 23{,}299 \cdot 10^6 \cdot 10^{-12}} = \underline{\underline{0{,}00127\ rad}}$$

Tabulka 5.1 *Součinitele pro výpočet kroucení prutů obdélníkového průřezu*

h/b	1	1,1	1,2	1,3	1,4	1,5	1,6	1,7
α	0,1406	0,154	0,166	0,177	0,187	0,196	0,204	0,211
β	0,208	0,214	0,219	0,223	0,227	0,231	0,234	0,237



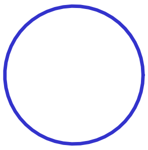
$$\frac{h}{b} = \frac{60}{40} = 1{,}5 \Rightarrow \alpha = 0{,}196$$
$$\beta = 0{,}231$$

$$I_t = \alpha b^3 h = 0{,}196 \cdot 0{,}04^3 \cdot 0{,}06 = 0{,}75264 \cdot 10^{-6}\, m^4$$

$$W_t = \beta b^2 h = 0{,}231 \cdot 0{,}04^2 \cdot 0{,}06 = 22{,}176 \cdot 10^{-6}\, m^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{22{,}176 \cdot 10^{-6}} = \underline{\underline{45{,}094\ MPa}}$$

$$\varphi_\ell = \frac{T\ell}{GI_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 0{,}75264 \cdot 10^{-6}} = \underline{\underline{0{,}039\ rad}}$$



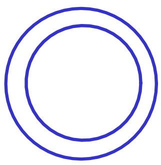
$d = 60\,mm$

$$I_t = \frac{\pi}{32} d^4 = \frac{\pi}{32} \cdot 0{,}06^4 = 1{,}272345 \cdot 10^{-6}\, m^4$$

$$W_t = \frac{I_t}{r} = \frac{2 \cdot 1{,}272345 \cdot 10^{-6}}{0{,}06} = 42{,}4115 \cdot 10^{-6}\, m^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{42{,}4115 \cdot 10^{-6}} = \underline{\underline{23{,}579\ MPa}}$$

$$\varphi_\ell = \frac{T\ell}{GI_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 1{,}272345 \cdot 10^{-6}} = \underline{\underline{0{,}0233\ rad}}$$



$d = 60\,mm$
$t = 5\,mm$
$d_s = 50\,mm$

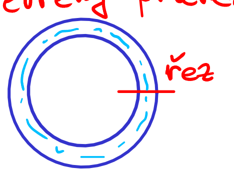
$$I_t = \frac{\pi}{32}(d^4 - d_s^4) = \frac{\pi}{32}(0{,}06^4 - 0{,}05^4) = 0{,}65875 \cdot 10^{-6}\, m^4$$

$$W_t = \frac{I_t}{r} = \frac{2 \cdot 0{,}65875 \cdot 10^{-6}}{0{,}06} = 21{,}9584 \cdot 10^{-6}\, m^3$$

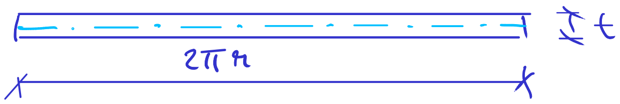
$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{21{,}9584 \cdot 10^{-6}} = \underline{\underline{45{,}54\ MPa}}$$

$$\varphi_\ell = \frac{T\ell}{GI_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 0{,}65875 \cdot 10^{-6}} = \underline{\underline{0{,}045\ rad}}$$

otevřený průřez



$d = 60\,mm$ $\quad t = 5\,mm$
$d_s = 50\,mm$



$$I_t = \frac{\eta}{3} \sum t_i^3 h_i$$

$$I_t = \frac{1}{3}\, 0{,}005^3 \cdot 2 \cdot \pi \cdot \frac{0{,}06 - 0{,}005}{2} = 7{,}19948 \cdot 10^{-9}\, m^4$$

$$W_t = \frac{I_t}{t_{max}} = \frac{7{,}19948 \cdot 10^{-9}}{0{,}005} = 1{,}4399 \cdot 10^{-6}\, m^3$$

$$\tau_{max} = \frac{T}{W_t} = \frac{10^3}{1{,}4399 \cdot 10^{-6}} = \underline{\underline{694{,}49\ MPa}}$$

$$\varphi_\ell = \frac{T\ell}{GI_t} = \frac{10^3 \cdot 2{,}4}{81 \cdot 10^9 \cdot 7{,}19948 \cdot 10^{-9}} = \underline{\underline{4{,}1155\ rad}}$$