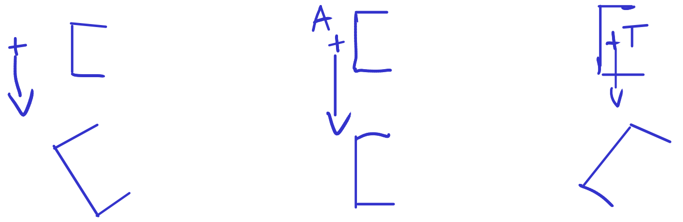
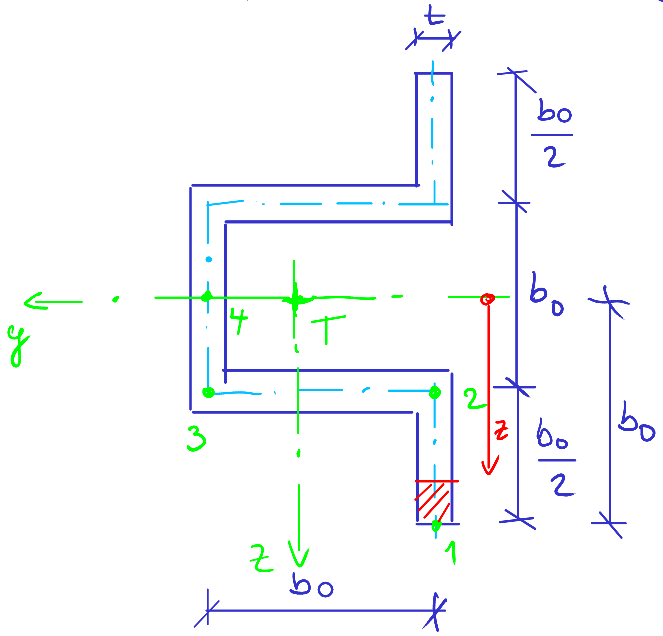
Střed smyku

- místo, kterým musí procházet výsledná rovina zatížení, aby nedocházelo ke kroucení průřezu



Určete polohu středu smyku



Průběh smykových napětí

$$\tau = \frac{V_z \cdot \bar{S}_y}{I_y \cdot t}$$

$$I_y = \frac{1}{12} t \cdot b_0^3 + \frac{1}{12} t \cdot (2b_0)^3 - \frac{1}{12} t b_0^3 + b_0 + \cancel{2 \cdot \frac{1}{12} \cdot b_0 \cdot t^3} + 2 \cdot b_0 \cdot t \left(\frac{b_0}{2}\right)^2 = \underline{\frac{7}{6} t b_0^3}$$

zanedbáme $t \ll b_0$

přečnívající části:

$$\bar{S}_{y_{1-2}} = t(b_0 - z)\left[\frac{b_0 - z}{2} + z\right] = \frac{1}{2} t \left(b_0^2 - z^2\right)$$

$$\left(\tau_{1-2} = \frac{V_z \cdot \frac{1}{2} t \left(b_0^2 - z^2\right)}{\frac{7}{6} t b_0^3 \cdot t} \right)$$

bod 1: $z = b_0$ $\quad \bar{S}_{y_1} = 0 \Rightarrow \tau_1 = 0$

bod 2: $z = \frac{b_0}{2}$ $\quad \bar{S}_{y_2} = \frac{1}{2} t \left(b_0^2 - \frac{b_0^2}{4}\right) = \frac{3}{8} t b_0^2$

$$\tau_2 = \frac{V_z \cdot \frac{3}{8} t b_0^2}{\frac{7}{6} t b_0^3 \cdot t} = \frac{9}{28} \frac{V_z}{b_0 t}$$

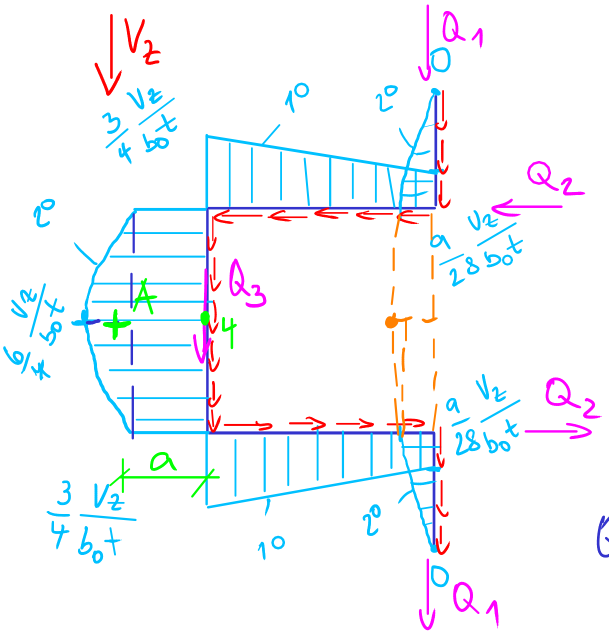
bod 3: $\bar{S}_{y_3} = \bar{S}_{y_2} + b_0 \cdot t \cdot \frac{b_0}{2} = \frac{7}{8} t b_0^2$

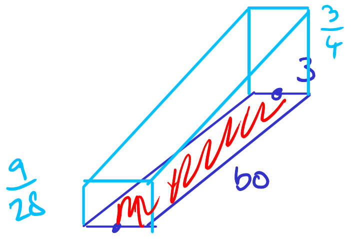
$$\tau_3 = \frac{V_z \cdot \frac{7}{8} t b_0^2}{\frac{7}{6} t b_0^3 \cdot t} = \frac{3}{4} \frac{V_z}{b_0 t}$$

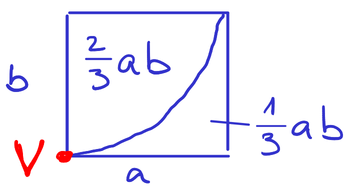
bod 4: $z = 0$

$$\bar{S}_{y_4} = \bar{S}_{y_3} + t \cdot \frac{b_0}{2} \cdot \frac{b_0}{4} = t b_0^2$$

$$\tau_4 = \frac{V_z \, t b_0^2}{\frac{7}{6} t b_0^3 \cdot t} = \frac{6}{4} \frac{V_z}{b_0 t}$$







$$Q_2 = \underbrace{\frac{\left(\frac{9}{28}+\frac{3}{4}\right)}{2}\,\frac{V_z}{b_0 t}\cdot b_0}\cdot t = \underline{\frac{15}{28}V_z}$$

$$Q_3 = t\left[\frac{3}{4}\frac{V_z}{b_0 t}\cdot b_0 + 2\cdot\frac{2}{3}\,\frac{b_0}{2}\left(\frac{6}{4}-\frac{3}{4}\right)\frac{V_z}{b_0 t}\right] = $$

$$= \underline{\frac{23}{28}V_z}$$

$$Q_1 = \int_{b_0/2}^{b_0} \tau_{1-2}\cdot t\,dz = \int_{b_0/2}^{b_0} \frac{V_z\,\frac{1}{2}t\left(b_0^2-z^2\right)}{\frac{4}{6}t\,b_0^3\quad t}\cdot t\,dz = \underline{\frac{5}{56}V_z}$$

<u>podmínky ekvivalence</u>

$\oplus\rightarrow$ $Q_2 - Q_2 = 0$ ✓

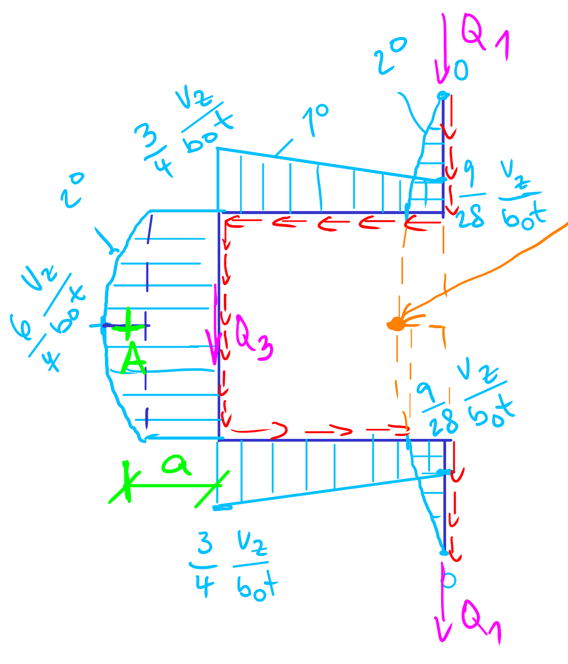
$2Q_1 + Q_3 = V_z$

$\frac{28}{28}V_z = V_z$ ✓

momentová podmínka k bodu 4:

$\oplus\circlearrowright$ $(Q_3\cdot 0) + 2\cdot Q_1\cdot b_0 - Q_2\cdot b_0 = -V_z\cdot a$

$$a = \frac{5}{14}b_0 \doteq \underline{0{,}357\,b_0}$$



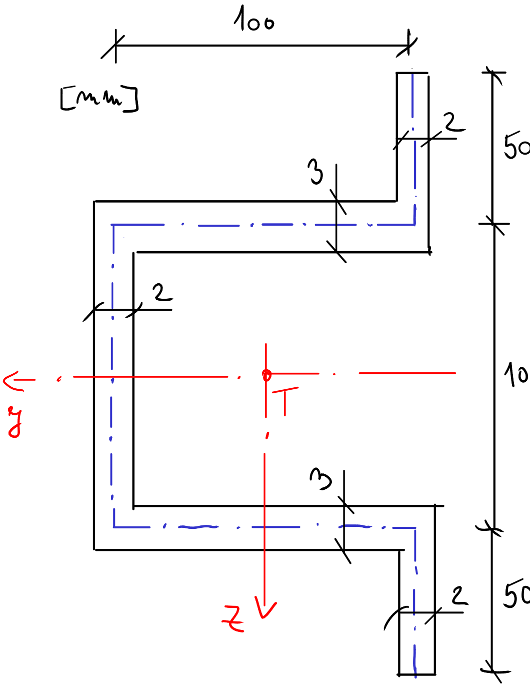
$z = 0$

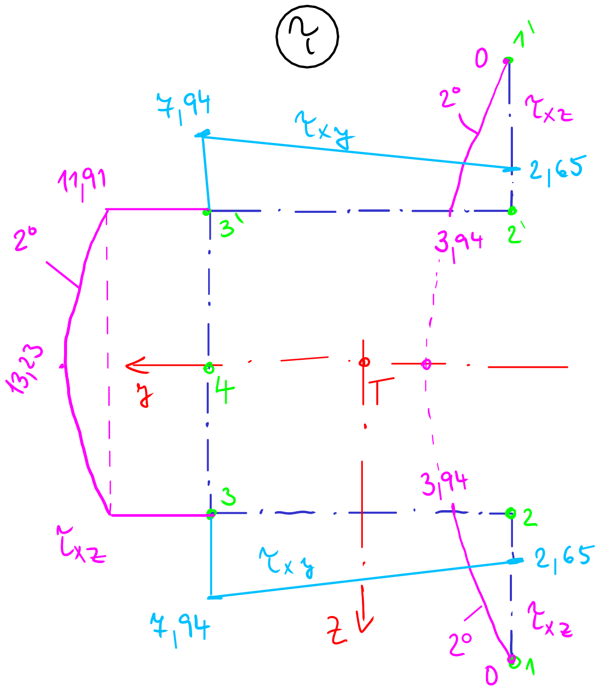
$$\tau' = \frac{V_z\cdot\frac{1}{2}t\left(b_0^2-0\right)}{\frac{4}{6}t\,b_0^3\cdot t} = \frac{3}{4}\frac{V_z}{b_0 t}$$

$$Q_1 = \left[\frac{2}{3}\left(\frac{3}{4}\frac{V_z}{b_0 t}\right)b_0 - \frac{9}{28}\frac{V_z}{b_0 t}\cdot\frac{b_0}{2} - \frac{2}{3}\,\frac{b_0}{2}\left(\frac{3}{4}-\frac{9}{28}\right)\frac{V_z}{b_0 t}\right]\cdot t$$

$$Q_1 = \frac{2}{4}V_z - \frac{9}{56}V_z - \frac{1}{28}V_z = \underline{\underline{\frac{5}{56}V_z}}$$

Vypočítejte smyková napětí pro $V_z = 3\,kN$ a určete střed smyku.





$$I_y = \overset{(3-3')}{\frac{1}{12}\, 2\cdot 100^3} + 2\left(\overset{(2-3)(2'-3')}{\frac{1}{12}\cdot 100\cdot 3^3 + 100\cdot 3\cdot 50^2}\right) + \overset{(1-1')}{\frac{1}{12}\cdot 2\cdot 200^3} - \overset{(2-2')}{\frac{1}{12}\cdot 2\cdot 100^3} = 2{,}83378\overline{3}\cdot 10^6\,mm^4 = 2{,}83378\overline{3}\cdot 10^{-6}\,m^4$$

$$\tau = \frac{V_z \bar{S}_y}{I_y \cdot t} = \frac{3\cdot 10^{-3}}{2{,}83378\overline{3}\cdot 10^{-6}}\,\frac{\bar{S}_y}{t} = 1058{,}65539\cdot\frac{\bar{S}_y}{t} \quad [m \to MPa]$$

τ_{xz} vodorovné řezy po výšce průřezu

$$\bar{S}_{y1} = \bar{S}_{y1'} = 0 \Rightarrow \tau_{xz1} = \tau_{xz1'} = 0$$

$$\bar{S}_{y2} = \bar{S}_{y2'} = 50\cdot 2\cdot 75 = 7500\,mm^3 = 7{,}5\cdot 10^{-6}\,m^3$$

$$\tau_{xz2} = \tau_{xz2'} = \frac{V_z}{I_y}\cdot\frac{\bar{S}_{y2}}{t_2} = 1058{,}65539\cdot\frac{7{,}5\cdot 10^{-6}}{2\cdot 10^{-3}} = \underline{3{,}97\,MPa}$$

$$\bar{S}_{y3} = S_{y3'} = \bar{S}_{y2} + 100\cdot 3\cdot 50 = 22{,}5\cdot 10^3\,mm^3 = 22{,}5\cdot 10^{-6}\,m^3$$

$$\tau_{xz3} = \tau_{xz3'} = \frac{V_z}{I_y}\cdot\frac{22{,}5\cdot 10^{-6}}{2\cdot 10^{-3}} = \underline{11{,}91\,MPa}$$

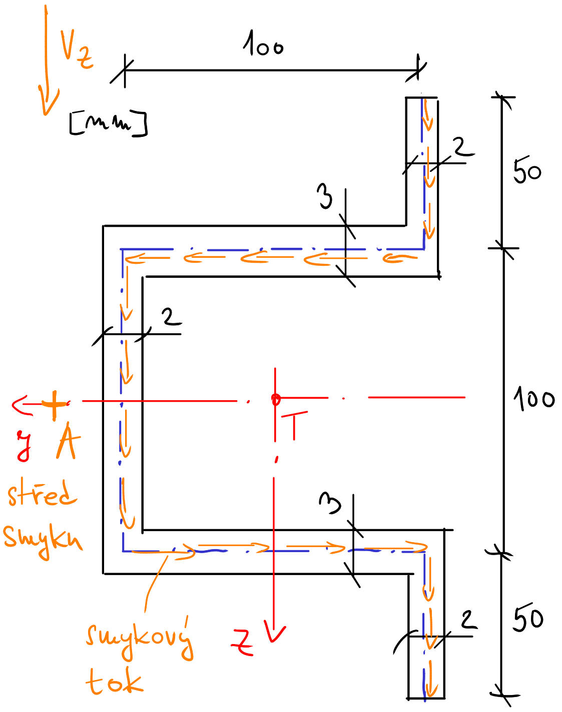
$$\bar{S}_{y4} = \bar{S}_{y3} + 50\cdot 2\cdot 25 = 25\cdot 10^3\,mm^3 = 25\cdot 10^{-6}\,m^3$$

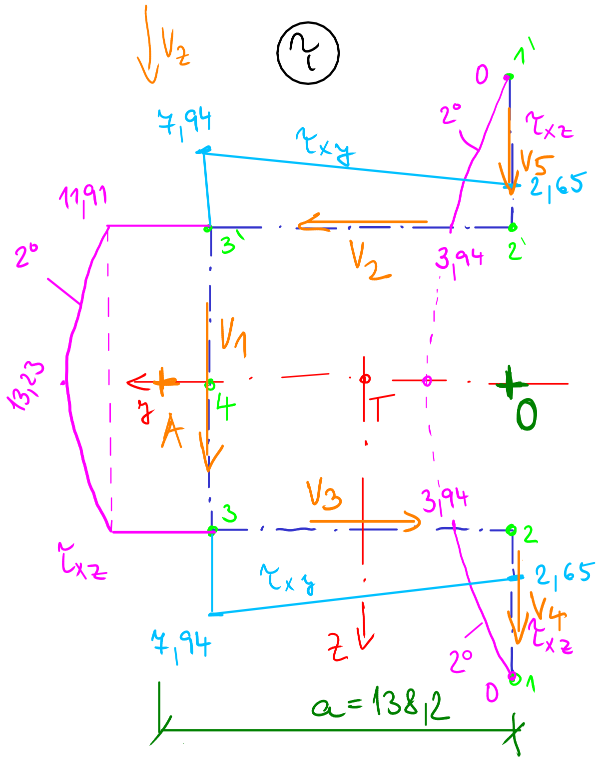
$$\tau_{xz4} = \frac{V_z}{I_y}\cdot\frac{25\cdot 10^{-6}}{2\cdot 10^{-3}} = \underline{13{,}23\,MPa}$$

τ_{xy} svislé řezy po šířce

$$\bar{S}_{y2} = \bar{S}_{y2'} = 7{,}5\cdot 10^{-6}\,m^3 \qquad \tau_{xy2} = \tau_{xy2'} = \frac{V_z}{I_y}\cdot\frac{7{,}5\cdot 10^{-6}}{3\cdot 10^{-3}} = \underline{2{,}65\,MPa}$$

$$\bar{S}_{y3} = \bar{S}_{y3'} = 22{,}5\cdot 10^{-6}\,m^3 \qquad \tau_{xy3} = \tau_{xy3'} = \frac{V_z}{I_y}\cdot\frac{22{,}5\cdot 10^{-6}}{3\cdot 10^{-3}} = \underline{4{,}94\,MPa}$$





$V \doteq \int_A \tau \, dA = A_\tau \cdot t$

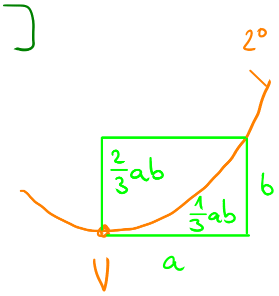
podmínka ekvivalence $\Sigma M_{V_i} = M_{V_z}$

Pozn. pokud otočíme směr smykového toku použijeme podmínku rovnováhy $\Sigma M_i = 0$

↳ vhodně zvolíme bod 0, ke kterému budeme počítat moment
[tím nemusíme počítat V_1 a V_2, které jsou pracnější na výpočet]

$V_2 = V_3 = \left[\frac{7{,}94 - 2{,}65}{2} \cdot 0{,}1\right] \cdot 3 \cdot 10^{-3} = 1{,}5885 \cdot 10^{-3}\, MN = \underline{1{,}5885\, kN}$

$V_1 = \left[11{,}91 \cdot 0{,}1 + \frac{2}{3}(13{,}23 - 11{,}91) \cdot 0{,}1\right] \cdot 2 \cdot 10^{-3} = 2{,}558 \cdot 10^{-3}\, MN = \underline{2{,}558\, kN}$

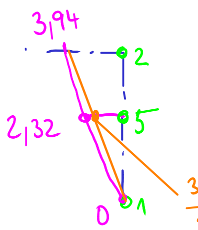


$\oplus \; \Sigma M_{V_i} = M_{V_z}: \quad V_1 \cdot 0{,}1 + V_2 \cdot 0{,}1 = V_z \cdot a$

$$a = \frac{2{,}558 \cdot 0{,}1 + 1{,}5885 \cdot 0{,}1}{3} = 0{,}13821\overline{6}\, m = \underline{\underline{138{,}2\, mm}}$$

(vzdálenost od těžiště $138{,}2 - 50 = 88{,}2\, mm$)

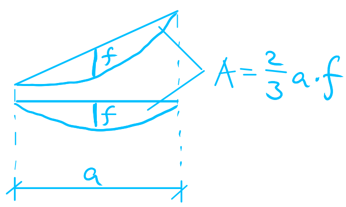
Výpočet V_4 a V_5 – integrací nebo protažením paraboly k těžišti viz předchozí př.
– výpočet hodnoty uprostřed a dopočet s jejím využitím



$S_{y5} = 25 \cdot 2 \cdot \left(100 - \frac{25}{2}\right) = 4{,}375 \cdot 10^3\, mm^3 = 4{,}375 \cdot 10^{-6}\, m^3$

$\tau_{xz5} = \frac{V_z}{I_y} \frac{4{,}375 \cdot 10^{-6}}{2 \cdot 10^{-3}} = \underline{2{,}32\, MPa}$

$\frac{3{,}94}{2} = 1{,}97$



$2{,}32 - 1{,}97$

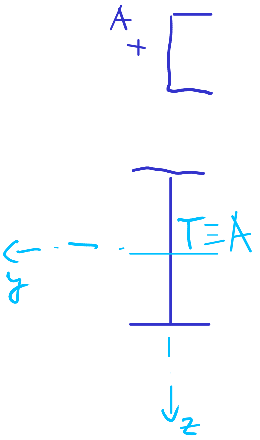
$V_4 = V_5 = \left[\frac{1}{2} 3{,}94 \cdot 0{,}05 + \frac{2}{3}(2{,}32 - 1{,}97) \cdot 0{,}05\right] \cdot 2 \cdot 10^{-3} = 0{,}220\overline{3} \cdot 10^{-3}\, MN = 0{,}220\overline{3}\, kN$

$\Sigma V_{yi} = 0: \quad V_2 - V_3 = 0 \;\checkmark$

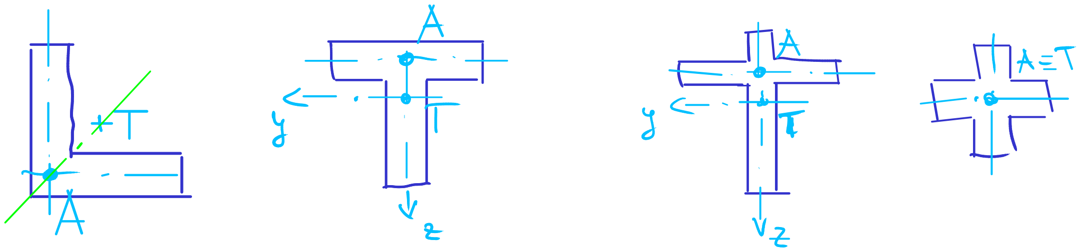
$\Sigma V_{zi} = V_z: \oplus\downarrow \quad V_1 + V_4 + V_5 \doteq V_z$

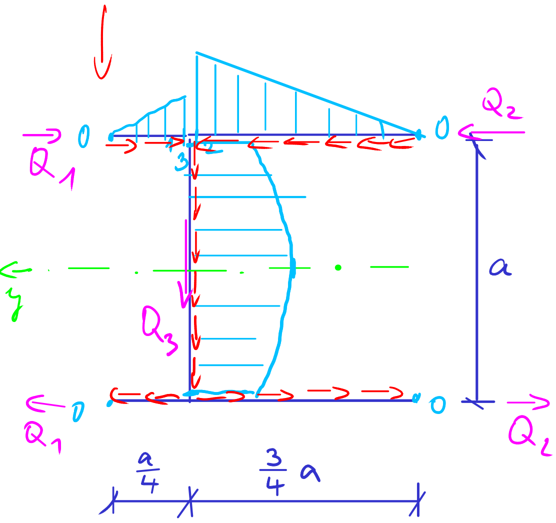
$2{,}558 + 2 \cdot 0{,}220\overline{3} = 3$

$\underline{2{,}998\overline{6} \doteq 3} \;\checkmark$



svazkové průřezy (střednice pásnic ∩ v jednom bodě)





tloušťka všech částí = t

zanedbáme

$$I_y = \frac{1}{12} t a^3 + \cancel{2\cdot\frac{1}{12} a\cdot t^3} + 2\cdot t\cdot a\left(\frac{a}{2}\right)^2$$

$$I_y = \frac{7}{12} t a^3$$

$$\bar{S}_{y_1} = t\cdot\frac{1}{4}a\left(\cdot\frac{1}{2}a\right) = \frac{1}{8} t a^2$$

$$\tau_1 = \frac{V_z\cdot\frac{1}{8} t a^2}{\frac{7}{12} t a^3\cdot t} = \frac{3}{14}\frac{V_z}{ta}$$