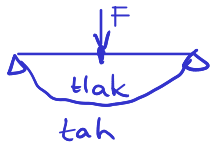
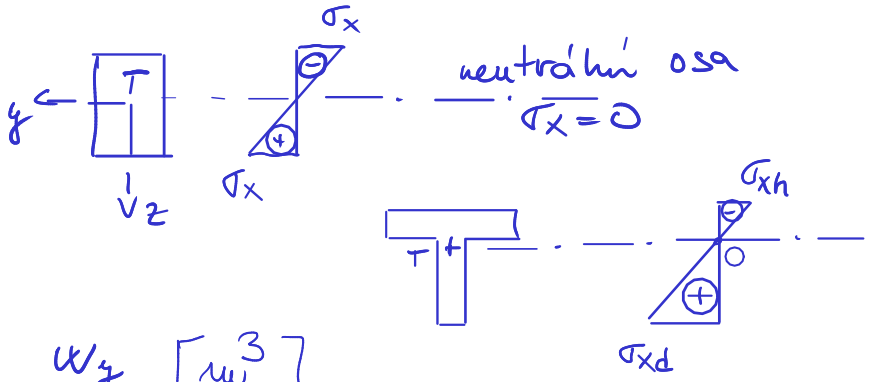
Prostý ohyb

- rovinný ohyb $N = V_y = 0$, $M_x = M_z = 0$, $\boxed{V_z \neq 0 \quad M_y \neq 0}$
- prostý ohyb $V_z = 0$ $\boxed{M_y \neq 0}$
- normálové napětí σ_x

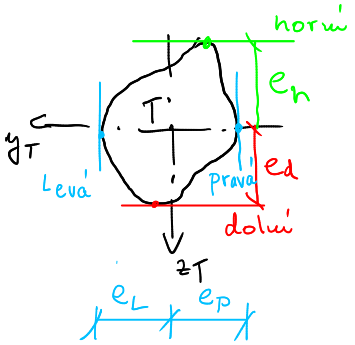
$$\sigma_x = \frac{M_y}{I_y} z \Rightarrow \text{lineární průběh po výšce průřezu}$$





- průřezový modul W_y $[m^3]$

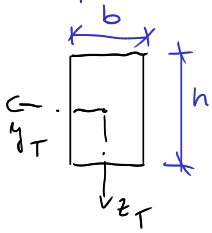
$$\sigma_x = \frac{M_y}{I_y} z \qquad W = \frac{I}{e} \rightarrow \text{vzdálenost ke krajnímu vláknům } \oplus$$



$$W_{y,h} = \frac{I_{yT}}{e_h} \qquad W_{y,d} = \frac{I_{yT}}{e_d}$$

$$W_{z,L} = \frac{I_{zT}}{e_L} \qquad W_{z,P} = \frac{I_{zT}}{e_p}$$

- průřezový modul: obdélník



$$W_y = \frac{I_y}{e} = \frac{1/12\, bh^3}{h/2} = \frac{1}{6} bh^2$$

$$W_z = \frac{1}{6} hb^2$$

$$\sigma_{x\,\min\atop\max} = \pm \frac{|M_y|}{W_y}$$

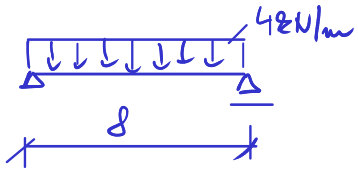
Návrh a posouzení

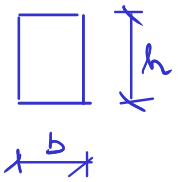
$$M_{Sd} \leq M_{Rd} = f_d \cdot W \qquad f_d = \frac{f_k}{\gamma_M}$$

nebo $\sigma_{xd} \leq f_d$ (lineárně pružný materiál)

Prostý ohyb

Navrhněte dřevěný trám II, $f_d = 13\,MPa$





volíme $\frac{b}{h} = \frac{5}{7}$

$b = \frac{5}{7}h$

$$M_{max} = \frac{1}{8}ql^2 = \frac{1}{8}4 \cdot 8^2 = \underline{32\,kNm} = M_{sd}$$

$$W = \frac{I_y}{z} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2 = \frac{1}{6}\left(\frac{5}{7}h\right)h^2 = \frac{5}{42}h^3 \Rightarrow$$

$$\Rightarrow h = \sqrt[3]{\frac{42}{5}W_y} = \sqrt[3]{\frac{42}{5}2{,}462 \cdot 10^{-3}} = \underline{0{,}274\,m}$$

$$M_{sd} \leq M_{Rd} = f_d W_y \Rightarrow W_y = \frac{M_{sd}}{f_d}$$

$$W_{min} = \frac{M_{sd}}{f_d} = \frac{32 \cdot 10^3}{13 \cdot 10^6} = \underline{2{,}462 \cdot 10^{-3}\,m^3}$$

návrh: $h = 280\,mm$

$b = \frac{5}{7}h = 200\,mm$

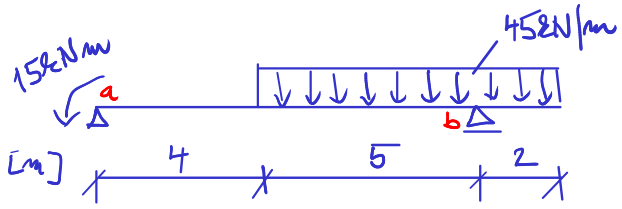
Posudek:

$$W_y = \frac{1}{6}bh^2 = \frac{1}{6}0{,}2 \cdot 0{,}28^2 = 2{,}61\overline{3} \cdot 10^{-3}\,m^3$$

$$M_{sd} = \underline{32 \cdot 10^3\,Nm} \leq M_{Rd} = f_d \cdot W_y = 13 \cdot 10^6 \cdot 2{,}61\overline{3} \cdot 10^{-3} = \underline{33{,}94\overline{3} \cdot 10^3\,Nm}$$

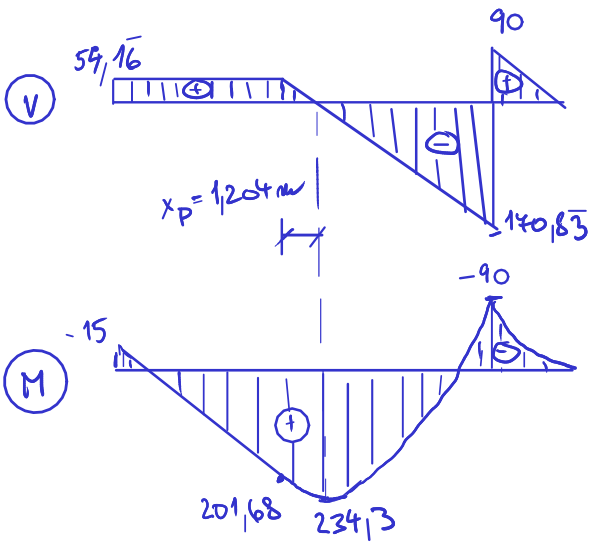
vyhoví

Navrhněte nosník ze 2 válcovaných I profilů, $f_d = 210\,MPa$



$R_a = 54{,}16\overline{2}\,kN$

$R_b = 260{,}8\overline{3}\,kN$



$M_{max} = M_{sd} = 234{,}3\,kNm$

$\sigma = \frac{M}{W}$

$I_{II} = 2\,I_I$

$W_{min\,II} = \frac{M_{sd}}{f_d} = \frac{234{,}3 \cdot 10^3}{210 \cdot 10^6} = 1{,}116 \cdot 10^{-3}\,m^3$

$W_{min\,I} = \frac{W_{min\,II}}{2} = 5{,}579 \cdot 10^{-4}\,m^3 = 557{,}9 \cdot 10^{+3}\,mm^3$

návrh: $\overline{I}\,300 \quad W_{yI} = 653{,}3 \cdot 10^3\,mm^3$

Posudek: $W_{y\,II} = 2\,W_{yI} = 1{,}3066 \cdot 10^{-3}\,m^3$

$M_{sd} = 234{,}3 \cdot 10^3\,Nm \leq M_{Rd} = f_d\,W_{y\,II} = 210 \cdot 10^6 \cdot 1{,}3066 \cdot 10^{-3} = 274{,}386 \cdot 10^3\,Nm$

✓ Vyhoví

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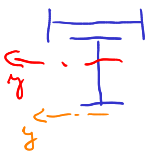
návrh: vyhoví předchozí profil $\overline{I}\,300$ i jako $\overline{\overline{I}}$?



$A = 6900\,mm^2 \quad t_w = 10{,}8\,mm \quad I_{yI} = 97{,}9 \cdot 10^6\,mm^4 \quad h = 300\,mm \quad b = 125\,mm$

$I_{y\,\overline{\overline{I}}} = I_{zI} = 4510 \cdot 10^3\,mm^3$

$z_T = \frac{6900 \cdot \left(-\frac{300}{2}\right) + 6900 \cdot \left(-300 + \frac{10{,}8}{2}\right)}{2 \cdot 6900} = -227{,}7\,mm$

$I_y = \left(I_y + A c^2\right) = 97{,}9 \cdot 10^6 + 6900 \cdot \left(227{,}7 - \frac{300}{2}\right)^2 + 4{,}51 \cdot 10^6 + 6900 \cdot \left(227{,}7 - 300 - \frac{10{,}8}{2}\right)^2 =$

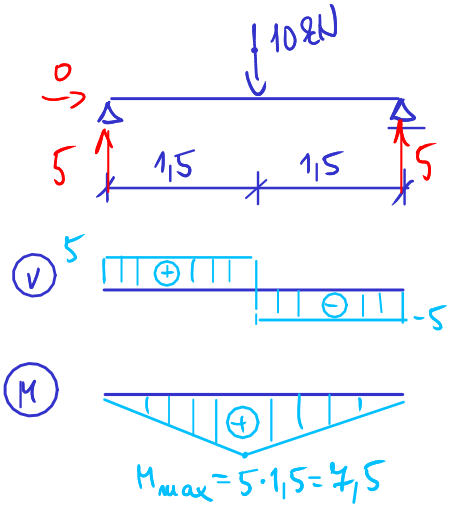
$= 185{,}725602 \cdot 10^6\,mm^4$

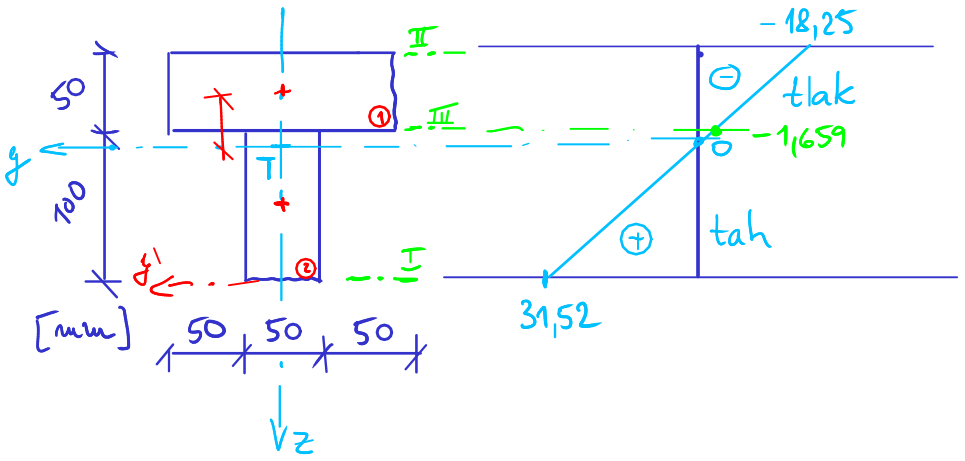
$\sigma_x = \frac{M}{W} \Leftrightarrow W_{min} = \frac{I}{z} \Leftrightarrow z_{max} = \begin{cases} 227{,}7\,mm \\ 140{,}2\,mm \end{cases}$

$W_{min} = \frac{185{,}725602 \cdot 10^6}{227{,}7} = 815{,}654818 \cdot 10^3\,mm^3$

$M_{sd} = 234{,}3 \cdot 10^3\,Nm \not\leq M_{Rd} = f_d\,W_{y\,II} = 210 \cdot 10^6 \cdot 815{,}655 \cdot 10^{-6} = 171{,}288 \cdot 10^3\,Nm$

✗ nevyhoví





$$\sigma_x = \frac{M}{W_y} = \frac{M}{I_y} z \qquad z_T = \frac{50 \cdot 150 \cdot (-125) + 50 \cdot 100 \cdot (-50)}{50 \cdot 150 + 50 \cdot 100} = \underline{-95\ mm}$$

$$I_y = \left| \frac{1}{12} bh^3 + Ac^2 \right| = \frac{1}{12} 150 \cdot 50^3 + 50 \cdot 150 \cdot (125 - 95)^2 + \frac{1}{12} 50 \cdot 100^3 + 50 \cdot 100 \cdot (50 - 95)^2 =$$

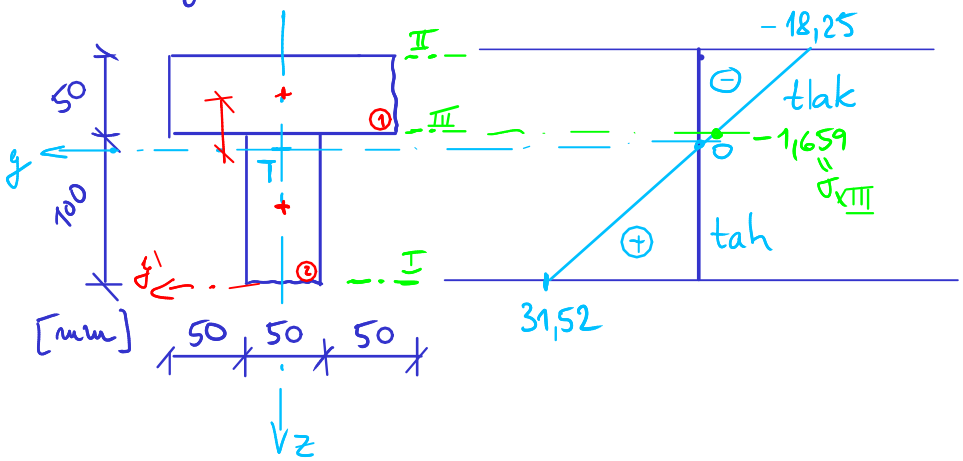
$$= \underline{22,60416 \cdot 10^6\ mm^4}$$

$$\sigma_{x\,I} = \frac{M}{I_y} z_I = \frac{7,5 \cdot 10^3}{22,60416 \cdot 10^{-6}} \left(+95 \cdot 10^{-3}\right) = 31,52 \cdot 10^6\ Pa = \underline{31,52\ MPa}$$

$$\sigma_{x\,II} = \frac{M}{I_y} z_{II} = \frac{7,5 \cdot 10^3}{22,60416 \cdot 10^{-6}} \cdot \left(-55 \cdot 10^{-3}\right) = \underline{-18,25\ MPa}$$

$$\sigma_{x\,T} = 0\ MPa \quad \text{(neutrální osa)}$$

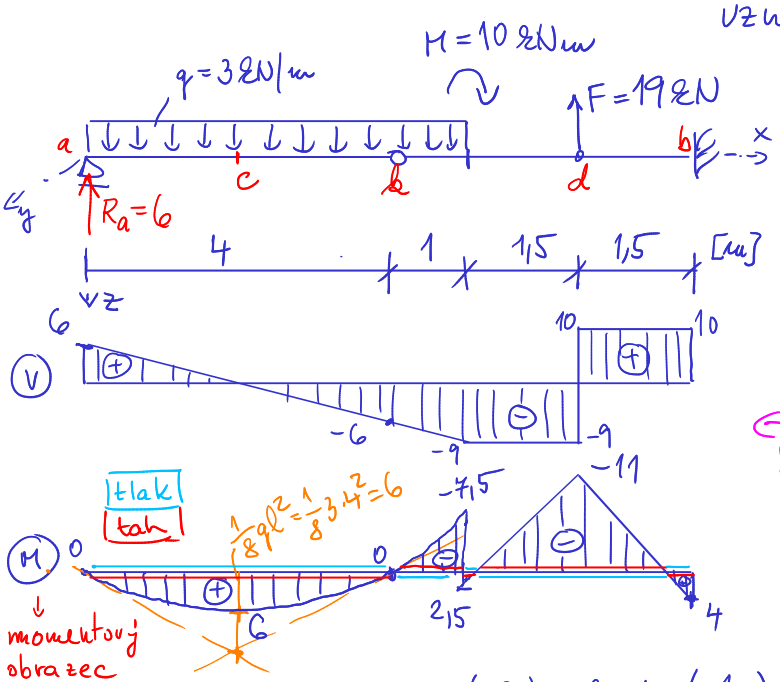
$$\sigma_{x\,III} = \frac{M}{I_y} z_{III} = \text{(—11—)} \cdot \left(-5 \cdot 10^{-3}\right) = \underline{-1,659\ MPa}$$



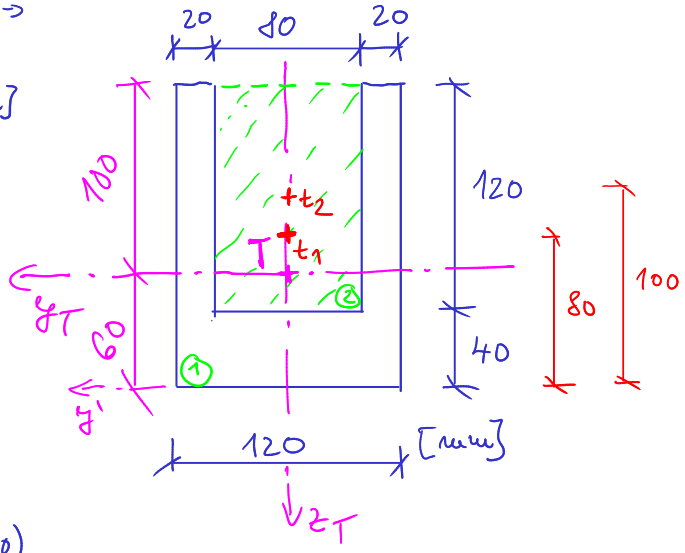
$$\frac{\sigma_{x\,III}}{-5} = \frac{31,52}{95} \left(= \frac{-18,25}{-55} \right)$$

$$\sigma_{x\,III} = \frac{31,52}{95} \cdot (-5) = \underline{-1,659\ MPa}$$

Určete místo na konstrukci a vlákna na průřezu, kde vznikne max. tlakové napětí.



$$\sum M_{z_i} = 0 \circlearrowright : \; R_a \cdot 4 - q \cdot \frac{4^2}{2} = 0 \Rightarrow R_a = 6\,\text{kN}$$



$$z_T = \frac{120 \cdot 160 \cdot (-80) - 80 \cdot 120 \cdot (-100)}{120 \cdot 160 - 80 \cdot 120} = -60\,\text{mm}$$

$$I_{y_T} = \sum_{i=1}^{2} \left(\frac{1}{12} b_i h_i^3 + A_i c_i^2 \right) = \frac{1}{12} \cdot 120 \cdot 160^3 + 120 \cdot 160 \cdot (80-60)^2 - \left(\frac{1}{12} 80 \cdot 120^3 + 80 \cdot 120 (100-60)^2 \right) =$$

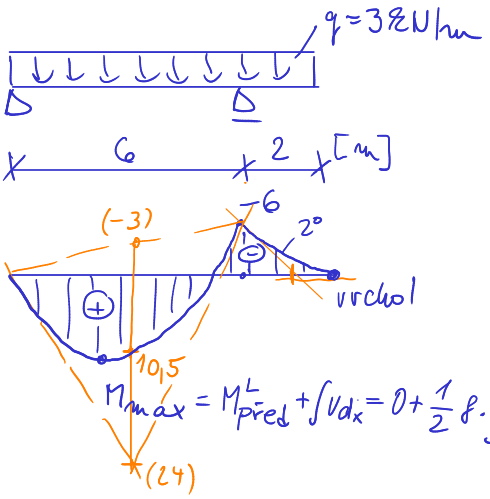
$$= 21{,}76 \cdot 10^6\,\text{mm}^4$$

$$\sigma_{x,c} = \frac{M_{yc}}{I_{y_T}} \cdot z_{horní} = \frac{+6 \cdot 10^3}{21{,}76 \cdot 10^{-6}} \cdot (-0{,}1) = -27{,}57 \cdot 10^6\,\text{Pa}$$

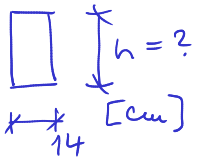
$$\sigma_{x,d} = \frac{M_{yd}}{I_{y_T}} \cdot z_{dolní} = \frac{-11 \cdot 10^3}{21{,}76 \cdot 10^{-6}} \cdot (+0{,}06) = \boxed{-30{,}33 \cdot 10^6\,\text{Pa}}$$

Maximální tlakové napětí bude v bodě d na spodních vláknech.

Jak to dopadne, když $M = 13\,\text{kNm}$ nebo $M = 14\,\text{kNm}$?



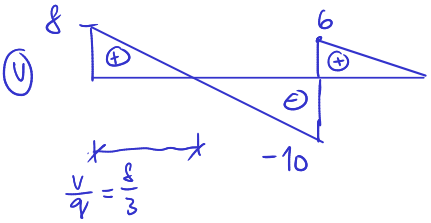
navrhněte výšku dřevěného trámu



$f_d = 12\,MPa$

$M_{max} = M^L_{před} + \int V_d dx = 0 + \frac{1}{2} \cdot 8 \cdot \frac{8}{3} = 10{,}6\overline{6}\,kNm$

vzepětí $\frac{1}{8} q l^2 = \frac{1}{8} \cdot 3 \cdot 6^2 = 13{,}5$



$$\sigma_x = \frac{|M_y|}{W_y} \le f_d \Rightarrow W_{y\,min} = \frac{|M_y|}{f_d}$$

$$W_{y\,min} = \frac{M_{max}}{f_d} = \frac{10{,}6\overline{6} \cdot 10^3}{12 \cdot 10^6} = 888{,}8 \cdot 10^{-6}\,m^3$$

$$W_y = \frac{1}{6} b h^2 \Rightarrow h_{min} = \sqrt{\frac{6 W_{y\,min}}{b}} = \sqrt{\frac{6 \cdot 888{,}8 \cdot 10^{-6}}{0{,}14}} = 0{,}19518\,m$$

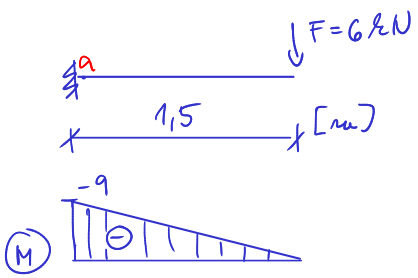
návrh: $h = 20\,cm$

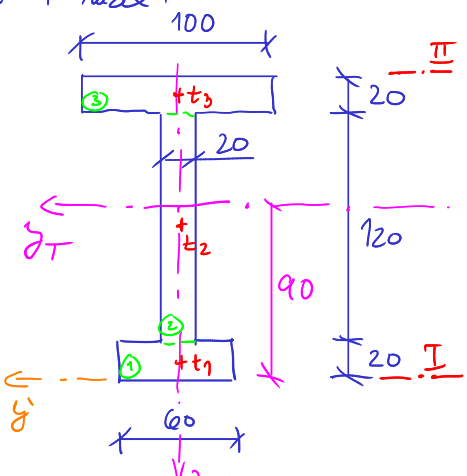
posouzení:

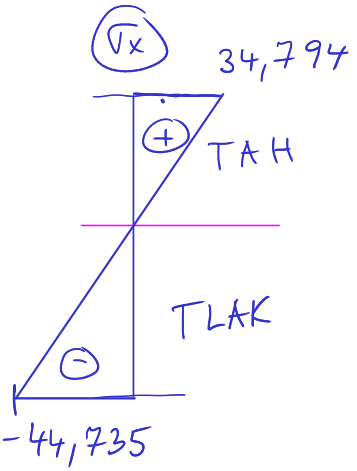
$$\sigma_x = \frac{|M_{max}|}{W} = \frac{6\,|M_{max}|}{b h^2} = \frac{6 \cdot 10{,}6\overline{6} \cdot 10^3}{0{,}14 \cdot 0{,}2^2} = 11{,}429\,MPa \not\le f_d = 12\,MPa$$

vyhoví

Průběh napětí v místě $|M_{max}|$







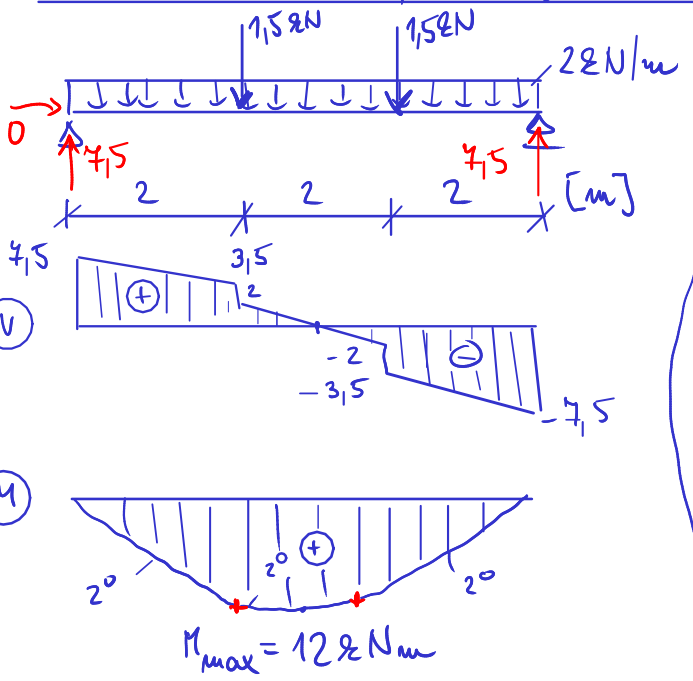
$$z_T = \frac{60 \cdot 20 \cdot (-10) + 20 \cdot 120 \cdot (-80) + 100 \cdot 20 \cdot (-150)}{60 \cdot 20 + 20 \cdot 120 + 100 \cdot 20} = 90\,mm$$

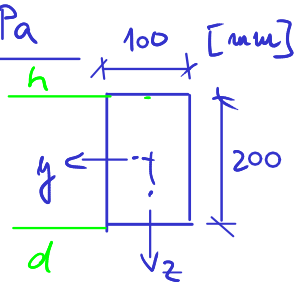
$$I_y = \Sigma\left(\frac{1}{12} b h^3 + A c^2\right) = \frac{1}{12} \cdot 60 \cdot 20^3 + 60 \cdot 20 \cdot 80^2 + \frac{1}{12} \cdot 20 \cdot 120^3 + 20 \cdot 120 \cdot 10^2 + \frac{1}{12} \cdot 100 \cdot 20^3 + 100 \cdot 20 \cdot 60^2 = 18{,}10\overline{6} \cdot 10^6\,mm^4$$

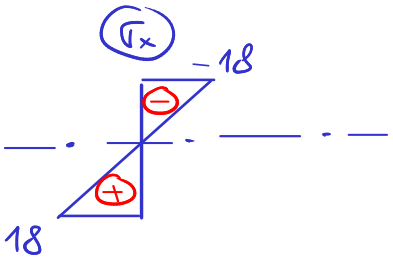
$$\sigma^{I}_{x,a} = \frac{-9 \cdot 10^3}{18{,}10\overline{6} \cdot 10^{-6}} \cdot (+0{,}090) = -44{,}735\,MPa$$

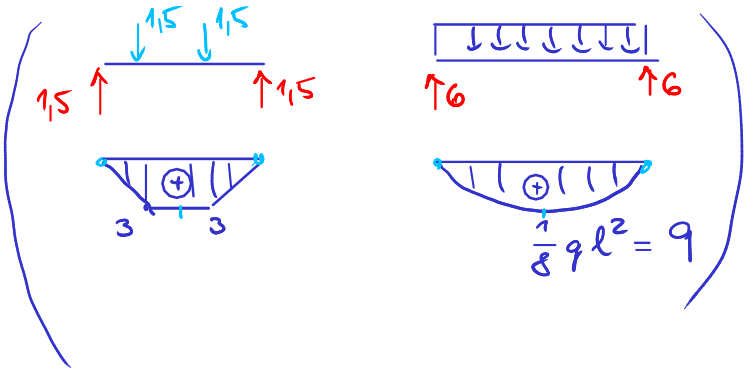
$$\sigma^{II}_{x,a} = \frac{-9 \cdot 10^3}{18{,}10\overline{6} \cdot 10^{-6}} \cdot (-0{,}070) = +34{,}794\,MPa$$

Posuďte nosník, je-li $f_d = 13\,MPa$









$M_{max} = 12\,kNm$

$$\sigma_x = \frac{M_y}{I_y} z = \frac{M}{W}$$

Posudek: $M_{Sd} \leq M_{Rd}$ $(\sigma_{max} \leq f_d)$

$$M_{Rd} = f_d W$$

$$W_y = \frac{I_y}{z} = \frac{I_y}{\frac{h}{2}} = \frac{\frac{1}{12}bh^3}{\frac{h}{2}} = \frac{1}{6}bh^2 = \frac{1}{6}\cdot 0{,}1\cdot 0{,}2^2 = \underline{6{,}\bar{6}\cdot 10^{-4}\,m^3}$$

$M_{Sd} = 12\,kNm \not\leq M_{Rd} = f_d \cdot W_y = 13\cdot 10^6 \cdot 6{,}\bar{6}\cdot 10^{-4} = 8{,}\bar{6}\cdot 10^3\,Nm = \underline{8{,}\bar{6}\,kNm}$

neuvyhoví

Návrh $b = 140\,mm$: $M_{Sd} = 12\,kNm \not\leq M_{Rd} = 12{,}1\bar{3}\,kNm$ vyhoví

$$\sigma_{x_{max}} = \frac{M}{W} = \frac{12\cdot 10^3}{6{,}\bar{6}\cdot 10^{-4}} = \underline{18\,MPa}$$

horní vlákna: $\sigma_x = \frac{M}{I} z = \frac{+12\cdot 10^3}{6{,}\bar{6}\cdot 10^{-5}}(-0{,}1) = \underline{-18\,MPa}$

dolní: $\sigma_x = \frac{+12\cdot 10^3}{6{,}\bar{6}\cdot 10^{-5}}\cdot(+0{,}1) = \underline{18\,MPa}$