

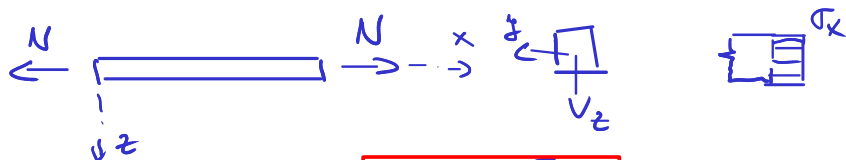
Prostý tah a tlak

- napětí při osovém tlaku a tahu

- $N > 0$ tah ; $N < 0$ tlak

- $V, M = 0$

- normálové napětí σ_x - v průřezu konstantní



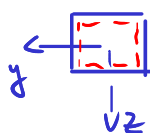
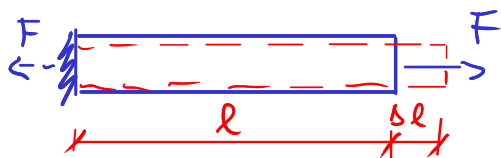
Hookův zákon

$$\varepsilon_x = \frac{\sigma_x}{E}$$

E ... modul pružnosti v tahu a tlaku (Youngův) [MPa]

$$N = \int_A \sigma_x dA = \sigma_x A \Rightarrow \sigma_x = \frac{N}{A}$$

Přetvoření prutu

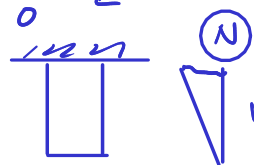


$$N = F$$

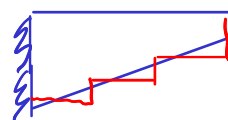
- celkové prodloužení

$$\Delta l = \int_0^l \varepsilon_x dx = \int_0^l \frac{\sigma_x}{E} dx = \int_0^l \frac{N}{EA} dx = \frac{Nl}{EA}$$

N, E, A jsou konstantní



vl. tíha $N(x)$



- prodloužení od změny teploty

$$\Delta l_T = \int_0^l \varepsilon_{xT} dx = \int_0^l \alpha_T \Delta T dx = \alpha_T \Delta T l$$

α_T ... teplotní součinitel roztažnosti
[°C⁻¹] [K⁻¹]

Návrh a posouzení

- návrhová normálová síla

$$N_{Rd} = A \cdot f_d = A \frac{f_k}{\gamma_M}$$

d ... návrhová hodnota

f_k ... charakteristická hodnota

γ_M ... dílčí součinitel spolehlivosti materiálu

posouzení

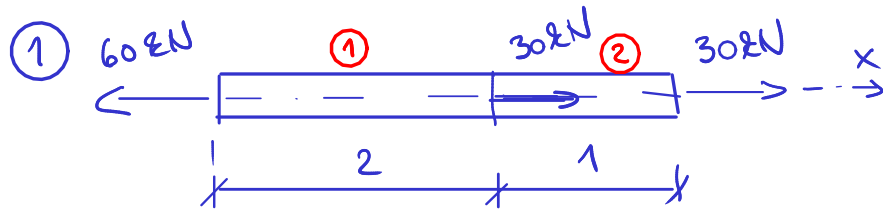
$$N_{Sd} \leq N_{Rd}$$

- nebo i pro lineárně pružný materiál

$$\sigma_{xd} \leq f_d$$

$$\sigma = \frac{N}{A} \quad [Pa] = \frac{[N]}{[m^2]}$$

$$[MPa] = \frac{[MN]}{[m^2]} = \frac{[N]}{[mm^2]}$$



$a = 20 \text{ mm}$
 $E = 210 \text{ GPa}$

(N)
[kN]



(σ_x)
[MPa]

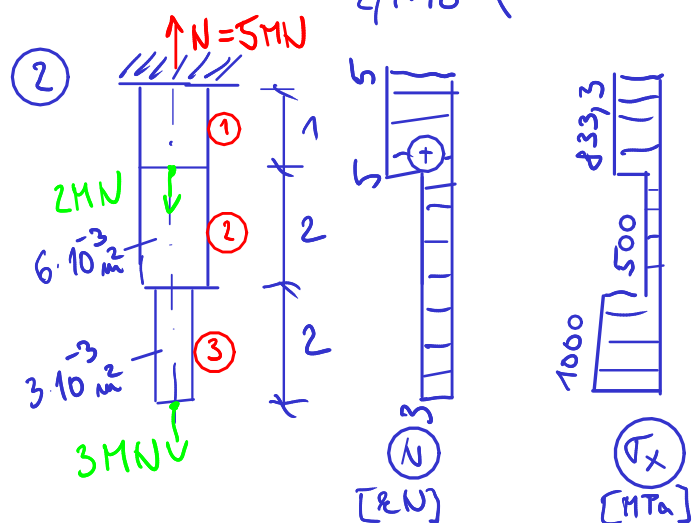


$$\sigma_1 = \frac{N_1}{A} = \frac{60 \cdot 10^3}{0,002^2} = 150 \cdot 10^6 \text{ Pa} \quad \parallel \underline{\underline{150 \text{ MPa}}}$$

$$\sigma_2 = \frac{N_2}{A} = \frac{30 \cdot 10^3}{20^2} = \underline{\underline{75 \text{ MPa}}}$$

$$\Delta l = \Delta l_1 + \Delta l_2 = \frac{N_1 l_1}{EA} + \frac{N_2 l_2}{EA} = \frac{\sigma_1 l_1}{E} + \frac{\sigma_2 l_2}{E} =$$

$$= \frac{1}{21 \cdot 10^5} (150 \cdot 2 + 75 \cdot 1) = 1,486 \cdot 10^{-3} \text{ m} = \underline{\underline{1,486 \text{ mm}}}$$



vl. tíhnu neuvažujeme $\sigma_x = ?$
 $E = 210 \text{ GPa}$ $\Delta l = ?$

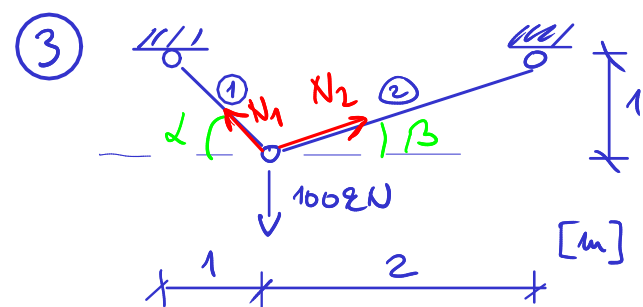
$$\sigma_1 = \frac{N_1}{A_1} = \frac{5}{6 \cdot 10^{-3}} = \underline{\underline{833,3 \text{ MPa}}}$$

$$\sigma_2 = \frac{N_2}{A_2} = \frac{3}{6 \cdot 10^{-3}} = \underline{\underline{500 \text{ MPa}}}$$

$$\sigma_3 = \frac{N_3}{A_3} = \frac{3}{3 \cdot 10^{-3}} = \underline{\underline{1000 \text{ MPa}}}$$

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3 = \frac{1}{E} (\sigma_1 l_1 + \sigma_2 l_2 + \sigma_3 l_3) =$$

$$= \frac{1}{21 \cdot 10^5} (833,3 \cdot 1 + 500 \cdot 2 + 1000 \cdot 2) = 0,01825 \text{ m} = \underline{\underline{18,25 \text{ mm}}}$$



$d_{1,2} = ?$ (průměr v celých mm)

S235 $f_z = f_y = 235 \text{ MPa}$

$\gamma_M = 1,0$

$?$ přemístění bodu?

$E = 210 \text{ GPa}$

výpočet osových sil

⊕ → $-N_{1x} + N_{2x} = 0$

⊕ ↑ $N_{1y} + N_{2y} - 100 = 0$

① $-N_1 \cos \alpha + N_2 \cos \beta = 0$

② $N_1 \sin \alpha + N_2 \sin \beta - 100 = 0$

$l_1 = \sqrt{2} \text{ m}$

$l_2 = \sqrt{2^2 + 1^2} = \sqrt{5} \text{ m}$

$\sin \alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin \beta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos \alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos \beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$$\textcircled{1} \Rightarrow N_1 = \frac{N_2 \cos \beta}{\cos \alpha} \Rightarrow \text{dosadím do } \textcircled{2} \Rightarrow \underline{N_2 = 74,54 \text{ kN}}$$

$$\Rightarrow \underline{N_1 = 94,28 \text{ kN}}$$

$$N_i \leq N_{Rd} = A_i f_d = A_i \frac{f_k}{\gamma_M}$$

$$A_{i,\min} = \frac{N_i}{f_d}$$

$$A_i = \pi r_i^2$$

$$A_{1,\min} = \frac{94,28 \cdot 10^3}{\frac{235}{1}} = \underline{401191 \cdot 10^{-6} \text{ m}^2}$$

$$r_1 = \sqrt{\frac{A_1}{\pi}} = 11,3 \text{ mm} \quad d_1 = 22,6 \text{ mm}$$

$$A_{2,\min} = \frac{74,54 \cdot 10^3}{\frac{235}{1}} = \underline{317191 \cdot 10^{-6} \text{ m}^2}$$

$$r_2 = \sqrt{\frac{A_2}{\pi}} = 10,05 \text{ mm} \quad d_2 = 20,1 \text{ mm}$$

navrh: $d_1 = 23 \text{ mm}$

$d_2 = 21 \text{ mm}$

- posouzení

$$N_1 = 94,28 \text{ kN} \leq \checkmark N_{Rd1} = A_1 \cdot \frac{f_k}{\gamma_M} = \pi \left(\frac{23}{2}\right)^2 \cdot \frac{235}{1} = \underline{97,637 \text{ kN}}$$

$$N_2 = 74,54 \text{ kN} \leq \checkmark N_{Rd2} = A_2 \cdot \frac{f_k}{\gamma_M} = \pi \left(\frac{21}{2}\right)^2 \cdot \frac{235}{1} = \underline{81,395 \text{ kN}}$$

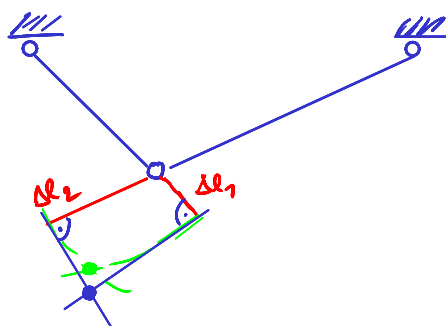
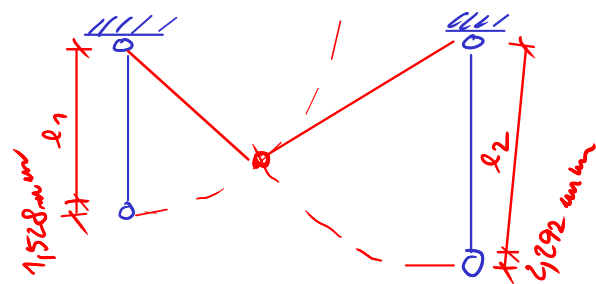
vyhoví

- protažení

$$\Delta l = \frac{Nl}{EA}$$

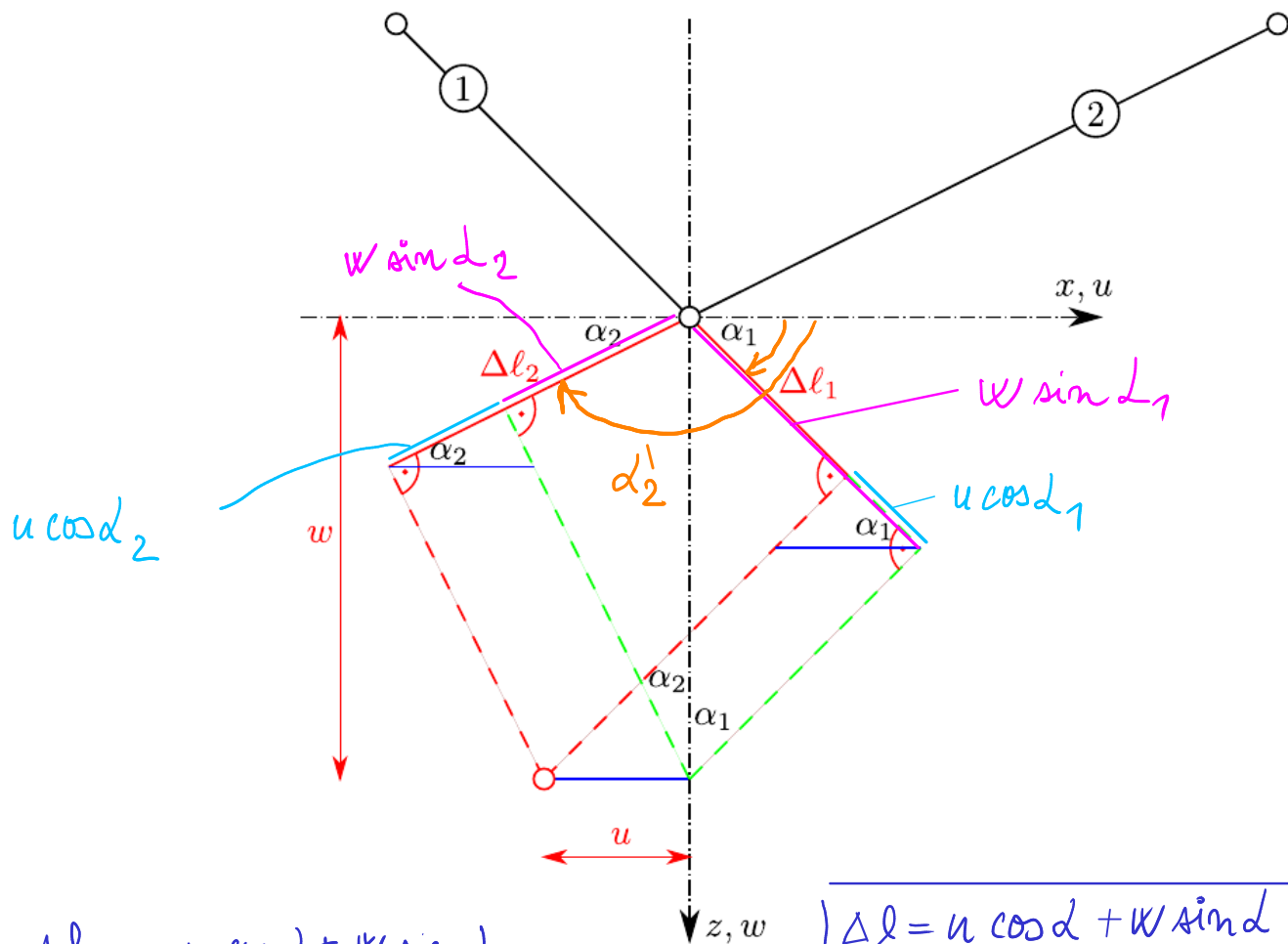
$$\Delta l_1 = \frac{N_1 l_1}{EA_1} = \frac{94,28 \cdot 10^3 \sqrt{2}}{2,1 \cdot 10^5 \pi \cdot \left(\frac{0,023}{2}\right)^2} = \underline{\underline{1,528 \text{ mm}}}$$

$$\Delta l_2 = \frac{N_2 l_2}{EA_2} = \frac{74,54 \cdot 10^3 \sqrt{5}}{2,1 \cdot 10^5 \pi \cdot \left(\frac{0,021}{2}\right)^2} = \underline{\underline{2,292 \text{ mm}}}$$



+S





$$\Delta l_2 = -u \cos d_2 + w \sin d_2$$

$$w \sin d_1 = \Delta l_1 - u \cos d_1$$

$$\Delta l_1 = 1,528 \text{ mm}$$

$$\Delta l_2 = 2,292 \text{ mm}$$

$$\sin d_1 = \frac{\sqrt{2}}{2} \quad \sin d_2 = \frac{\sqrt{5}}{5}$$

$$\cos d_1 = \frac{\sqrt{2}}{2} \quad \cos d_2 = \frac{2\sqrt{5}}{5}$$

$$2,292 = -\frac{2\sqrt{5}}{5} u + \frac{\sqrt{5}}{5} w$$

$$-1,528 = -\frac{\sqrt{2}}{2} u - \frac{\sqrt{2}}{2} w$$

$$u = -0,988 \text{ mm}$$

$$w = 3,149 \text{ mm}$$

$$\Delta l = u \cos d + w \sin d$$

$d \dots$ směrové úhly

$$\sin d_1 = \frac{\sqrt{2}}{2}$$

$$\cos d_1 = \frac{\sqrt{2}}{2}$$

$$\alpha'_2 = 180^\circ - d_2$$

$$\sin d'_2 = \frac{\sqrt{5}}{5}$$

$$\cos d'_2 = -\frac{2\sqrt{5}}{5}$$

$$\Delta l_1 = \cos d_1 u + \sin d_1 w$$

$$\Delta l_2 = \cos d'_2 u + \sin d'_2 w$$

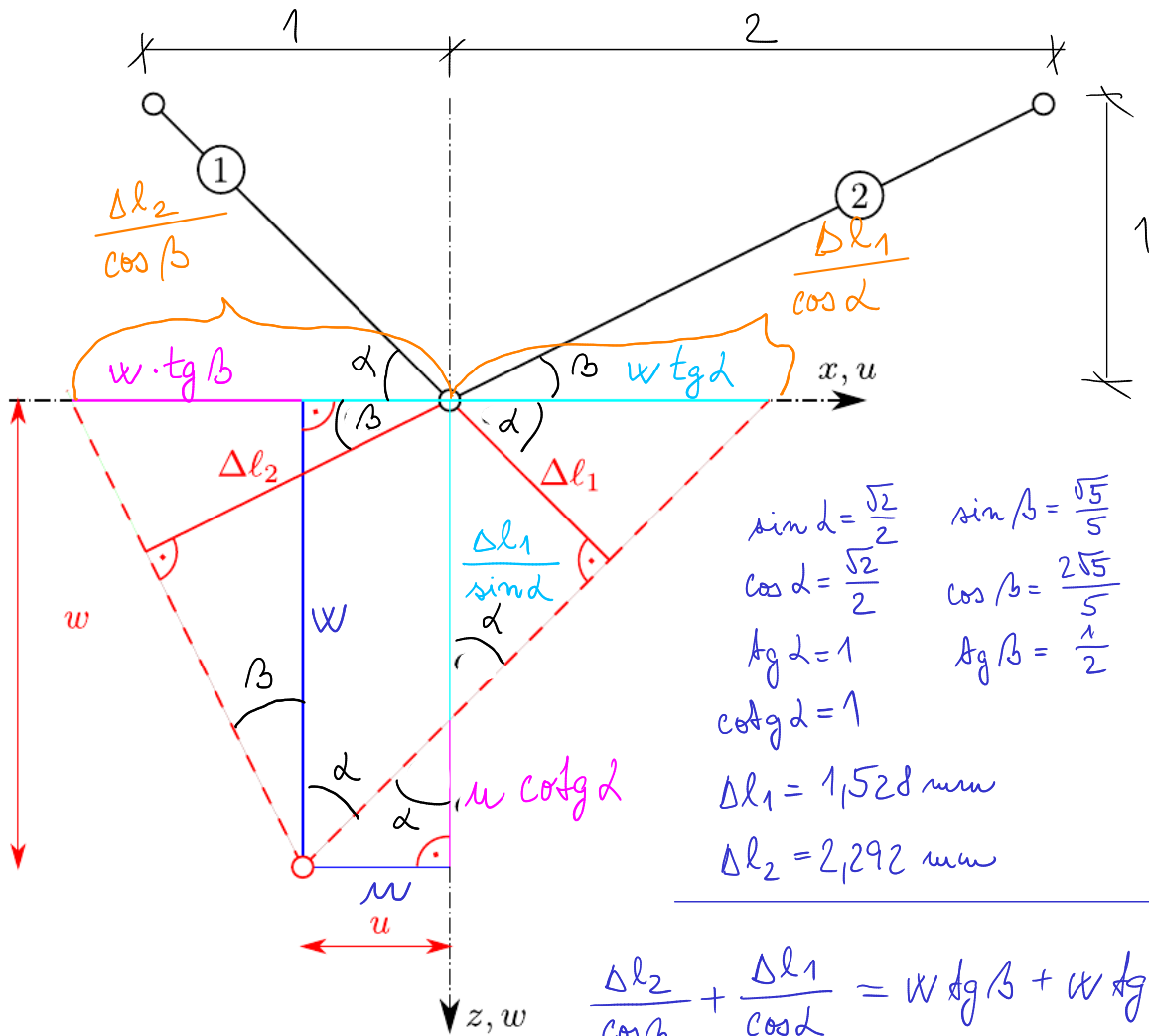
$$1,528 = \frac{\sqrt{2}}{2} u + \frac{\sqrt{2}}{2} w$$

$$2,292 = -\frac{2\sqrt{5}}{5} u + \frac{\sqrt{5}}{5} w$$

$$u = -0,988 \text{ mm}$$

$$w = 3,149 \text{ mm}$$

souřadnice nové polohy
stýčnicku



$$\frac{\Delta l_2}{\cos \beta} + \frac{\Delta l_1}{\cos \alpha} = w \tan \beta + w \tan \alpha$$

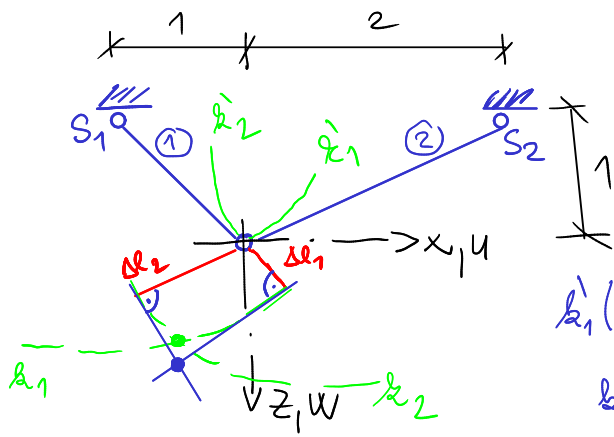
$$\frac{2,292 \cdot 5}{2\sqrt{5}} + \frac{1,528 \cdot 2}{\sqrt{2}} = w \left(\frac{1}{2} + 1 \right)$$

$$\underline{\underline{w = 3,149 \text{ mm}}}$$

$$w = u \cot \alpha + \frac{\Delta l_1}{\sin \alpha}$$

$$u = \frac{w - \Delta l_1 / \sin \alpha}{\cot \alpha} = \frac{3,149 - 1,528 \cdot 2 / \sqrt{2}}{1}$$

$$\underline{\underline{u = 0,988 \text{ mm}}}$$



Výpočet pomocí kružnic

kontrola:

$$l_1(S_1; l_1): (x+1)^2 + (z+1)^2 = l_1^2 \quad (1)$$

$$l_2(S_2; l_2): (x-2)^2 + (z+1)^2 = l_2^2 \quad (2) \quad / \cdot (-1)$$

$$l_1(S_1; l_1 + \Delta l_1):$$

$$(1) (x+1)^2 + (z+1)^2 = (\sqrt{2} + 1,528 \cdot 10^{-3})^2$$

$$l_2(S_2; l_2 + \Delta l_2):$$

$$(2) (x-2)^2 + (z+1)^2 = (\sqrt{5} + 2,292 \cdot 10^{-3})^2 \quad / \cdot (-1)$$

$$(x+1)^2 - (x-2)^2 = -3,0059312$$

$$x^2 + 2x + 1 - x^2 + 4x - 4 = -3,0059312$$

$$6x = -5,9312 \cdot 10^{-3}$$

$$x = -0,988536 \cdot 10^{-3} \text{ m}$$

$$(1) (-0,988536 \cdot 10^{-3} + 1)^2 + (z+1)^2 = (\sqrt{2} + 1,528 \cdot 10^{-3})^2$$

$$(z+1)^2 = 1,0063$$

$$z = \pm \sqrt{1,0063} - 1$$

$$z = \pm 3,145 \cdot 10^{-3} \text{ m}$$

$$z = \pm 2,003145 \text{ m}$$

$$l_1 \cap l_2 = [-0,988536 \cdot 10^{-3}; 3,145 \cdot 10^{-3}]$$

$$(x+1)^2 - (x-2)^2 = l_1^2 - l_2^2$$

$$x^2 + 2x + 1 - x^2 + 4x - 4 = 2 - 5$$

$$6x = 0$$

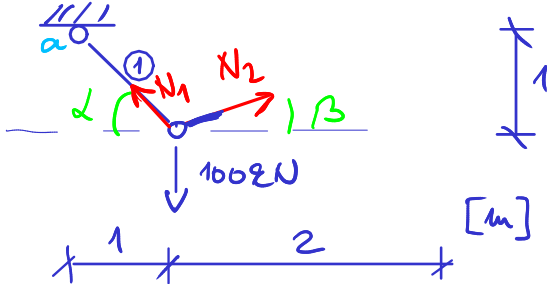
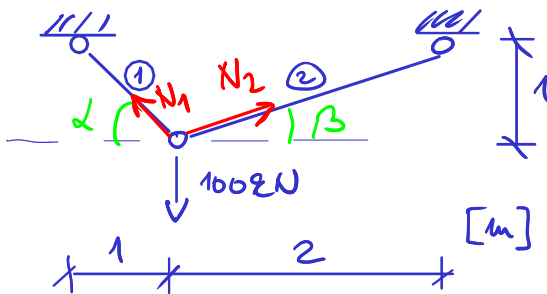
$$x = 0$$

$$(1) 1^2 + (z+1)^2 = 2$$

$$(z+1)^2 = 1$$

$$z = 0$$

$$l_1 \cap l_2 = [0, 0]$$



Výpočet N bez soustavy rovnic

- odříznou prut ②

$$\sum M_{ai} = 0 \quad \oplus \downarrow$$

$$100 \cdot 10^3 \cdot 1 - N_{2x} \cdot 1 - N_{2z} \cdot 1 = 0$$

$$100 \cdot 10^3 - N_2 \cos \beta - N_2 \sin \beta = 0$$

$$N_2 = \frac{100 \cdot 10^3}{\cos \beta + \sin \beta} = \frac{100 \cdot 10^3}{\left(2 \frac{\sqrt{5}}{5} + \frac{\sqrt{5}}{5}\right)} = \underline{\underline{74,54 \text{ kN}}}$$

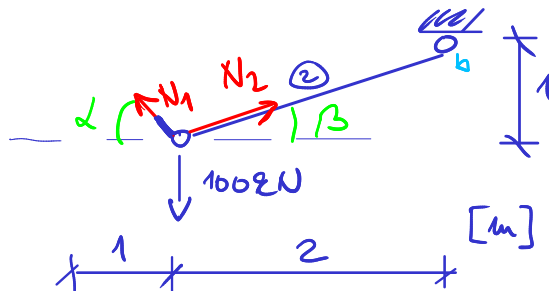
- odříznou prut ①

$$\sum M_{bi} = 0 \quad \oplus \uparrow$$

$$100 \cdot 10^3 \cdot 2 - N_{1x} \cdot 1 - N_{1z} \cdot 2 = 0$$

$$200 \cdot 10^3 - N_1 \cdot \cos \alpha - 2N_1 \sin \alpha = 0$$

$$N_1 = \frac{200 \cdot 10^3}{\cos \alpha + 2 \sin \alpha} = \frac{200 \cdot 10^3}{\left(\frac{\sqrt{2}}{2} + 2 \frac{\sqrt{2}}{2}\right)} = \underline{\underline{94,28 \text{ kN}}}$$

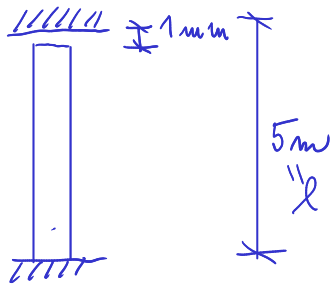


Kontrola: $\sum F_{xi} = 0 \quad \sum F_{zi} = 0$

$$\oplus \rightarrow -N_{1x} + N_{2x} = 0$$

$$-94,28 \cdot \frac{\sqrt{2}}{2} + 74,54 \cdot 2 \cdot \frac{\sqrt{5}}{5} = 0 \quad \checkmark$$

$$\oplus \downarrow 100 - N_{1z} - N_{2z} = 0 \quad \checkmark$$



□ 50
50 [Cm]

a) $\Delta T = ?$ dotek stropu

$E = 21 \text{ GPa}$

b) $\Delta T = 30^\circ\text{C}$ $\sigma_x = ?$

$\alpha_T = 1,2 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$

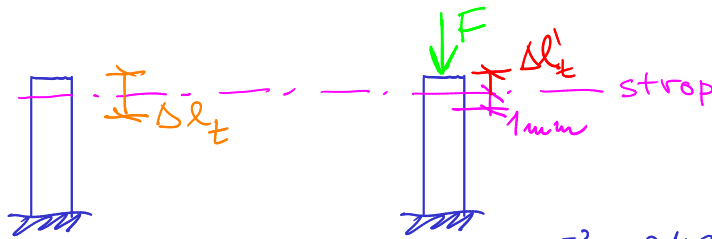
c) $\Delta T = ?$ $\sigma_x = -5 \text{ MPa}$

$$A = 0,5 \cdot 0,5 = 0,25 \text{ m}^2$$

a) $\Delta l = 1 \text{ mm}$ $\Delta l = \alpha_T \Delta T l$

$$\Delta T = \frac{\Delta l}{\alpha_T l} = \frac{10^{-3}}{1,2 \cdot 10^{-5} \cdot (5 \cdot 10^{-3})} = \underline{\underline{16,67^\circ\text{C}}}$$

b) $\Delta l_t = \alpha_T \Delta T l = 1,2 \cdot 10^{-5} \cdot 30 \cdot (5 \cdot 10^{-3}) = 1,79964 \cdot 10^{-3} \text{ m}$



$$\Delta l_t' = \Delta l_t - 1 \text{ mm} = 1,79964 \cdot 10^{-3} - 1 \cdot 10^{-3} = 0,79964 \cdot 10^{-3} \text{ m}$$

$$l_t = l + \Delta l_t' = 5 + 0,79964 \cdot 10^{-3} = 5,00079964 \text{ m}$$

$$\Delta l_t' = \frac{N l_t}{EA} \Rightarrow N = \frac{\Delta l_t' EA}{l_t} = \frac{-0,79964 \cdot 10^{-3} \cdot 21 \cdot 10^9 \cdot 0,25}{5,00079964}$$

$$\underline{\underline{N = -839,488 \text{ N}}}$$

$$\sigma_x = \frac{N}{A} = \frac{-839,488}{0,25} = \underline{\underline{-3,358 \text{ MPa}}}$$

c) $N = \sigma_x A = -5 \cdot 10^6 \cdot 0,25 = -1,25 \cdot 10^6 \text{ N}$

$$\Delta l_t' = \frac{N l_t}{EA} \quad l_t = l + \Delta l_t'$$

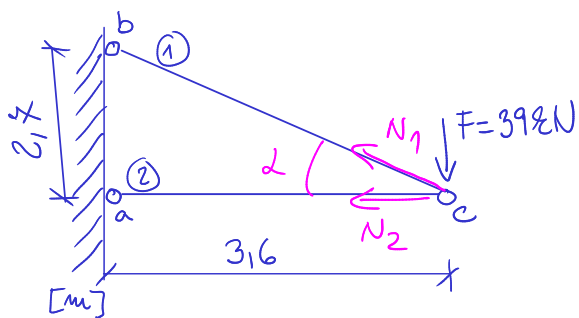
$$EA \Delta l_t' = N l + N \Delta l_t'$$

$$\Delta l_t' = \frac{N l}{EA - N} = \frac{-1,25 \cdot 10^6 \cdot 5}{21 \cdot 10^9 \cdot 0,25 + 1,25 \cdot 10^6} = -1,19019 \cdot 10^{-3} \text{ m}$$

$$\Delta l_t = 1 \text{ mm} + 1,19019 = 2,19019 \text{ mm}$$

$$\Delta l_t = \alpha_T \Delta T l \Rightarrow \Delta T = \frac{\Delta l}{\alpha_T l} = \frac{2,19019 \cdot 10^{-3}}{1,2 \cdot 10^{-5} \cdot (5 \cdot 10^{-3})} = \underline{\underline{36,51^\circ\text{C}}}$$

Návrhové



① $\odot d$ (po 2 mm)

$$f_{d1} = 210 \text{ MPa}$$

$$E_1 = 210 \text{ GPa}$$

② $\square a$ (po 2 cm)

$$f_{d2} = 8 \text{ MPa}$$

$$E_2 = 10 \text{ GPa}$$

$$l_1 = \sqrt{3,6^2 + 2,7^2} = 4,5 \text{ m}$$

$$\oplus \sum M_{bi} = 0: 39 \cdot 3,6 + N_2 \cdot 2,7 = 0 \Rightarrow N_2 = -\frac{39 \cdot 3,6}{2,7} = -52 \text{ kN}$$

$$\ominus \sum M_{ai} = 0: 39 \cdot 3,6 - N_1 \cdot 3,6 = 0 \Rightarrow N_1 = 39 \text{ kN}$$

$$\sin \alpha = \frac{2,7}{4,5}$$

$$N_1 \sin \alpha = 39$$

$$N_1 = \frac{39}{\sin \alpha} = \frac{4,5 \cdot 39}{2,7} = 65 \text{ kN}$$

$$\frac{N_1}{A_1} \leq f_{d1}$$

$$A_{1, \min} = \frac{N_1}{f_{d1}} = \frac{65 \cdot 10^3}{210} = 309,524 \cdot 10^{-6} \text{ m}^2$$

$$A_1 = \pi \frac{d^2}{4} \Rightarrow d_{\min} = \sqrt{\frac{A_1 \cdot 4}{\pi}} = \sqrt{\frac{309,524 \cdot 10^{-6} \cdot 4}{\pi}}$$

$$d_{\min} = 19,852 \cdot 10^{-3} \text{ m}$$

$$\frac{N_2}{A_2} \leq f_{d2}$$

$$A_{2, \min} = \frac{N_2}{f_{d2}} = \frac{52 \cdot 10^3}{8} = 6,5 \cdot 10^{-3} \text{ m}^2$$

$$A_2 = a^2 \Rightarrow a = \sqrt{A_2} = \sqrt{6,5 \cdot 10^{-3}}$$

$$a = 80,623 \cdot 10^{-3} \text{ m}$$

návrh: $d = 20 \text{ mm}$

$a = 100 \text{ mm}$

$$A_1 = \pi \frac{20^2}{4} = 314,159 \text{ mm}^2$$

$$A_2 = a^2 = 10000 \text{ mm}^2$$

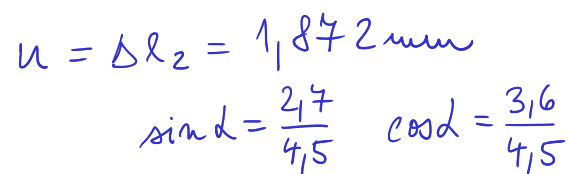
posudek:

$$\sigma_{x1} = \frac{N_1}{A_1} = \frac{65 \cdot 10^3}{314,159} = 206,9 \text{ MPa} \not\leq f_{d1} = 210 \text{ MPa} \quad \text{vyhoví!}$$

$$\sigma_{x2} = \frac{N_2}{A_2} = \frac{52 \cdot 10^3}{10000} = 5,2 \text{ MPa} \not\leq f_{d2} = 8 \text{ MPa} \quad \text{vyhoví!}$$

$$\Delta l_1 = \frac{N_1 l_1}{E_1 A_1} = \frac{65 \cdot 10^3 \cdot 4,5}{210 \cdot 10^9 \cdot 314,159 \cdot 10^{-6}} = 4,433606 \cdot 10^{-3} \text{ m}$$

$$\Delta l_2 = \frac{N_2 l_2}{E_2 A_2} = \frac{-52 \cdot 10^3 \cdot 3,6}{10 \cdot 10^9 \cdot 10000 \cdot 10^{-6}} = -1,872 \cdot 10^{-3} \text{ m}$$



$$\Delta L_1 = w \sin d - u \cos d$$

$$4,433606 = w \frac{2,7}{4,5} - 1,872 \cdot \frac{3,6}{4,5}$$

$w = 9,885 \text{ mm}$

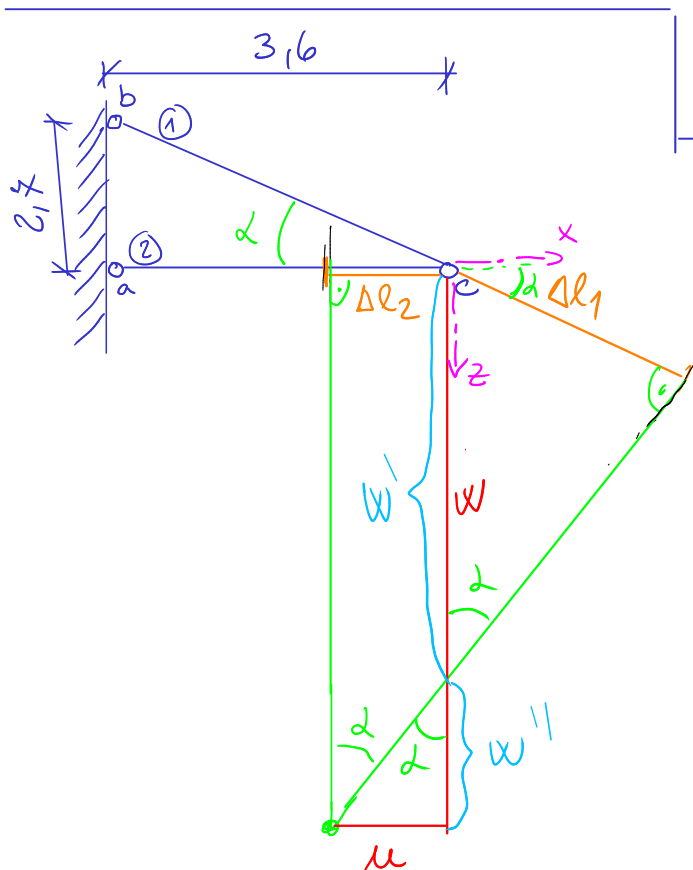
$$\textcircled{1} \quad z_1: (x+3,6)^2 + (z+2,7)^2 = (4,5 + 4,433606 \cdot 10^{-3})^2$$

$$\textcircled{2} b_2: (x+3,6)^2 + z^2 = (3,6 - 1,872 \cdot 10^{-3})^2 \quad | \cdot (-1)$$

$$z^2 + 5,4z + 2,7^2 - z^2 = 7,343397$$

$$z = 9,888 \cdot 10^{-3} \text{ m}$$

②. $x = \begin{cases} -1,886 \cdot 10^{-3} \text{ m} \\ -3,598 \text{ m} \end{cases}$



$$\operatorname{tg} \alpha = \frac{2,7}{3,6}$$

$$W = W' + W''$$

$$u = \Delta l_2$$

$$\frac{\Delta l_1}{w'} = \sin \alpha \quad \frac{u}{w''} = \tan \alpha$$

$$W = \frac{\Delta l_1}{\sin \alpha} + \frac{\Delta l_2}{\tan \alpha}$$

$$w = \frac{4,433606}{2,7} \cdot 4,5 + \frac{1,872}{2,7} \cdot 3,6$$

$W = 9,855 \text{ mm}$