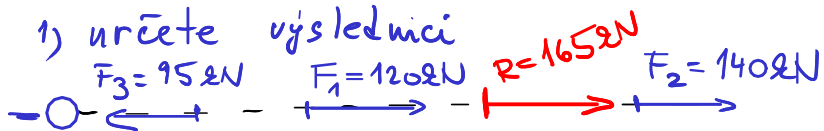


SÍLY VE SPOLEČNÉM PÁPRSKU

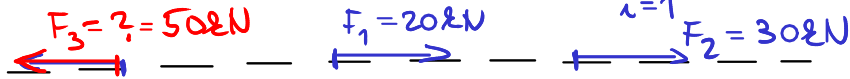
1) určete výslednici



výslednice $R = \sum_{i=1}^n F_i$ $\oplus \rightarrow R = \sum_{i=1}^3 F_i = F_1 + F_2 - F_3 = 120 + 140 - 95 = \underline{\underline{165 \text{ N}}}$

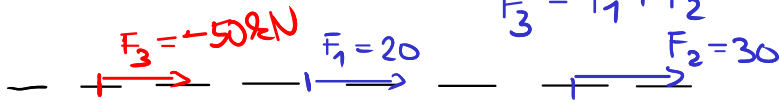
2) Určete velikost síly tak, aby soustava sil byla v rovnováze

soustava v rovnováze $\Leftrightarrow R = \sum_{i=1}^n F_i = 0$



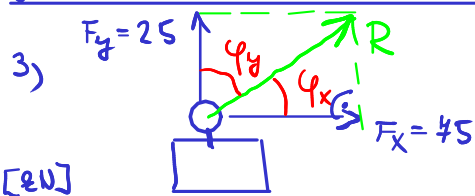
$\oplus \rightarrow \sum_{i=1}^3 F_i = 0 : F_1 + F_2 - F_3 = 0$

$F_3 = F_1 + F_2 = 20 + 30 = \underline{\underline{50 \text{ N}}}$



$\oplus \rightarrow \sum_{i=1}^3 F_i = 0 : F_1 + F_2 + F_3 = 0 \Rightarrow F_3 = -F_1 - F_2 = -20 - 30 = \underline{\underline{-50 \text{ N}}}$

SKLÁDÁNÍ A ROZKLAD SÍLY DO PRAVOUHLÝCH SLOŽEK



[N]

určete výslednici

$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{45^2 + 25^2} = \sqrt{6250}$$

$$R = \underline{\underline{79,057 \text{ N}}}$$

Určete úhly výslednice a sil F_x, F_y

$$\cos \varphi_x = \frac{F_x}{R} = \sin \varphi_y$$

$$\sin \varphi_x = \frac{F_y}{R} = \cos \varphi_y$$

$$\cos \varphi_x = \frac{45}{79,057} \Rightarrow \varphi_x = 18,435^\circ$$

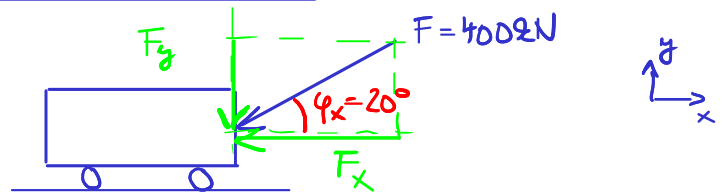
$$\sin \varphi_y = \frac{45}{79,057} \Rightarrow \varphi_y = 41,565^\circ$$

kontrola:

$$\varphi = \varphi_x + \varphi_y = 90^\circ$$
$$18,435^\circ + 41,565^\circ = 90^\circ \checkmark$$

Axiom 1 výslednice R je úhlopříčka rovnoběžníku

4)



Rozložte sílu F do dvou $\perp$ směrů $\parallel$ s x a y

$$\frac{F_x}{F} = \cos \varphi_x$$

$$F_x = F \cdot \cos \varphi_x = 400 \cdot \cos 20^\circ$$

$$F_x = \underline{\underline{375,844 \text{ N}}}$$

$$\frac{F_y}{F} = \sin \varphi_x$$

$$F_y = F \sin \varphi_x = 400 \cdot \sin 20^\circ$$

$$F_y = \underline{\underline{136,808 \text{ N}}}$$

kontrola:

$$F = \sqrt{F_x^2 + F_y^2}$$

$$400 = \sqrt{375,844^2 + 136,808^2} \checkmark$$

SKLA'DA'NI' A ROZKLAD DVOU OBEČNÝCH SIL

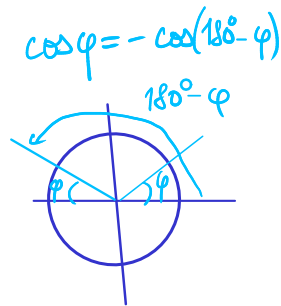
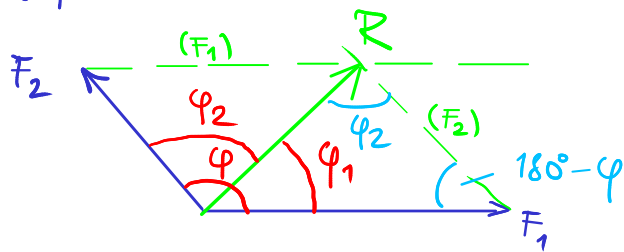
5) a) určete výslednici sil (4 platné číslice)

$$F_1 = 90 \text{ kN}$$

$$F_2 = 40 \text{ kN}$$

$$\varphi = 125^\circ$$

$$R = ?$$



KOSINOVÁ VĚTA

$$R = \sqrt{F_1^2 + F_2^2 + 2 F_1 F_2 \cos \varphi} = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cos (180^\circ - \varphi)}$$

$$R = \sqrt{90^2 + 40^2 + 2 \cdot 90 \cdot 40 \cdot \cos 125^\circ} = (45,9498 \text{ kN}) = \underline{\underline{45,98 \text{ kN}}}$$

b) určete úhly φ_1 a φ_2

SINOVÁ VĚTA

$$\frac{R}{\sin (180^\circ - \varphi)} = \frac{R}{\sin \varphi} = \frac{F_1}{\sin \varphi_2} = \frac{F_2}{\sin \varphi_1}$$

$$\frac{R}{\sin \varphi} = \frac{F_2}{\sin \varphi_1} \Rightarrow \sin \varphi_1 = \frac{F_2 \sin \varphi}{R} = \frac{40 \cdot \sin 125^\circ}{45,98}$$

$$\varphi_1 = (48,994819^\circ) = \underline{\underline{49^\circ}}$$

$$\frac{R}{\sin \varphi} = \frac{F_1}{\sin \varphi_2} \Rightarrow \sin \varphi_2 = \frac{F_1 \sin \varphi}{R} = \frac{90 \cdot \sin 125^\circ}{45,98} \Rightarrow \varphi_2 = 46^\circ$$

$$49^\circ + 46^\circ = 125^\circ \checkmark$$

KONTROLA: $\varphi_1 + \varphi_2 = \varphi$

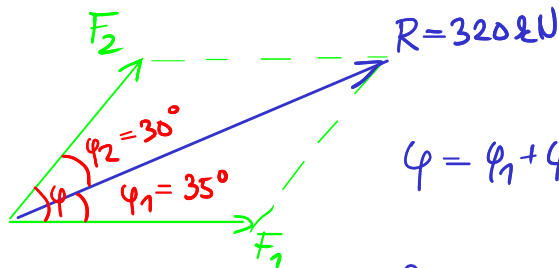
c) rozložte R do dvou zadaných směrů

$$R = 320 \text{ kN}$$

$$\varphi_1 = 35^\circ$$

$$\varphi_2 = 30^\circ$$

$$F_1, F_2 = ?$$



$$\varphi = \varphi_1 + \varphi_2 = 35 + 30 = 65$$

$$\frac{R}{\sin \varphi} = \frac{F_1}{\sin \varphi_2} \Rightarrow F_1 = \frac{R \sin \varphi_2}{\sin \varphi} = \frac{320 \cdot \sin 30^\circ}{\sin 65^\circ} = \underline{\underline{146,5 \text{ kN}}}$$

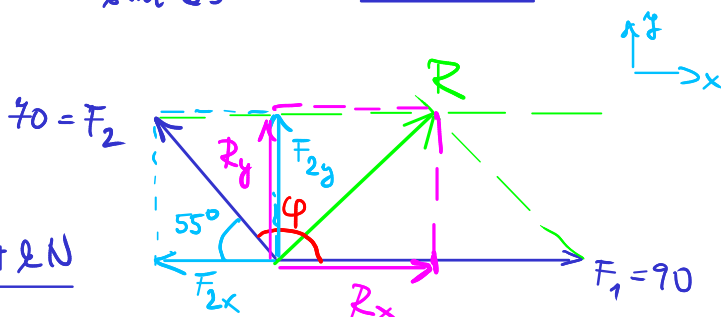
$$\frac{R}{\sin \varphi} = \frac{F_2}{\sin \varphi_1} \Rightarrow F_2 = \frac{R \sin \varphi_1}{\sin \varphi} = \frac{320 \cdot \sin 35^\circ}{\sin 65^\circ} = \underline{\underline{202,5 \text{ kN}}}$$

add a)

$$\frac{F_{2x}}{F_2} = \cos 55^\circ \Rightarrow F_{2x} = F_2 \cos 55^\circ$$

$$F_{2x} = 40,15 \text{ kN}$$

$$\frac{F_{2y}}{F_2} = \sin 55^\circ \Rightarrow F_{2y} = F_2 \sin 55^\circ = 54,34 \text{ kN}$$



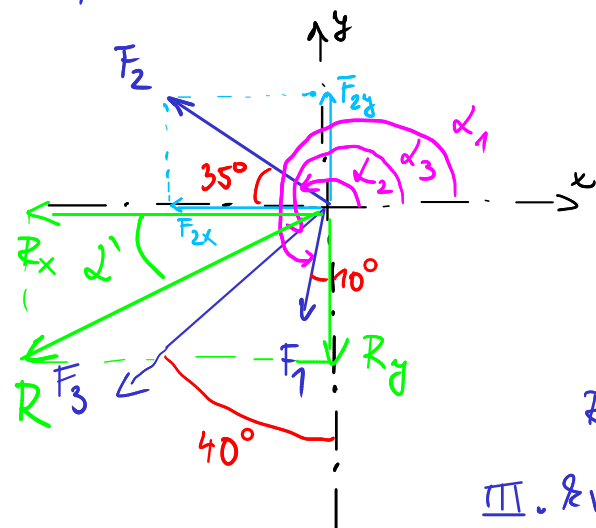
$$\oplus \rightarrow R_x = \sum F_{ix} = F_1 - F_{2x} = 49,85 \text{ kN}$$

$$\oplus \uparrow R_y = \sum F_{iy} = F_{2y} = 54,34 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{49,85^2 + 54,34^2}$$

$$R = 45,98 \text{ kN}$$

6) a) určete výslednici



		směrové úhly		
		F_{ix}	F_{iy}	
i	F_i	α_i	$F_i \cos \alpha_i$	$F_i \sin \alpha_i$
1	10	260	-1,4365	-9,8487
2	20	145	-16,3830	11,4715
3	30	230	-19,2836	-22,9813
			$R_x = \sum F_{ix}$	$R_y = \sum F_{iy}$
			-37,402 N	-21,362 N

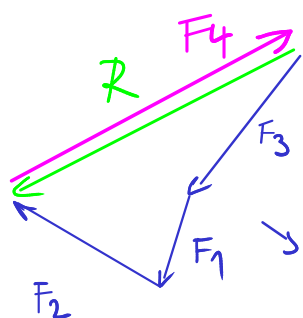
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-37,4)^2 + (-21,36)^2} = \underline{\underline{43,07 \text{ kN}}}$$

III. kvadrant:

$$\frac{|R_x|}{R} = \cos \alpha' \Rightarrow \cos \alpha' = \frac{37,4}{43,07}$$

$$\alpha' = 29,73^\circ$$

$$\alpha = 180^\circ + \alpha' = \underline{\underline{209,73^\circ}}$$



→ graficky - složkový (Cremourov) obrazec
→ při rovnováze smysl všech sil obíhá

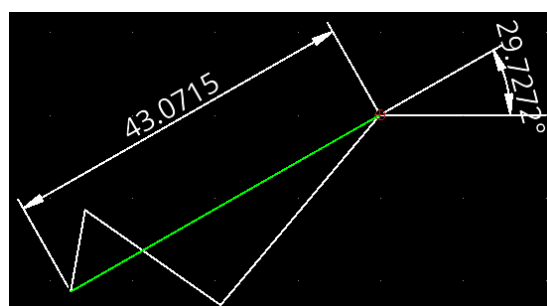
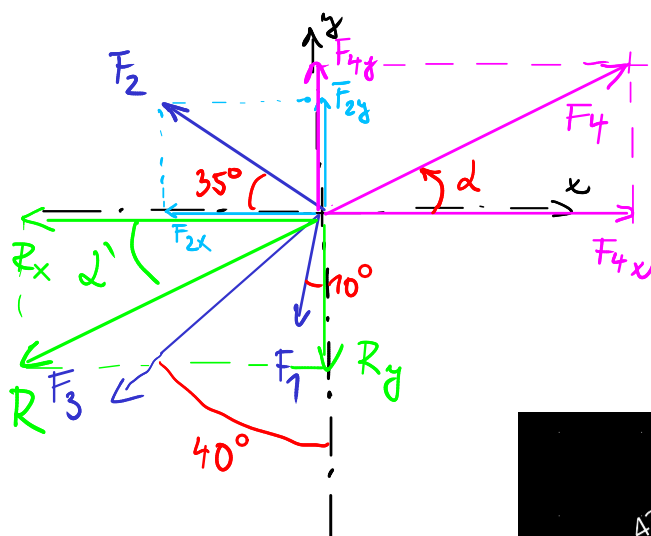
b) určete sílu F_4 , aby soustava byla v rovnováze

$$R = \sum F_i = 0 \Rightarrow F_4 = R = 43,07 \text{ kN}$$

než R z a)
má opačný směr (I. kvadrant)
 $\alpha = 29,73^\circ$

$$R_x = \sum F_{ix} = 0 \Rightarrow F_{4x} = -R_x = 37,4 \text{ kN}$$

$$R_y = \sum F_{iy} = 0 \Rightarrow F_{4y} = -R_y = 21,36 \text{ kN}$$



platne' cislice

45,94959

4pl.c. 45,98

0,00446123

0,004461

4,46123 · 10⁻³

4,461 · 10⁻³

0,000446123

0,446123 · 10⁻⁴

ENG 446,123 · 10⁻⁶

0,446123 · 10⁻³

1pl.c 8

0,004

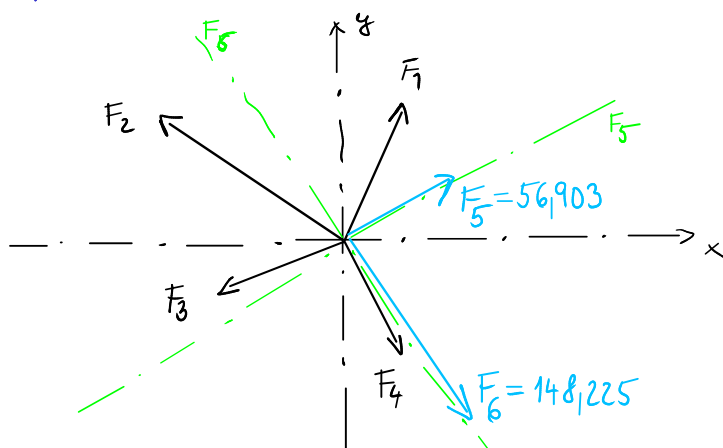
4 · 10⁻³

$$\frac{1}{3} = 0,\overline{3}$$

~~(0,333 | 0,3 | 0,334 | 0,333)~~

Uvedte do rovnováhy svazek sil pomocí dvou sil F_5 a F_6 .

i	$[N]$ F_i	$[^\circ]$ α_i	$[N]$ $F_{xi} = F_i \cos \alpha_i$	$[N]$ $F_{yi} = F_i \sin \alpha_i$
1	98	66	39,860	89,527
2	156	142	-122,930	96,043
3	76	204	-69,429	-30,912
4	62	298	29,107	-54,743
5	F_5	30	$F_5 \cos 30^\circ$	$F_5 \sin 30^\circ$
6	F_6	120	$F_6 \cos 120^\circ$	$F_6 \sin 120^\circ$
			$\Sigma F_{xi} = 0$	$\Sigma F_{yi} = 0$



$$\Sigma F_{xi} = 0 \Rightarrow -123,392 + F_5 \cos 30^\circ + F_6 \cdot \cos 120^\circ = 0 \quad / \cdot \sin 30^\circ$$

$$\Sigma F_{yi} = 0 \Rightarrow 99,915 + F_5 \sin 30^\circ + F_6 \cdot \sin 120^\circ = 0 \quad / \cdot (-\cos 30^\circ)$$

$$-123,392 \cdot \sin 30^\circ - 99,915 \cdot \cos 30^\circ + F_6 \cdot \cos 120^\circ \cdot \sin 30^\circ - F_6 \cdot \sin 120^\circ \cdot \cos 30^\circ = 0$$

$$F_6 = \frac{123,392 \cdot \sin 30^\circ + 99,915 \cdot \cos 30^\circ}{\cos 120^\circ \sin 30^\circ - \sin 120^\circ \cos 30^\circ} = -148,225 \text{ N}$$

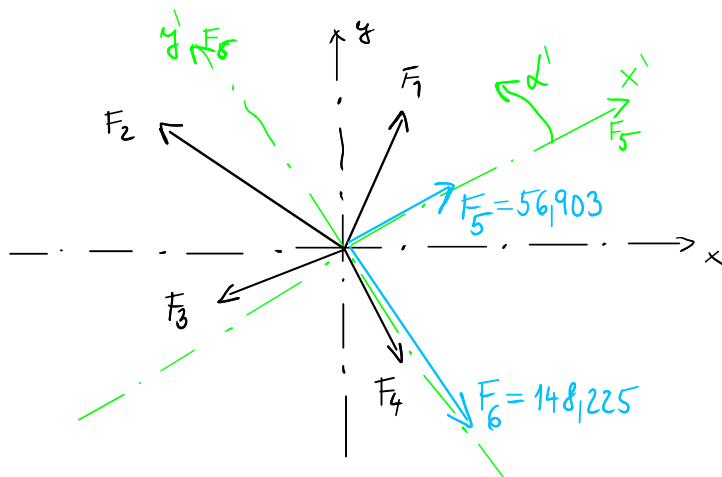
$$\textcircled{1} -123,392 + F_5 \cdot \cos 30^\circ - 148,225 \cos 120^\circ = 0$$

$$F_5 = \frac{123,392 + 148,225 \cdot \cos 120^\circ}{\cos 30^\circ} = 56,903 \text{ N}$$

ŘEŠENÍ BEZ PLNÉ SOUSTAVY ROVIC

- využijeme kolmosti F_5 a F_6 a souřadný systém otočíme do směrů F_5 a F_6
- upravíme směrové úhly $\alpha' = \alpha - 30^\circ$

i	$[N]$ F_i	$[^\circ]$ α'_i	$[N]$ $F_{xi} = F_i \cos \alpha'_i$	$[N]$ $F_{yi} = F_i \sin \alpha'_i$
1	98	36	79,284	57,603
2	156	112	-58,439	144,641
3	76	174	-75,584	4,944
4	62	268	-2,164	-61,962
5	F_5	0	$F_5 \cdot 1$	$F_5 \cdot 0$
6	F_6	90	$F_6 \cdot 0$	$F_6 \cdot 1$
			$\Sigma F_{xi} = 0$	$\Sigma F_{yi} = 0$



$$\Sigma F_{xi} = 0 \Rightarrow -56,903 + F_5 = 0 \Rightarrow F_5 = 56,903 \text{ N}$$

$$\Sigma F_{yi} = 0 \Rightarrow 148,226 + F_6 = 0 \Rightarrow F_6 = -148,226 \text{ N}$$

Maticový zápis

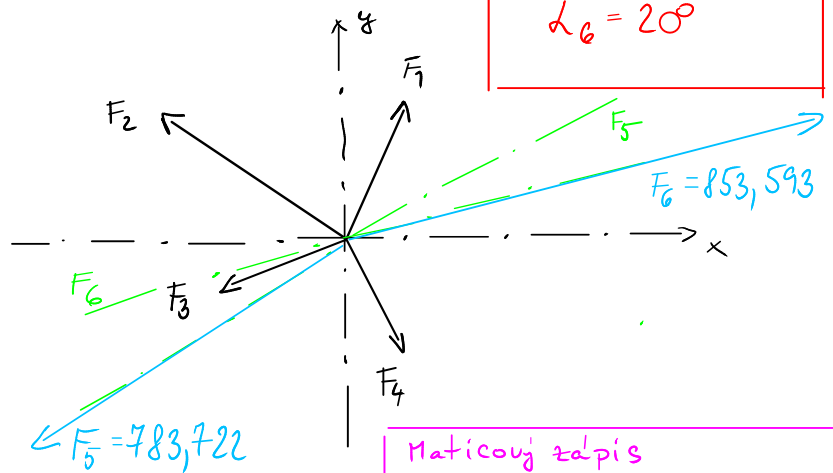
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} 56,903 \\ -148,226 \end{Bmatrix}$$

soustava 2 rovnic, které můžeme řešit nezávisle na sobě

Uvedte do rovnováhy svazek sil pomocí dvou sil F_5 a F_6

! změna
 $\alpha_6 = 20^\circ$

i	[kN]	[°]	[kN]	[kN]
	F_i	α_i	$F_{xi} = F_i \cos \alpha_i$	$F_{yi} = F_i \sin \alpha_i$
1	98	66	39,860	89,527
2	156	142	-122,930	96,043
3	76	204	-69,429	-30,912
4	62	298	29,107	-54,743
5	F_5	30	$F_5 \cos 30^\circ$	$F_5 \sin 30^\circ$
6	F_6	20	$F_6 \cos 20^\circ$	$F_6 \sin 20^\circ$
			$\Sigma F_{xi} = 0$	$\Sigma F_{yi} = 0$



$$\Sigma F_{xi} = 0 \Rightarrow -123,392 + F_5 \cos 30^\circ + F_6 \cos 20^\circ = 0 \quad / \cdot \sin 30^\circ$$

$$\Sigma F_{yi} = 0 \Rightarrow 99,915 + F_5 \sin 30^\circ + F_6 \sin 20^\circ = 0 \quad / \cdot (-\cos 30^\circ)$$

$$-123,392 \cdot \sin 30^\circ - 99,915 \cdot \cos 30^\circ + F_6 \cos 20^\circ \sin 30^\circ - F_6 \sin 20^\circ \cos 30^\circ = 0$$

$$F_6 = \frac{123,392 \cdot \sin 30^\circ + 99,915 \cdot \cos 30^\circ}{\cos 20^\circ \sin 30^\circ - \sin 20^\circ \cos 30^\circ} = 853,593 \text{ kN}$$

$$\textcircled{1} -123,392 + F_5 \cos 30^\circ + 853,593 \cos 20^\circ = 0$$

$$F_5 = \frac{123,392 - 853,593 \cos 20^\circ}{\cos 30^\circ} = -783,722 \text{ kN}$$

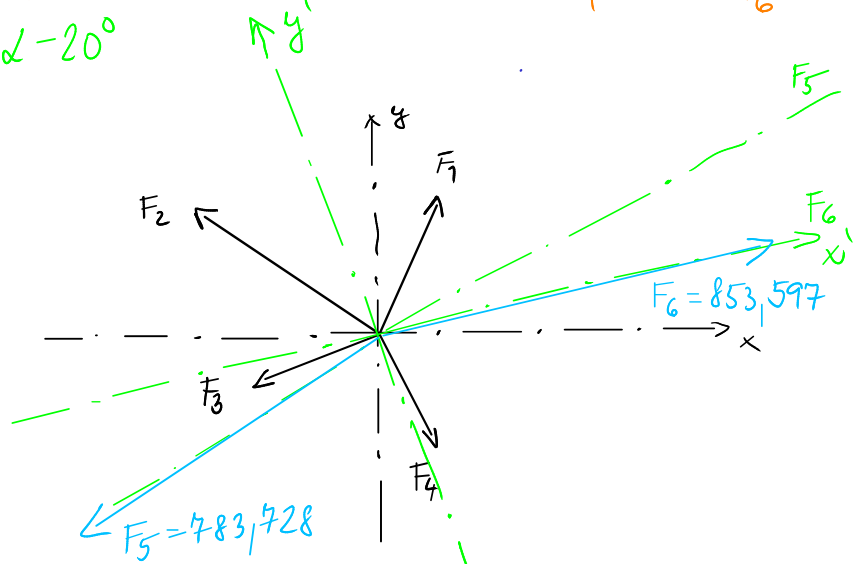
Maticový zápis

$$\begin{bmatrix} \cos 30^\circ & \cos 20^\circ \\ \sin 30^\circ & \sin 20^\circ \end{bmatrix} \begin{Bmatrix} F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} 123,392 \\ -99,915 \end{Bmatrix}$$

ŘEŠENÍ PLNĚ SOUSTAVY ROVNIC

- souřadný systém, do kterého budeme rozkládat natočíme tak, že $x \equiv \vec{F}_6$
- upravíme směrové úhly $\alpha' = \alpha - 20^\circ$

i	[kN]	[°]	[kN]	[kN]
	F_i	α'_i	$F_{xi} = F_i \cos \alpha'_i$	$F_{yi} = F_i \sin \alpha'_i$
1	98	46	68,047	70,495
2	156	122	-82,667	132,296
3	76	184	-75,815	-5,301
4	62	278	8,629	-61,397
5	F_5	10	$F_5 \cos 10^\circ$	$F_5 \sin 10^\circ$
6	F_6	0	$F_6 \cdot 1$	$F_6 \cdot 0$
			$\Sigma F_{xi} = 0$	$\Sigma F_{yi} = 0$



$$\Sigma F_{yi} = 0 \Rightarrow 136,093 + F_5 \sin 10^\circ = 0$$

$$F_5 = -\frac{136,093}{\sin 10^\circ} = -783,728 \text{ kN}$$

$$\Sigma F_{xi} = 0 \Rightarrow -81,776 + F_5 \cos 10^\circ + F_6 \cdot 1 = 0$$

$$F_6 = \frac{81,776 - (-783,728) \cos 10^\circ}{1} = 853,597 \text{ kN}$$

Maticový zápis

$$\begin{bmatrix} \cos 10^\circ & 1 \\ \sin 10^\circ & 0 \end{bmatrix} \begin{Bmatrix} F_5 \\ F_6 \end{Bmatrix} = \begin{Bmatrix} -136,093 \\ 81,776 \end{Bmatrix}$$

soustava 2 rovnic, kde nejprve vyřešíme F_5 a z 2. rovnice dopočítáme F_6